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ENGINEERING MECHANICS
BAA1113

Chapter 4: Force System Resultants (Static)

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Chapter Description

- Aims
  - To explain the Moment of Force (2D-scalar formulation & 3D-Vector formulation)
  - To explain the Principle Moment
  - To explain the Moment of a Couple
  - To explain the Simplification of a Force and Couple System
  - To explain the Reduction of Simple Distributed Loading

- Expected Outcomes
  - Able to solve the problems of MOF and COM in the mechanics applications by using principle of moments

- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14^{th} Edition

Chapter Outline

4.1 Moment of Force (MOF) –Part I

4.2 Principle of Moment –Part II

4.3 Moment of Couple (MOC) Part III

4.4 Simplification of a Force and Couple System

4.5 Reduction of Simple Distributed Loading- part IV

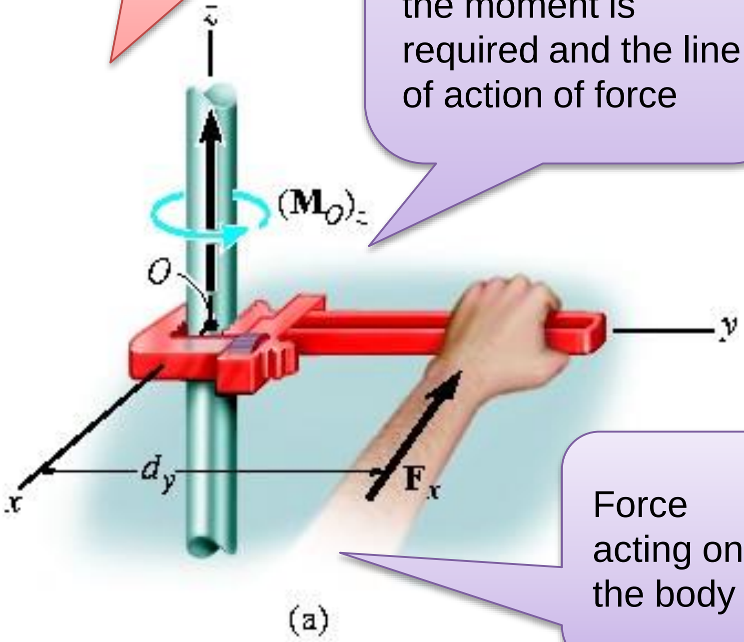
4.1 Moment of a Force

- Moment can be defined as turning force
- The tendency of a force to rotate a rigid body about any defined axis is called the moment of the force about the axis
- It is also called a torque or twist moment that tendency of a force to rotate a body about the axis
- It is a vector, so its has both magnitude and direction (right handrule)
- +ve CCW & -ve CW
- Unit used is N.m

- In a 2-D case, the magnitude of the moment

What is Moment?

Perpendicular distance between the point about which the moment is required and the line of action of force

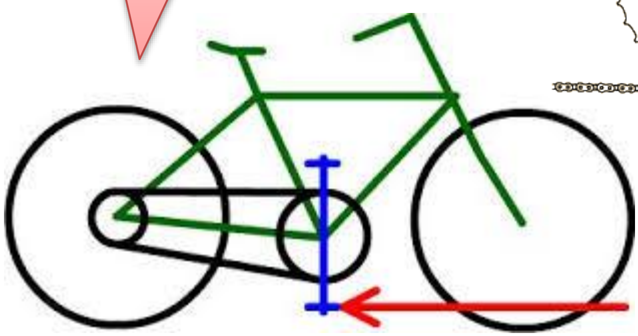


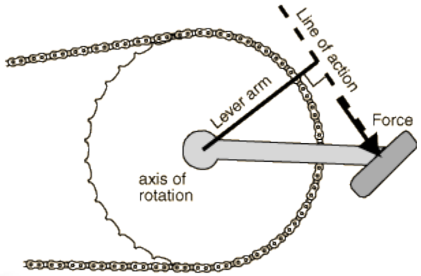
Force acting on the body

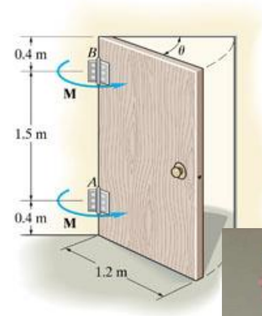
$$M_o = F\ d$$

Application of Moment (turning effect)

