# ENGINEERING MECHANICS BAA1113

## Chapter 4: Force System Resultants (Static)

**by**
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# Chapter Description

- Aims
  - To explain the Moment of Force (2D-scalar formulation & 3D-Vector formulation)
  - To explain the Principle Moment
  - To explain the Moment of a Couple
  - To explain the Simplification of a Force and Couple System
  - To explain the Reduction of Simple Distributed Loading

- Expected Outcomes
  - Able to solve the problems of MOF and COM in the mechanics applications by using principle of moments

- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14$^{th}$ Edition

# Chapter Outline

4.1 Moment of Force (MOF) –Part I

4.2 Principle of Moment –Part II

4.3 Moment of Couple (MOC) Part III

4.4 Simplification of a Force and Couple System

4.5 Reduction of Simple Distributed Loading- part IV

# 4.2 Principle of Moment

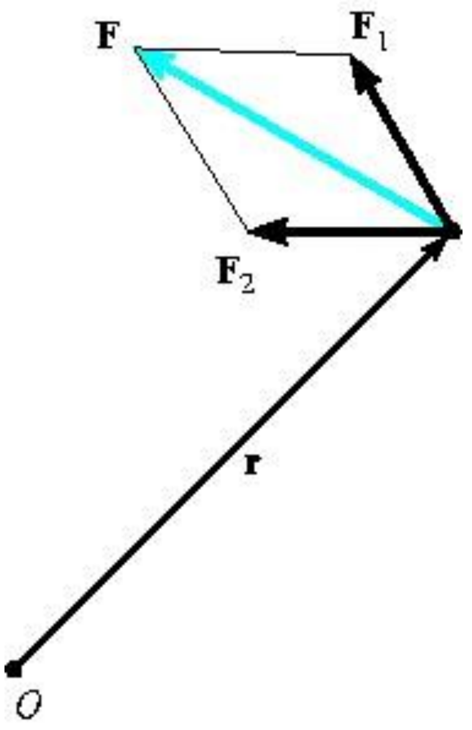
Varignon's Theorem states that "**Moment of a force** about **a point** is **equal** to the **sum of the moments of the forces**' components about the point"

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2,$$

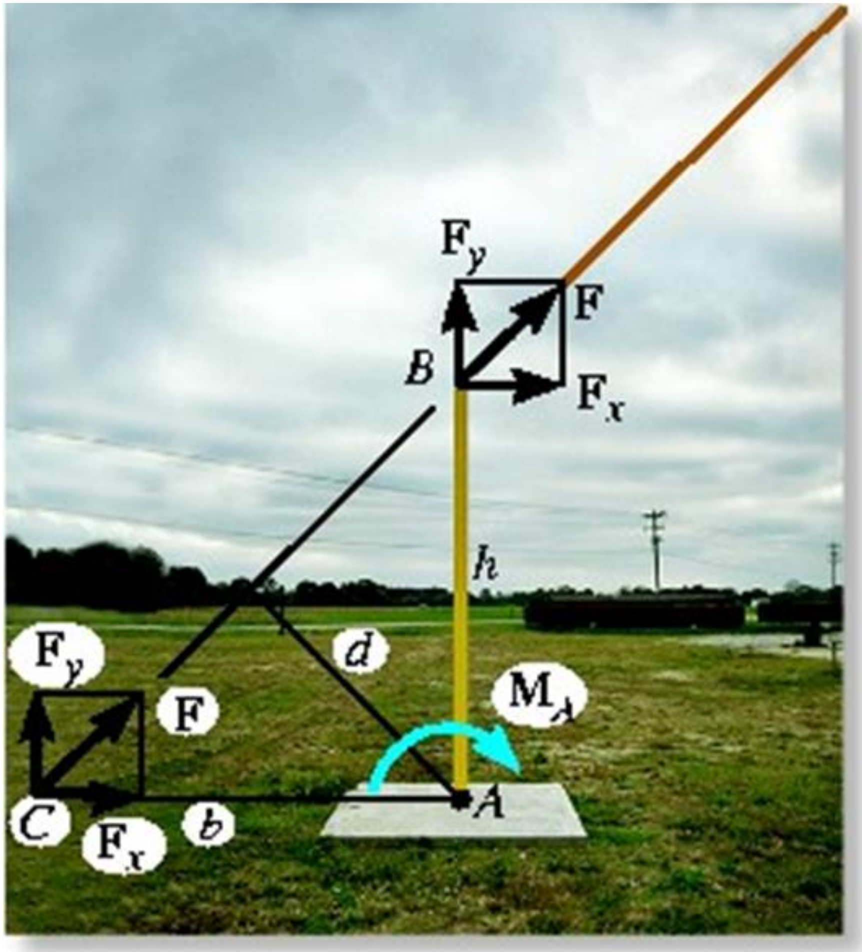
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

$$= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2)$$

$$= \mathbf{r} \times \mathbf{F}$$



# 4.2 Principle of Moment




- The guy cable exerts a force **F** on the pole and creates a moment about the base at A

$$M_A = Fd$$

- If the force is replaced by $\mathbf{F}_x$ and $\mathbf{F}_y$ at point B where the cable acts on the pole, the sum of moment about point A yields the same resultant moment

- $\mathbf{F}_y$ create zero moment about A

$$M_A = F_x h$$

- Apply principle of transmissibility and slide the force where line of action intersects the ground at C, $\mathbf{F}_x$ create zero moment about A

$$M_A = F_y b$$

# Example 4.9

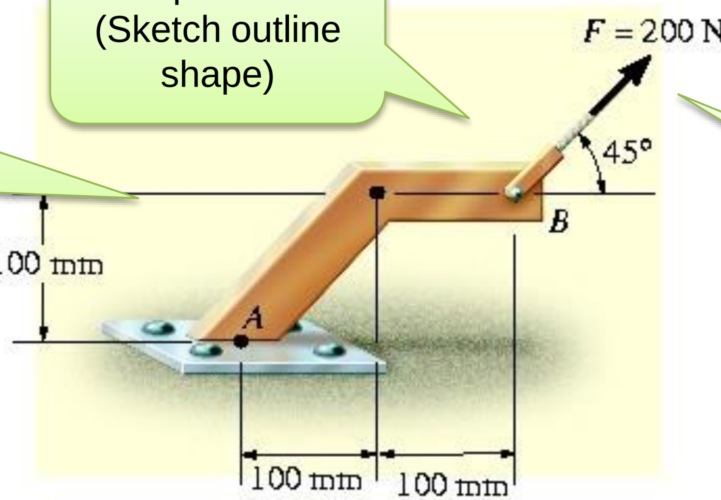
A 200 N force acts on the bracket. Determine the MOF about point A

Step 1: FBD (Sketch outline shape)

Step 2: det. The line of action/ moment arm (d)

Step 3: assume tendency to rotate/ moment

Step 4: use formula




(a)

$M_O = Fd$

# Solution Example 4.9

A 200 N force acts on the bracket. Determine the MOF about point A

**Method 1:**

From trigonometry using triangle BCD,

$CB = d = 100\cos 45^\circ = 70.71\text{mm}$

$= 0.07071\text{m}$

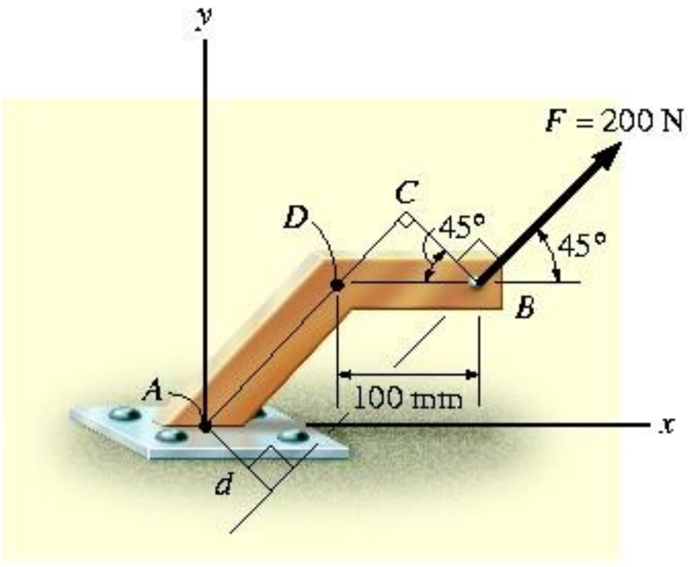
Thus,

$M_A = Fd = 200\text{N}(0.07071\text{m})$

$= 14.1\text{N.m (CCW)}$

As a Cartesian vector,

$\mathbf{M}_A = \{14.1\mathbf{k}\}\text{N.m}$




(b)

# Solution Example 4.9

A 200 N force acts on the bracket. Determine the MOF about point A

- Resolve 200N force into x and y components
- Principle of Moments

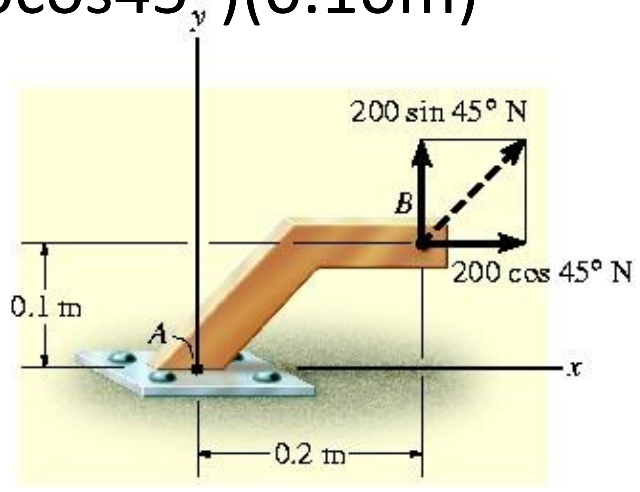
$M_A = \sum Fd$

$M_A = (200\sin 45°N)(0.20m) - (200\cos 45°)(0.10m)$

$= 14.1$ N.m (CCW)

Thus,

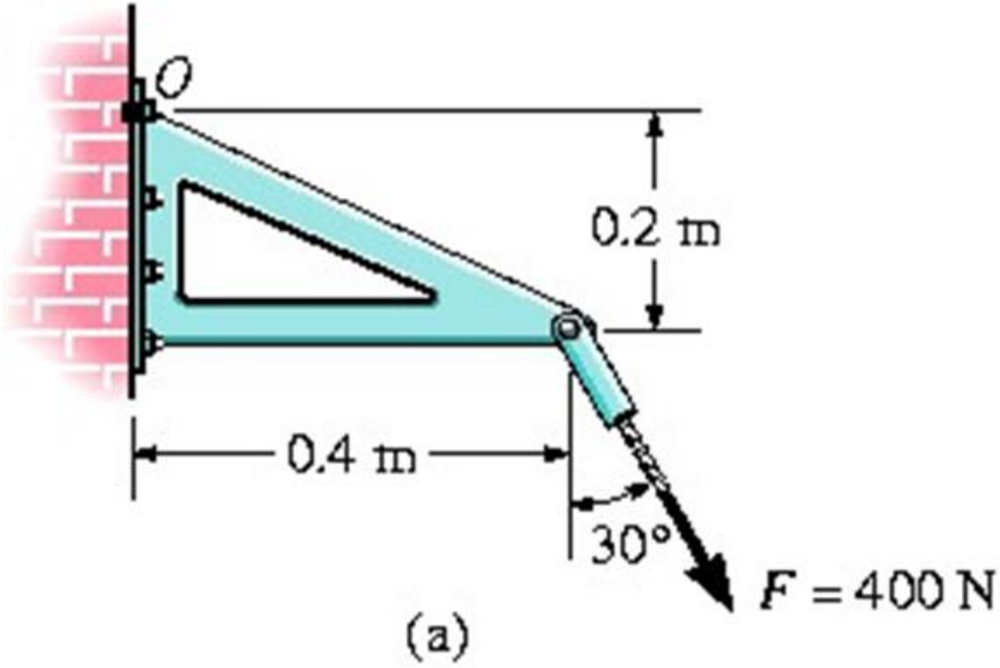
$\mathbf{M}_A = \{14.1\mathbf{k}\}$N.m




(c)

# Example 4.10

The force **F** acts at the end of the angle bracket. Determine the moment of the force about point O



# Solution Example 4.10

Method 1

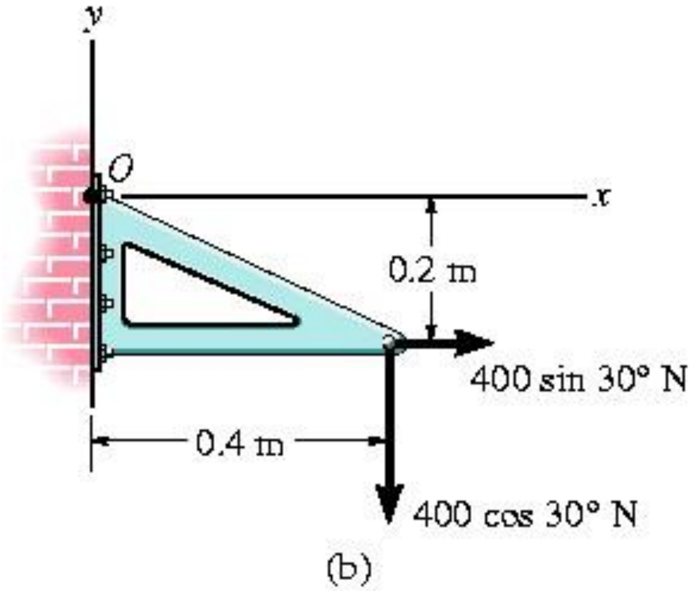
$M_O = 400\sin30°N(0.2m) - 400\cos30°N(0.4m)$

$= -98.6N.m$

$= 98.6N.m$ (CW)

As a Cartesian vector,

$\mathbf{M}_O = \{-98.6\mathbf{k}\}N.m$



# Solution Example 4.10

Method 2:

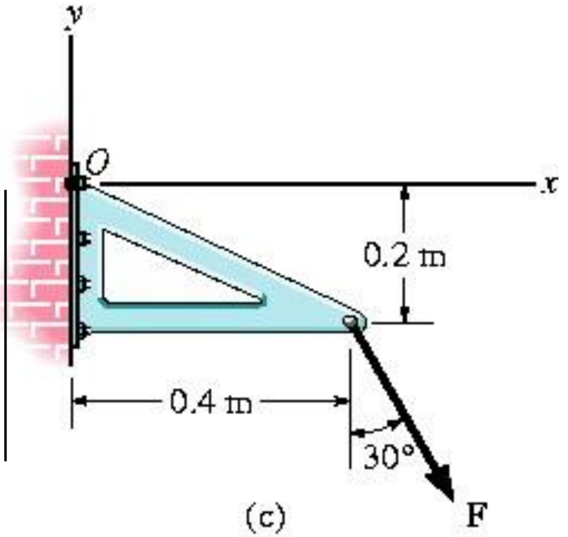
- Express as Cartesian vector

**r** = {0.4**i** – 0.2**j**}N

**F** = {400sin30°**i** – 400cos30°**j**}N

= {200.0**i** – 346.4**j**}N

For moment,

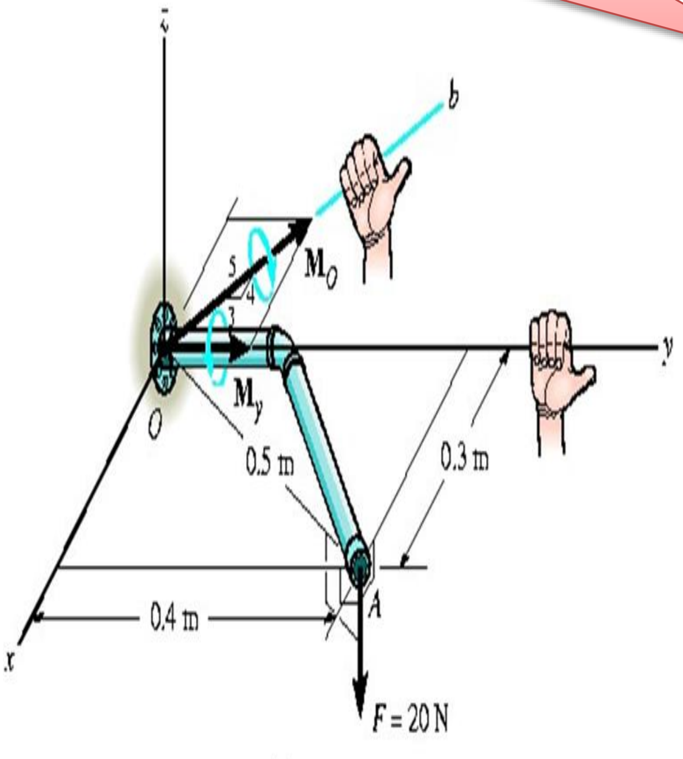



$$\vec{M}_O = \vec{r} X \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.4 & -0.2 & 0 \\ 200.0 & -346.4 & 0 \end{vmatrix}$$

$$= \left\{-98.6\vec{k}\right\} N.m$$

# Moment of a Force about specified axis

**What is Moment of a force** about **a specified axis**




(a)

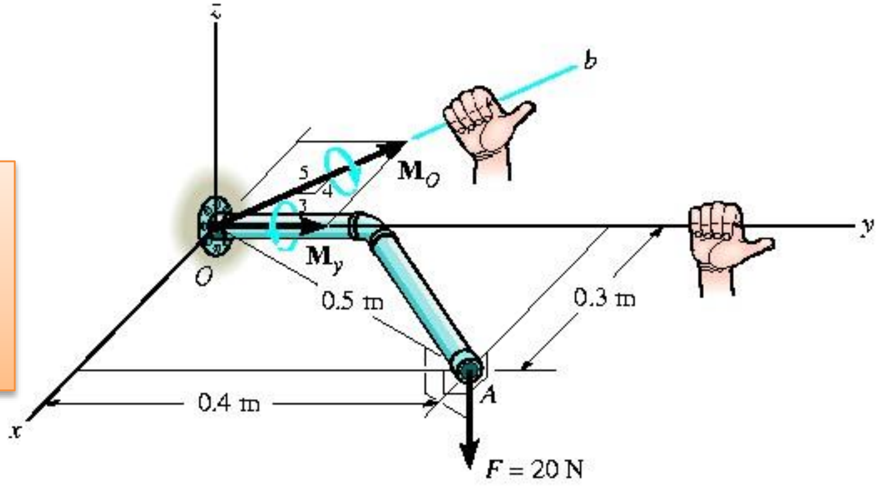
- For moment of a force about a point, the **moment** and its **axis** is always **perpendicular** to the plane containing the **force** and the **moment arm**
- A **scalar** or **vector** analysis is used to find the component of the moment along a specified axis that passes through the point

# Scalar analysis

- Consider the pipe assembly that lies in the horizontal plane and is subjected to the vertical force of F = 20N applied at point A.

- For magnitude of moment,

  $M_O$ = (20N)(0.5m) = 10N.m
- For direction of moment, apply right hand rule

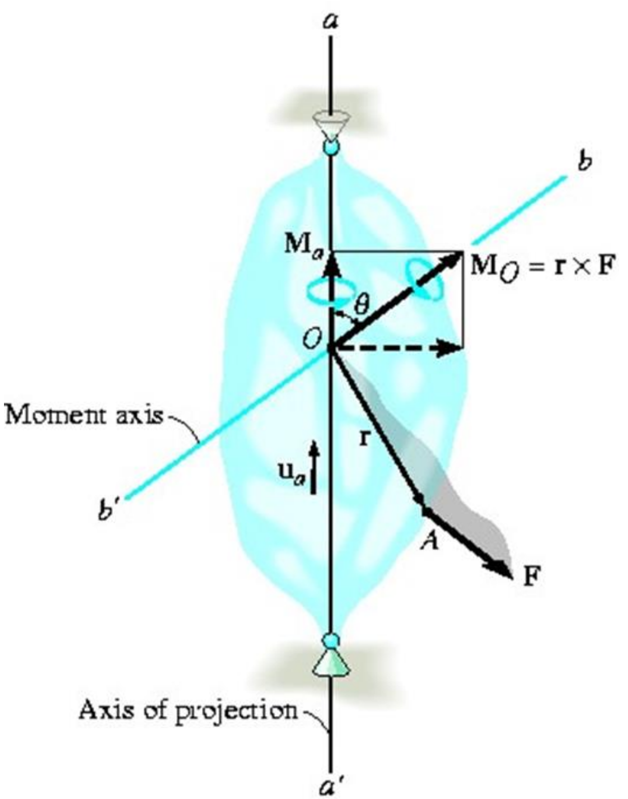


(a)

- Determine the component of $\mathbf{M}_O$ about the y axis, $\mathbf{M}_y$ since this component tend to unscrew the pipe from the flange at O
- For magnitude of $\mathbf{M}_y$,

  $M_y$ = 3/5(10N.m) = 6N.m
- For direction of $\mathbf{M}_y$, apply right hand rule

# Vector Analysis




- Consider body subjected to force **F** acting at point A
- To determine moment, $\mathbf{M}_a$,

  - For moment of **F** about any arbitrary point O that lies on the aa' axis

$$\mathbf{M}_O = \mathbf{r} \times F$$

where **r** is directed from O to A

  - $\mathbf{M}_O$ acts along the moment axis bb', so projected $\mathbf{M}_O$ on the aa' axis is $\mathbf{M}_A$