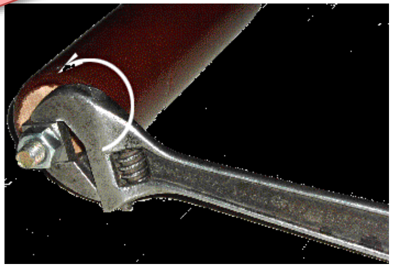
Causes of motion

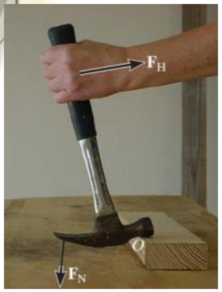




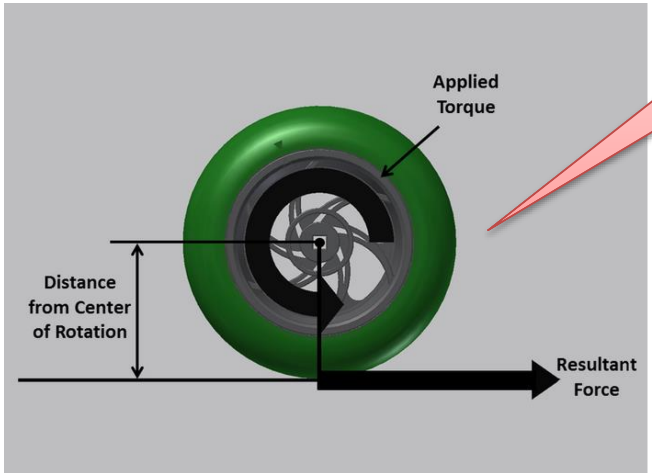


Day life activity-moment arm

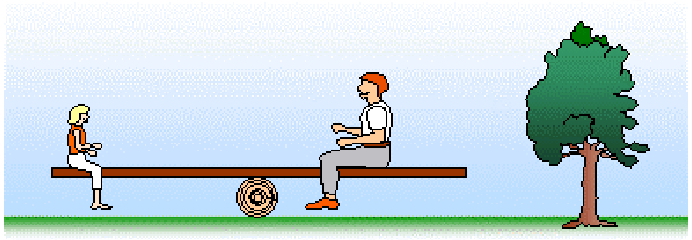




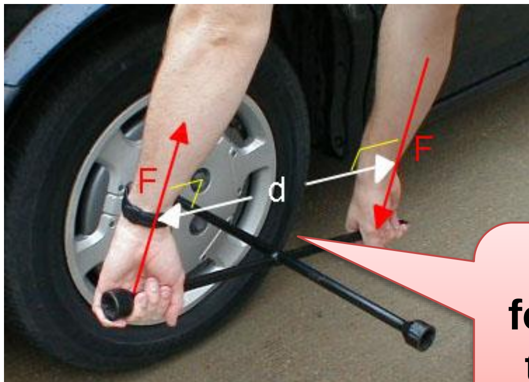
How does wheel size affect performance?



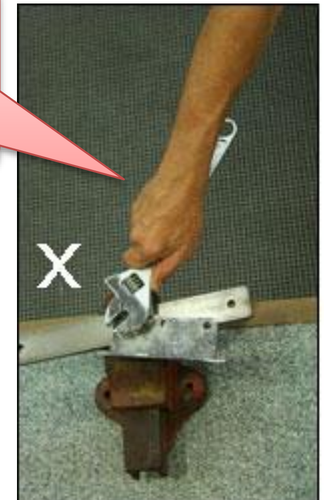
Seesaw-how to balance?