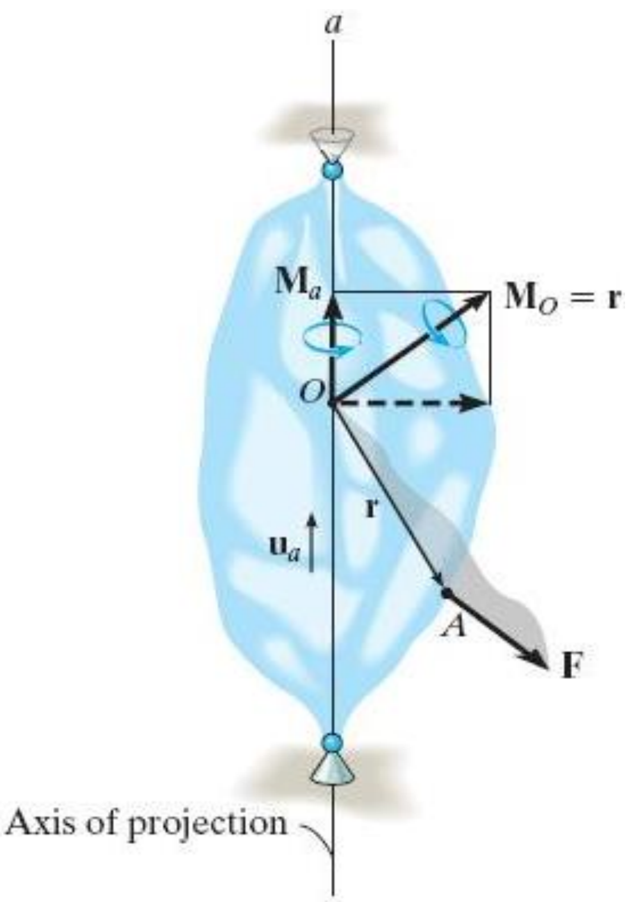
- For magnitude of $\mathbf{M}_A$,

$$M_A = M_O \cos\theta = \mathbf{M}_O \cdot \mathbf{u}_a$$

- where $\mathbf{u}_a$ is a unit vector that defines the direction of aa' axis

$$M_A = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$$

# Vector Analysis




$M_a$ can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$u_a$ represents the unit vector along the *a*-axis,

*r* is the position vector from any point on the *a*-axis to any point A on the line of action of the force, and

*F* is the force vector.

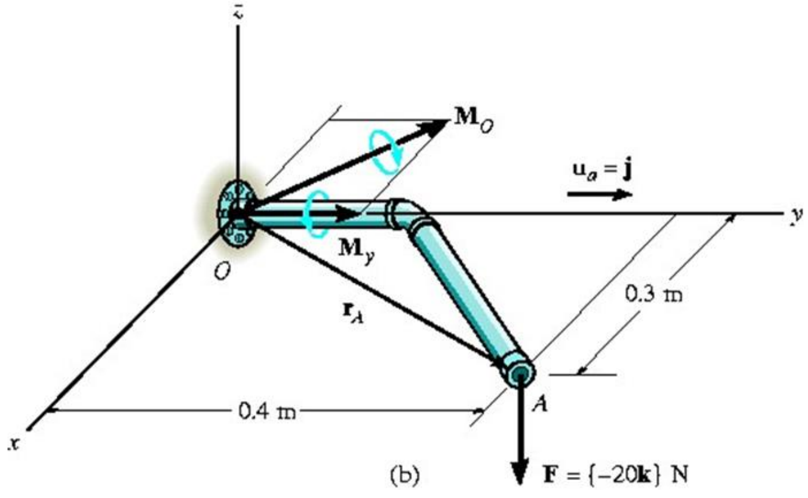
# Vector Analysis

- Determine the component of $\mathbf{M}_O$ about the y axis, $\mathbf{M}_y$ since this component tend to unscrew the pipe from the flange at O

$\mathbf{M}_O = \mathbf{r}_A \times F$
$= (0.3\mathbf{i} + 0.4\mathbf{j}) \times (-20\mathbf{k})$
$= \{-8\mathbf{i} + 6\mathbf{j}\}$N.m

Since unit vector for this axis is $\mathbf{u}_a = \mathbf{j}$,

$M_y = \mathbf{M}_O . \mathbf{u}_a$
$= (-8\mathbf{i} + 6\mathbf{j}) \cdot \mathbf{j} = 6$N.m



# Moment of a Force about specified axis

## Scalar

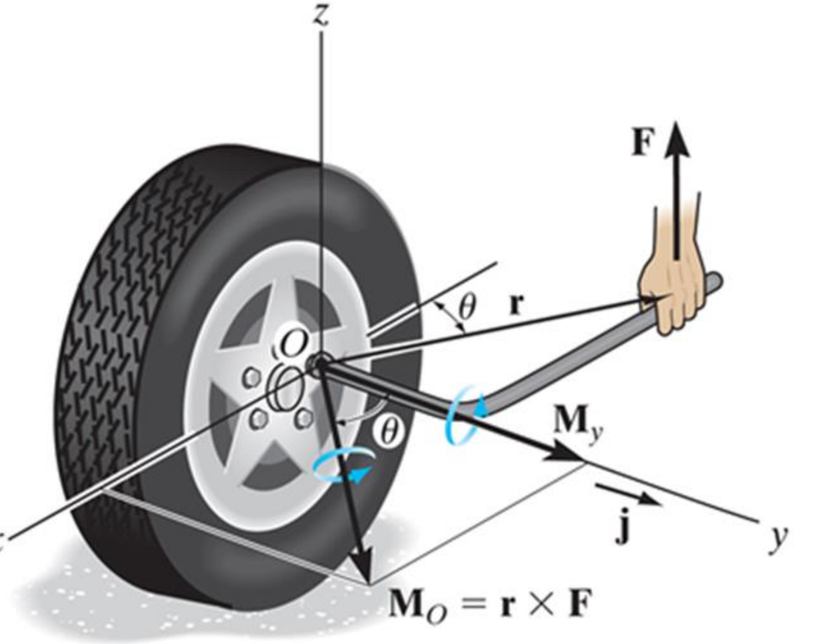
- MOF about any point O is
- $M_O = F\, d_O$
- Now finding moment about an axis using
- $M_a = Fd_a$
- $d_a$ is the perpendicular or shortest distance from the force line of action to the axis (any specified axis aa)
- No moment about a specified axis if the force line of action is parallel or passes through the axis

## Vector

- MOF about any arbitrary point O is
- $\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$
- Now find the moment along the a-axis using the dot product
- $M_a = \mathbf{u_a} \cdot \mathbf{M_O}$
- $M_a = \mathbf{u_a} \cdot (r \times F)$ (triple product)
- $\mathbf{u_a}$ Defines the direction of the axis
- r is directed from any point on the axis to any point on the line of action of the force
- Sign of scalar indicates the direction of $M_a$ *(if +ve,* $M_a$ *has same sense as* $\mathbf{u_a}$*, if -ve,* $M_a$ *act opposite to* $\mathbf{u_a}$

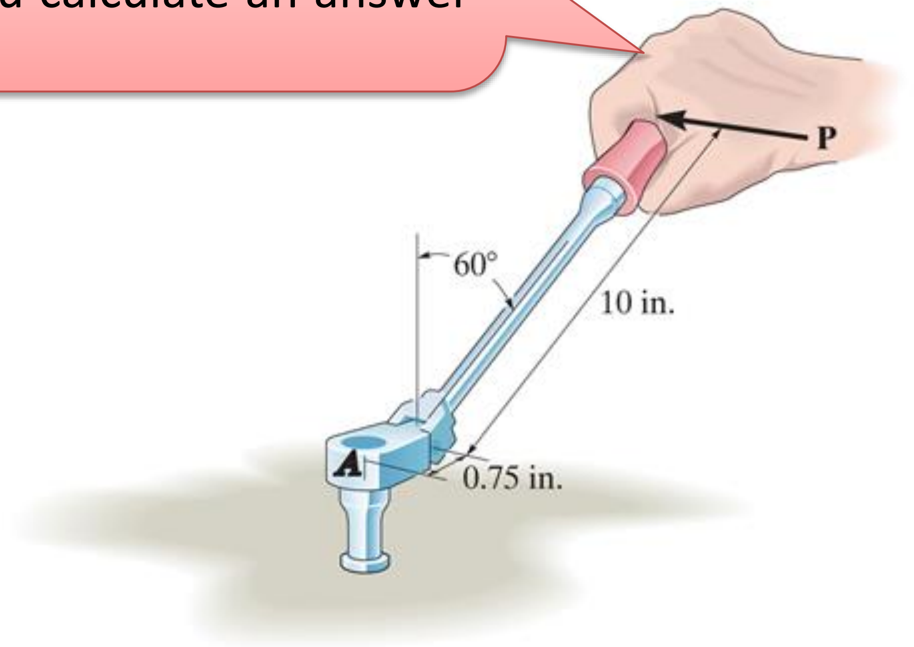
# Application (Scalar analysis)

- The moment about the y-axis would be
- $M_y = F_z (d_x) = F (r \cos \theta)$
- If force can easily be broken into components and the "$d_x$" found quickly
- such calculations are not always trivial and vector analysis may be much easier (and less likely to produce errors)



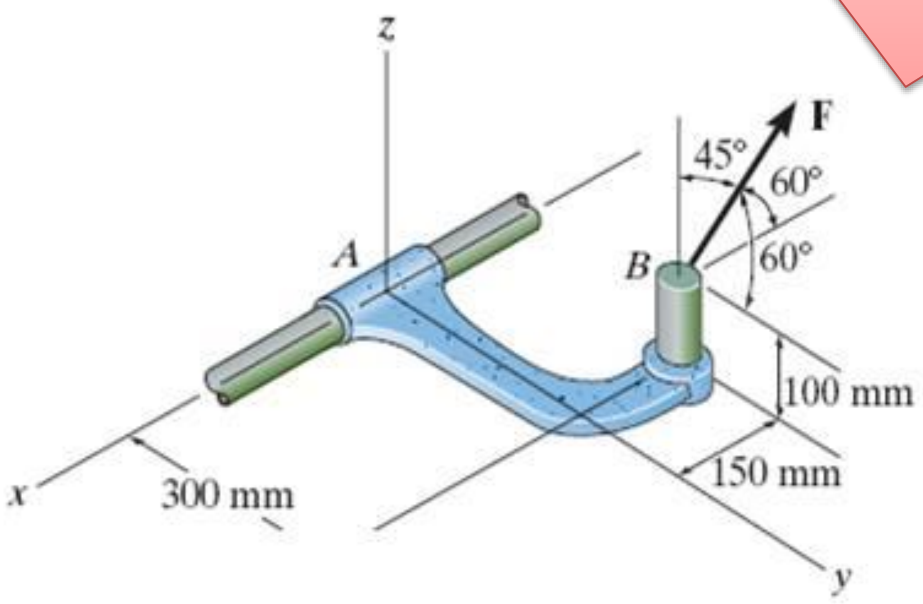
# Application

With the force ***P***, a person is creating a moment $M_A$ using this flex-handle socket wrench. Does all of $M_A$ act to turn the socket? How would you calculate an answer to this question?



# Application

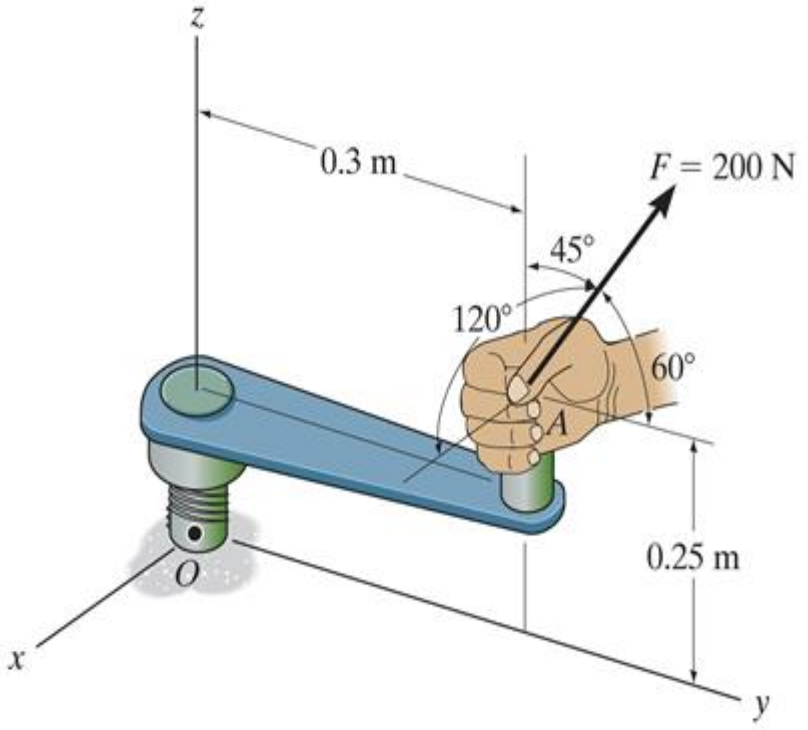
Sleeve A of this bracket can provide a maximum resisting moment of 125 N·m about the x-axis. How would you determine the maximum magnitude of ***F*** before turning about the x-axis occurs?



# Example 4.11

A force is applied to the tool as shown. Determine the magnitude of the moment of this force about the x axis of the value.

1) Use $M_z$ = ***u*** • (***r*** × ***F*** )
2) First, write ***F*** in Cartesian vector form
3) Note that ***u*** = 1 ***i*** in this case
4) The vector ***r*** is the position vector from O to A



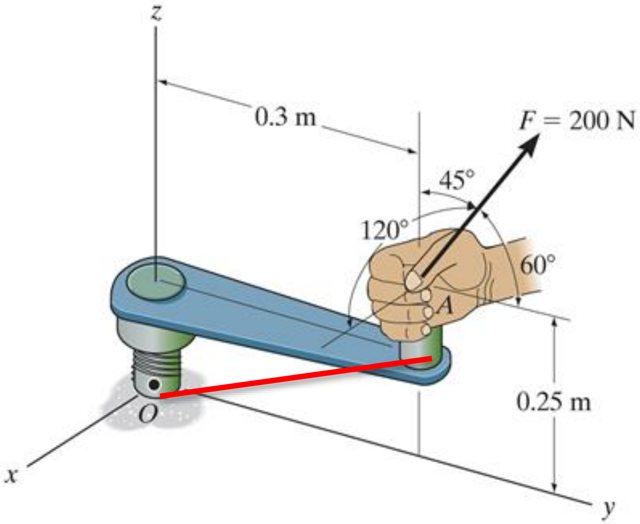
# Solution Example 4.11

$\boldsymbol{u} = 1\,\boldsymbol{i}$

$\boldsymbol{r}_{OA} = \{0\,\boldsymbol{i} + 0.3\,\boldsymbol{j} + 0.25\,\boldsymbol{k}\}$ m

$\boldsymbol{F} = 200\,(\cos 120\,\boldsymbol{i} + \cos 60\,\boldsymbol{j} + \cos 45\,\boldsymbol{k})$ N

$= \{-100\,\boldsymbol{i} + 100\,\boldsymbol{j} + 141.4\,\boldsymbol{k}\}$ N




Now find $M_z = \boldsymbol{u} \bullet (\boldsymbol{r}_{OA} \times \boldsymbol{F})$

$$M_z = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix} = 1\{0.3\,(141.4) - 0.25\,(100)\}\ \text{N·m}$$