Application of Moment (turning effect)

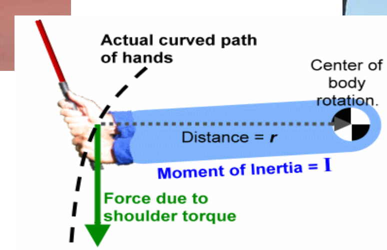


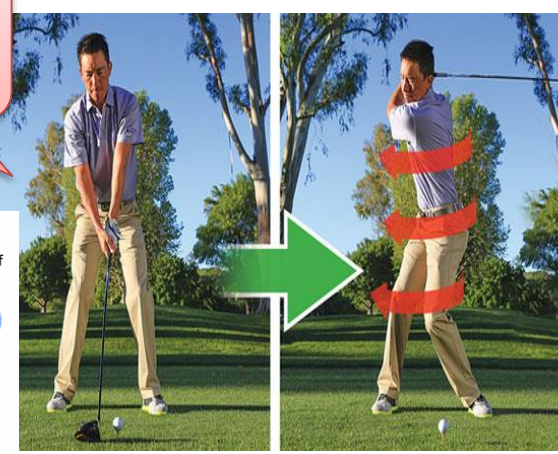
Measure the **moment arm(length)** to produce **rotary power**



Measure the **forces (weight transfer)** and **moment arm**

Measure the **forces/effort** to make sure **good swing**

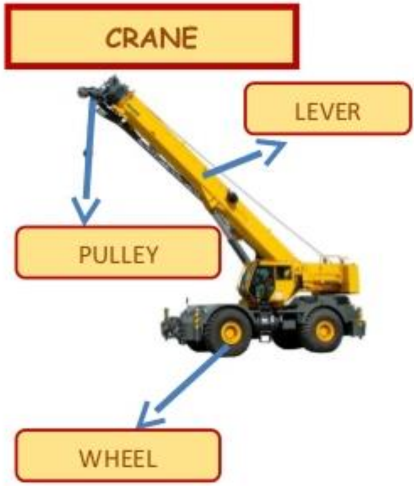




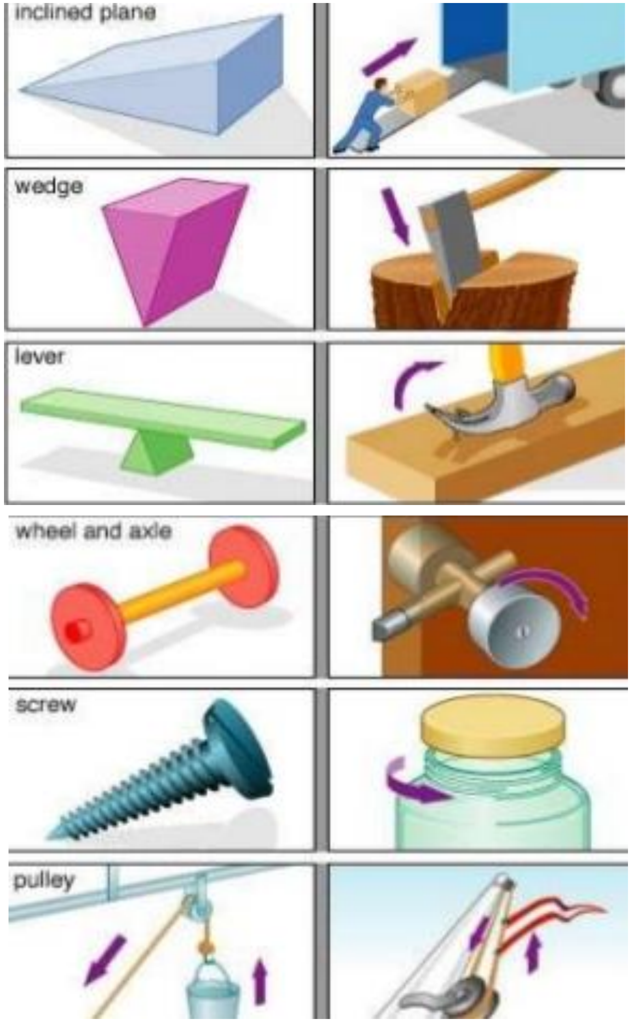
Measure the **forces/effort** to make sure **good swing**

Application of Moment (turning effect)