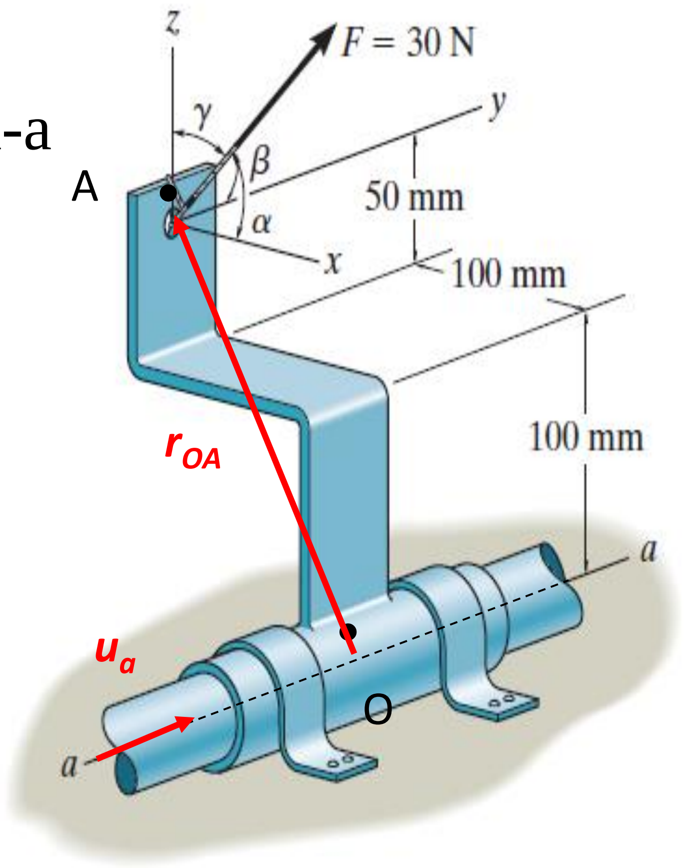
$M_z = 17.4$ N·m CCW

# Example 4.12

The force of F = 30 N acts on the bracket.

$\alpha = 60°, \beta = 60°, \gamma = 45°$

Determine the moment of ***F*** about the a-a axis.




1) Find $\boldsymbol{u_a}$ and $\boldsymbol{r_{OA}}$

2) Write ***F*** in Cartesian vector form

3) Use $M_a = \boldsymbol{u_a} \bullet (\boldsymbol{r_{OA}} \times \boldsymbol{F})$

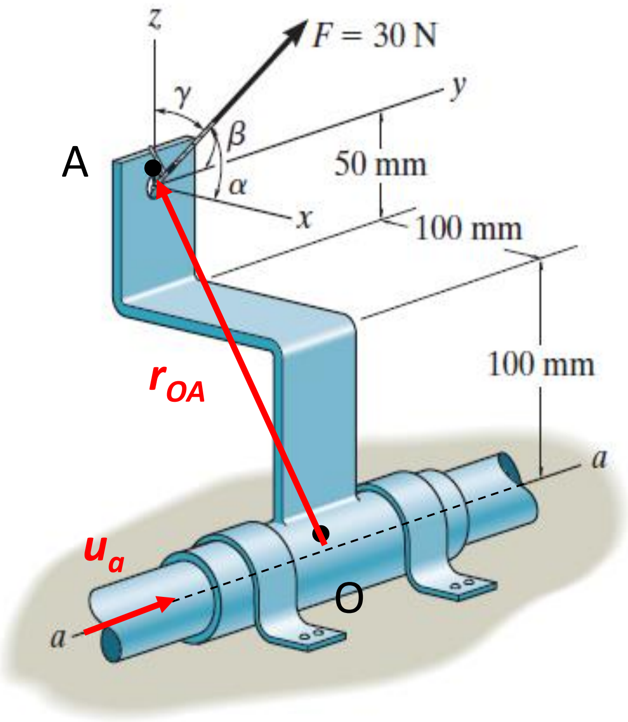
# Solution Example 4.12

$\boldsymbol{u_a} = \boldsymbol{j}$

$\boldsymbol{r_{OA}} = \{-0.1\ \boldsymbol{i} + 0.15\ \boldsymbol{k}\}\ \text{m}$

$\boldsymbol{F} = 30\ \{\cos 60°\ \boldsymbol{i} + \cos 60°\ \boldsymbol{j} + \cos 45°\ \boldsymbol{k}\}\ \text{N}$

$\boldsymbol{F} = \{\ 15\ \boldsymbol{i} + 15\ \boldsymbol{j} + 21.21\ \boldsymbol{k}\}\ \text{N}$



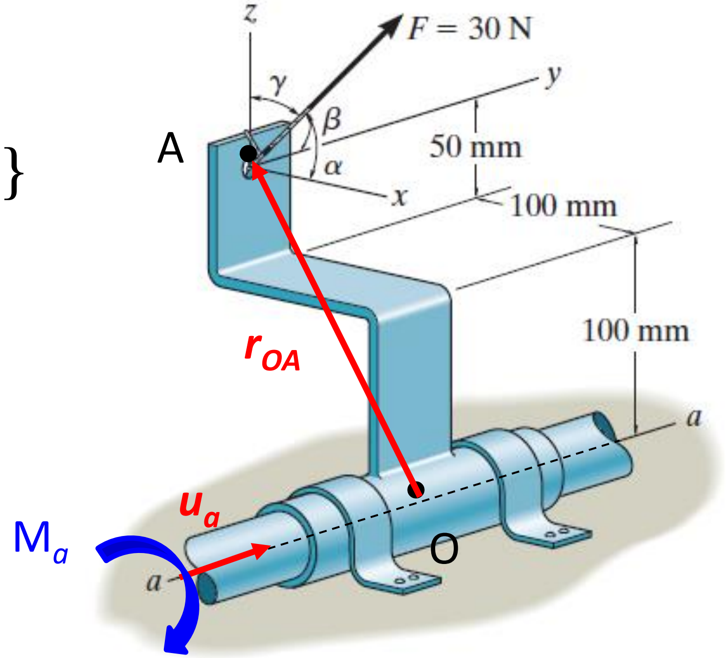
# Solution Example 4.12

Now find the triple product, $M_a = \mathbf{u_a} \cdot (\mathbf{r_{OA}} \times \mathbf{F})$

$$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} \ \text{N·m}$$

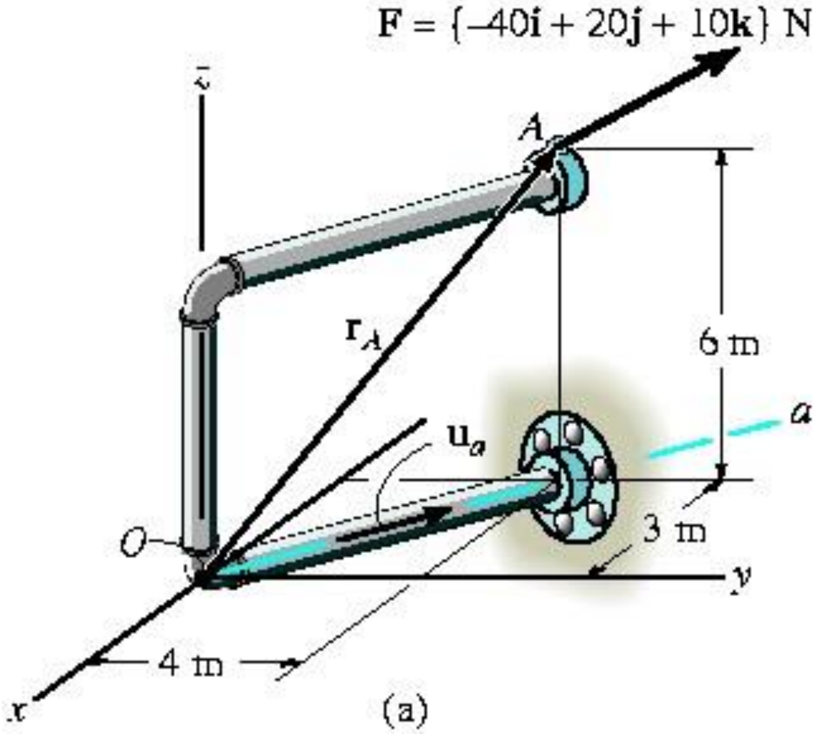
$$M_a = -1\ \{-0.1\ (21.21) - 0.15\ (15)\}$$

$$= 4.37\ \text{N·m}$$



# Example 4.13

The force **F** = {-40**i** + 20**j** + 10**k**} N acts on the point A. Determine the moments of this force about the x and a axes



# Solution Example 4.13

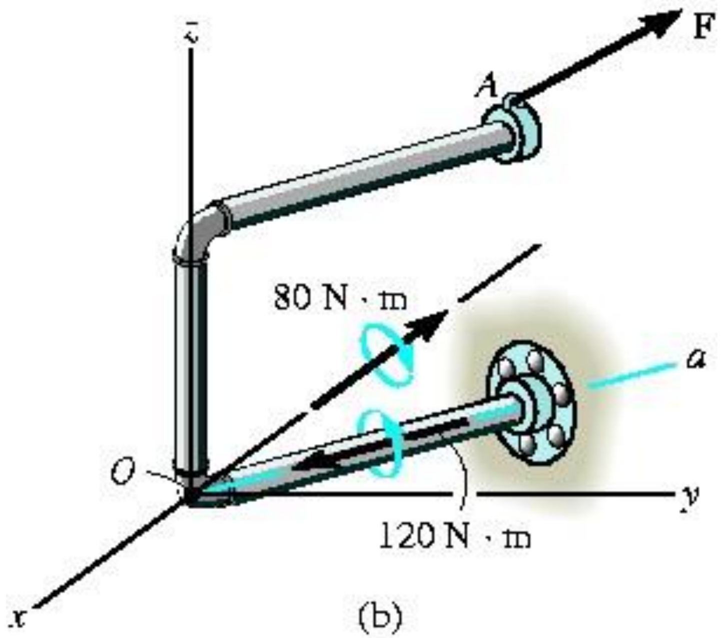
Method 1

$$\vec{r}_A = \{-3\vec{i} + 4\vec{j} + 6\vec{k}\}m$$

$$\vec{u}_x = \vec{i}$$

$$\left|\vec{M}_x\right| = \vec{i} \cdot (\vec{r}_A X \vec{F}) = \begin{vmatrix} 1 & 0 & 0 \\ -3 & 4 & 6 \\ -40 & 20 & 10 \end{vmatrix}$$

$$= -80 N.m$$




Negative sign indicates that sense of $\mathbf{M}_x$ is opposite to **i**

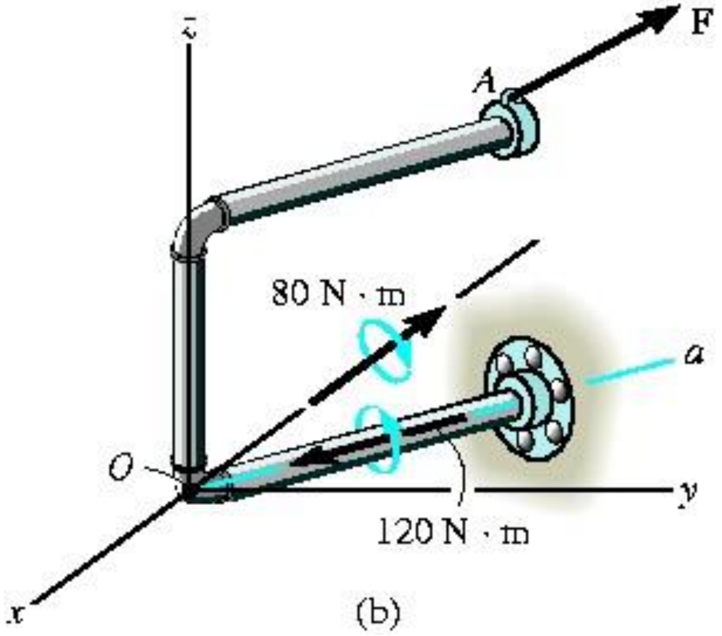
# Solution Example 4.13

We can also compute $M_a$ using $\mathbf{r}_A$ as $\mathbf{r}_A$ extends from a point on the a axis to the force

$$\vec{u}_A = -\tfrac{3}{5}\vec{i} + \tfrac{4}{5}\vec{j}$$

$$\left|\vec{M}_a\right| = \vec{u}_A \cdot (\vec{r}_A X \vec{F}) = \begin{vmatrix} -\tfrac{3}{5} & \tfrac{4}{5} & 0 \\ -3 & 4 & 6 \\ -40 & 20 & 10 \end{vmatrix}$$

$$= -120\,N.m$$



# Solution Example 4.13

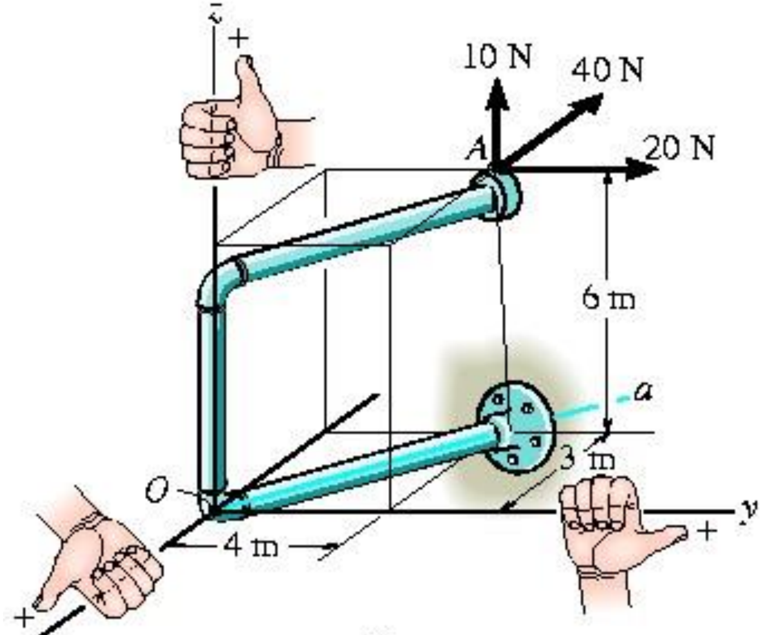
Method 2

- Only 10N and 20N forces contribute moments about the x axis
- Line of action of the 40N is parallel to this axis and thus, moment = 0
- Using right hand rule