Measure the input **forces** and **level** to make sure **output force**

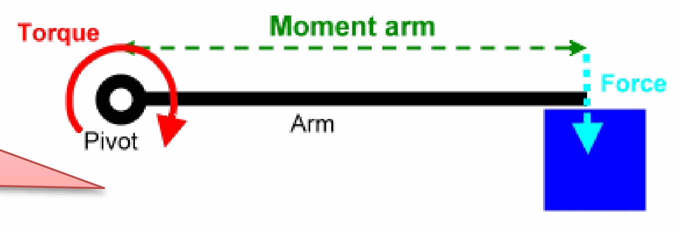


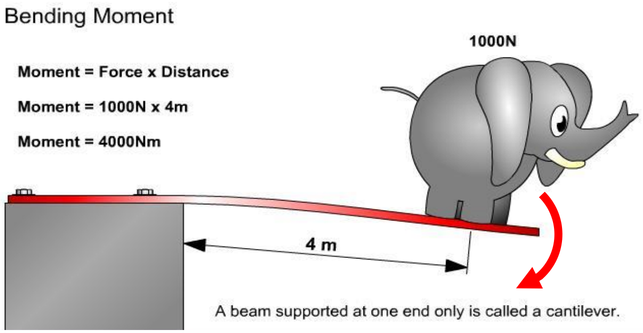
Measure the **effort/ load** to make easy work



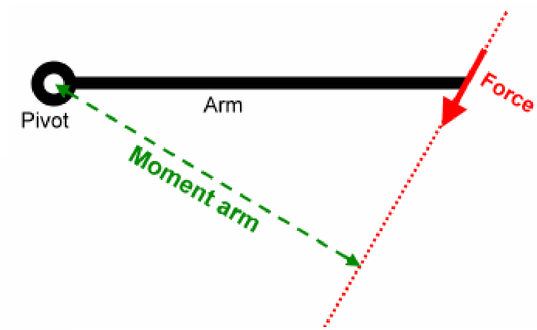
Moment factor

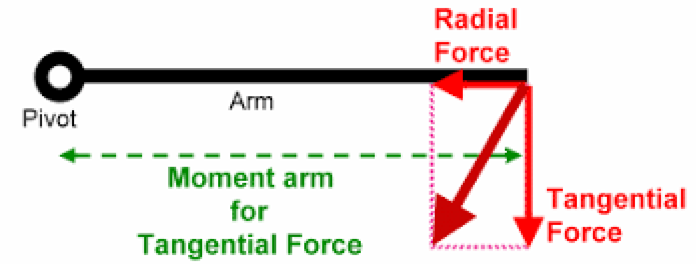
MOF is bigger if the force is bigger



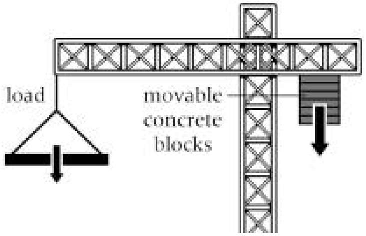


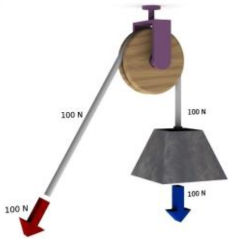
MOF is bigger if acts further from the pivot

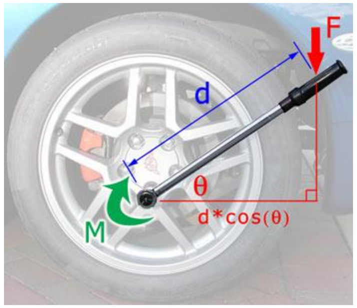




MOF is bigger if it acts at 90 to the body it acts on



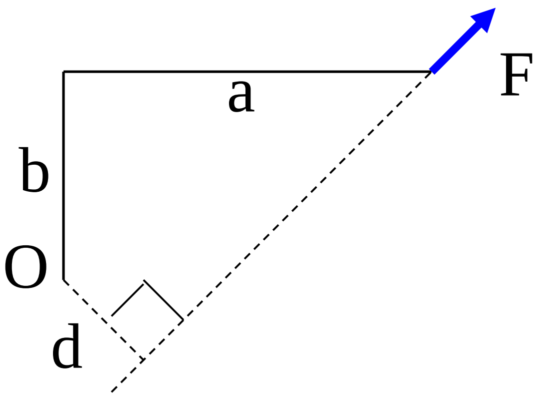




Moment of a force in 2-D (scalar formulation)

Magnitude

- $\mathbf{M}_O = Fd$
- d is the perpendicular distance from point O to the line of action of the force