  $M_x$ = (10N)(4m) – (20N)(6m) = -80N.m

  $M_y$ = (10N)(3m) – (40N)(6m) = -210N.m

  $M_z$ = (40N)(4m) – (20N)(3m) = 100N.m

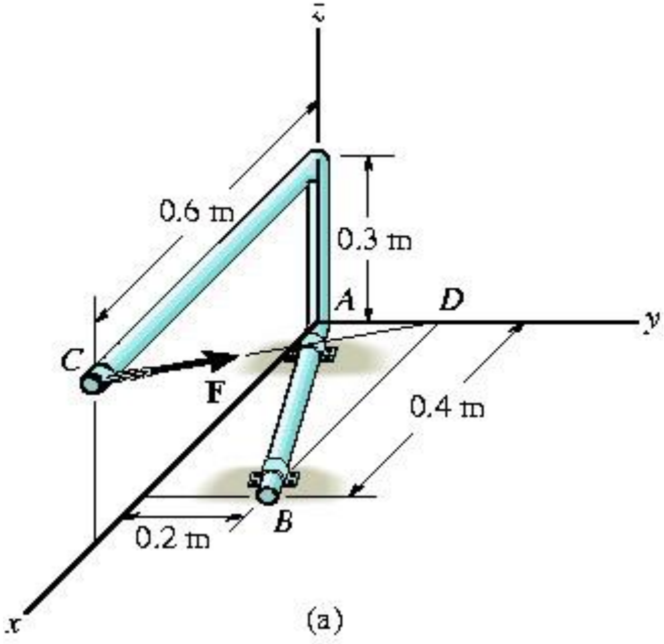



(c)

# Example 4.14

The rod is supported by two brackets at A and B. Determine the moment $\mathbf{M}_{AB}$ produced by

**F** = {-600**i** + 200**j** – 300**k**}N, which tends to rotate the rod about the AB axis.



# Solution Example 4.14

Vector analysis chosen as moment arm from line of action of **F** to the AB axis is hard to determine

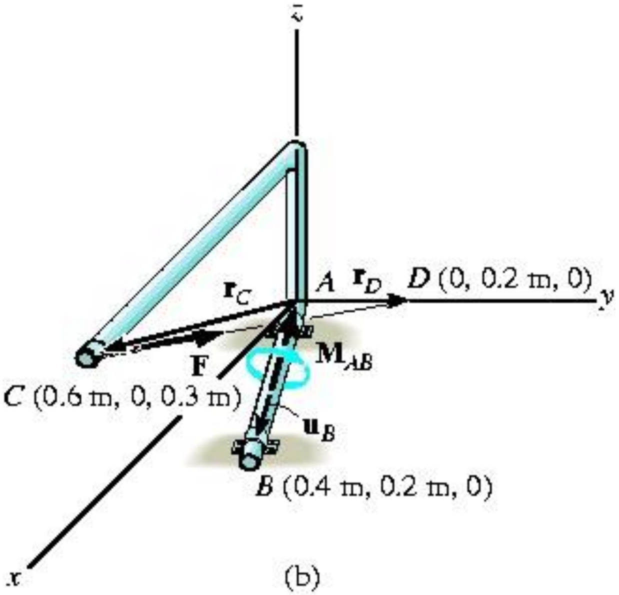
- For unit vector defining direction of AB axis of the rod,

- For simplicity, choose $\mathbf{r}_D$

$$\left|\vec{M}_{AB}\right| = \vec{u}_B \cdot (\vec{r} X \vec{F})$$

$$\vec{u}_B = \frac{\vec{r}_B}{\left|\vec{r}_B\right|} = \frac{0.4\vec{i} + 0.2\vec{j}}{\sqrt{(0.4)^2 + (0.2)^2}}$$

$$= 0.894\vec{i} + 0.447\vec{j}$$



# Solution Example 4.14

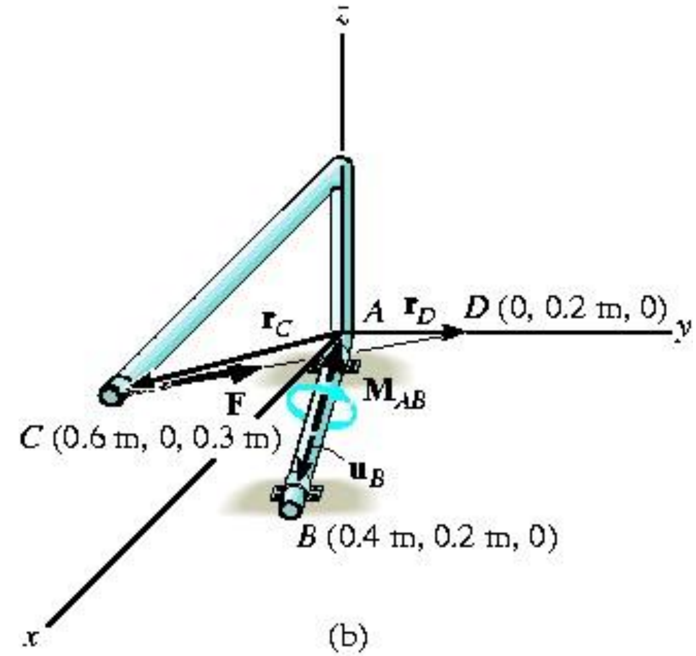
- For force,

$$\vec{F} = \{-600\vec{i} + 200\vec{j} - 300\vec{k}\}N$$

- In determinant form,

$$\left|\vec{M}_{AB}\right| = \vec{u}_B \cdot (\vec{r}_D X \vec{F}) = \begin{vmatrix} 0.894 & 0.447 & 0 \\ 0 & 0.2 & 0 \\ -600 & 200 & -300 \end{vmatrix}$$

$$= -53.67\,N.m$$



(b)

Negative sign indicates $\mathbf{M}_{AB}$ is opposite to $\mathbf{u}_B$

# Solution Example 4.14

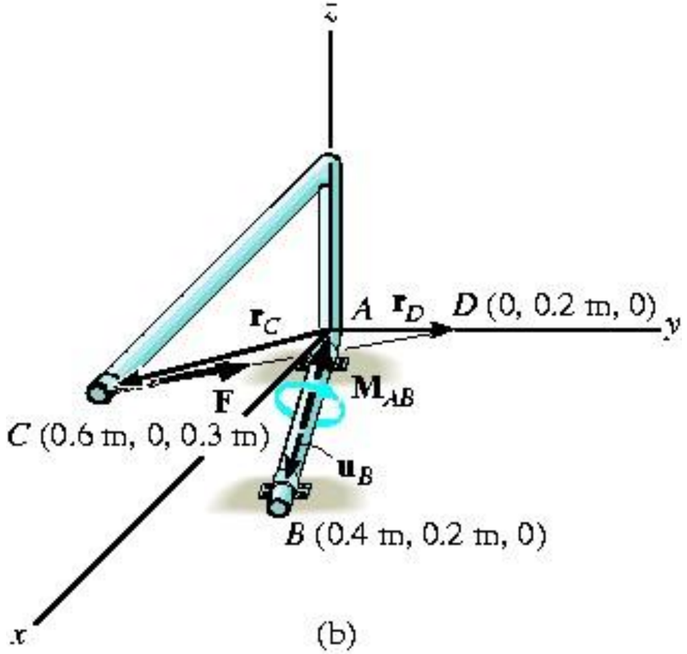
- In Cartesian form,

$$\vec{M}_{AB} = \left|\vec{M}_{AB}\right| \vec{u}_B = (-53.67 N.m)(0.894\vec{i} + 0.447\vec{j})$$

$$= \{-48.0\vec{i} - 24.0\vec{j}\} N.m$$

*Note: if axis AB is defined using unit vector directed from B towards A, the above formulation $-\mathbf{u}_B$ should be used.

$\mathbf{M}_{AB} = M_{AB}(-\mathbf{u}_B)$




(b)

# Conclusion of The Chapter 4

- Conclusions
  - The Principle Moment has been identified
  - The triple product vector have been implemented to solve Moment problems in specified axis

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