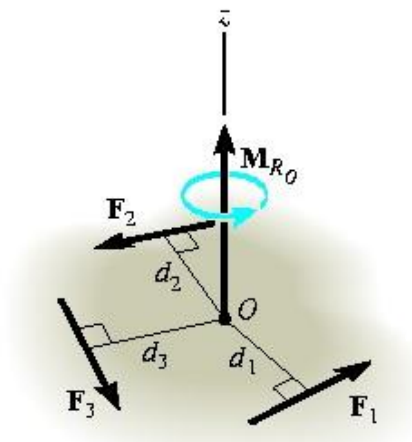


$M_O = F\,d$

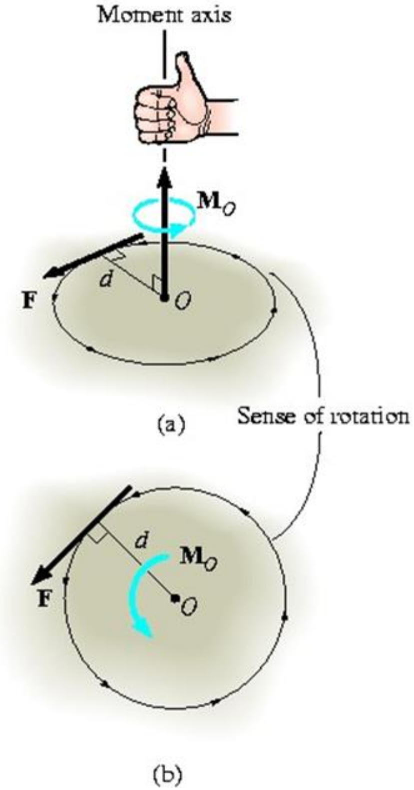
direction is counter-clockwise.

Direction

- Direction of $\mathbf{M}_O$ is specified by using "right hand rule"
- direction of M_O is either clockwise (CW) or counter-clockwise (CCW), depending on the tendency for rotation



$\mathbf{M}_{Ro} = \sum Fd$

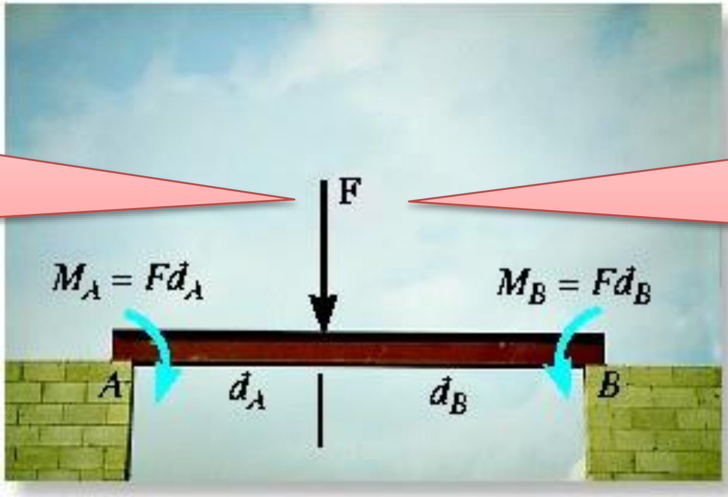


Moment of a force in 2-D (scalar formulation)

Moment of a force does not always cause rotation

Force F tends to rotate the beam clockwise about A with moment

$M_A = F\, d_A$



Force F tends to rotate the beam clockwise about B with moment

$M_B = F\, d_B$

Hence support at A prevents the rotation

Example 4.1

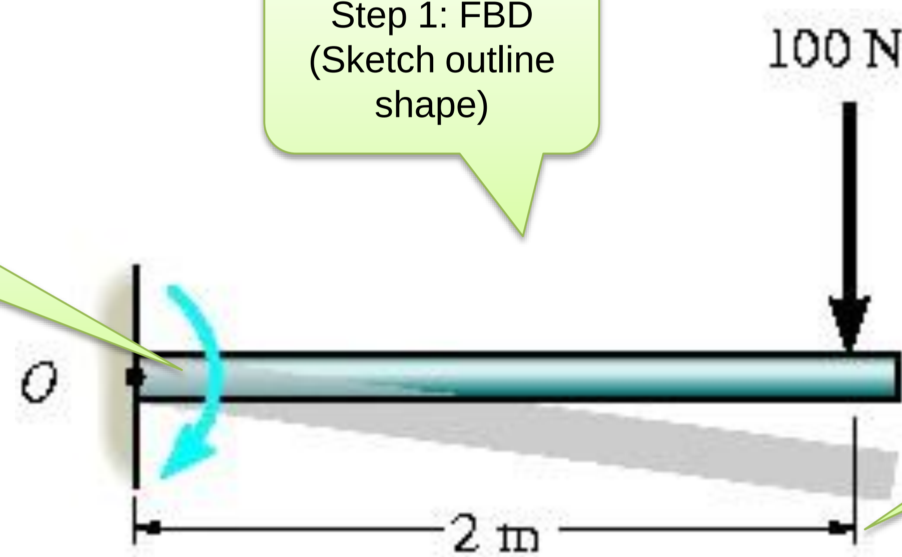
This is an example of a 2-D or coplanar force system. Determine the MOF about point O

Step 1: FBD (Sketch outline shape)

Step 3: assume tendency to rotate/ moment

Step 2: det. The line of action/ moment arm (d)

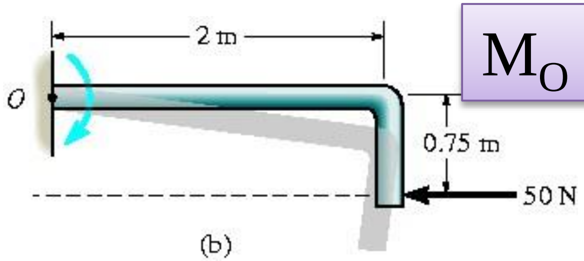
Step 4: use formula



$$M_O = Fd = (100\ N)\ (2\ m) = 200\ Nm \ \text{(CW)}$$

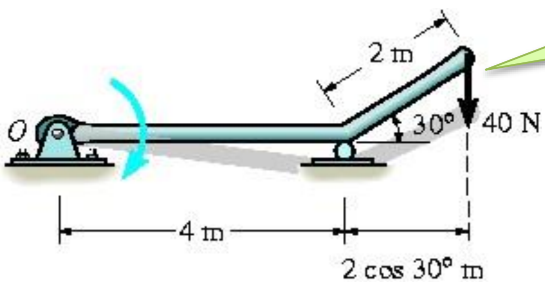
Solution Example 4.1

This is an example of a 2-D or coplanar force system. Determine the MOF about point O



$M_O = Fd = (50\text{ N})\,(0.75\text{ m}) = 37.5\text{ Nm}$ (CW)

Step 2: det. The line of action/ moment arm (d)

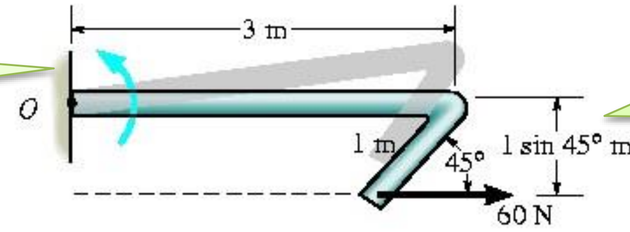


$M_O = Fd = (40\text{ N})\,(4\text{ m} + 2\cos 30°\text{ m}) = 229\text{ Nm}$ (CW)

Solution Example 4.1

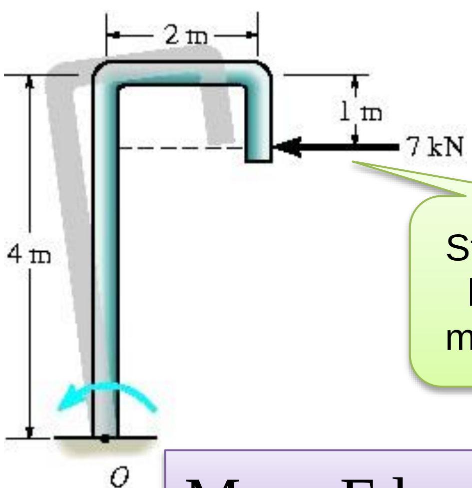
This is an example of a 2-D or coplanar force system. Determine the MOF about point O

Step 3: assume tendency to rotate/ moment



Step 2: det. The line of action/ moment arm (d)

M_O = Fd = (60 N) (1 sin 45° m) = 42.4 Nm (CCW)



Step 2: det. The line of action/ moment arm (d)

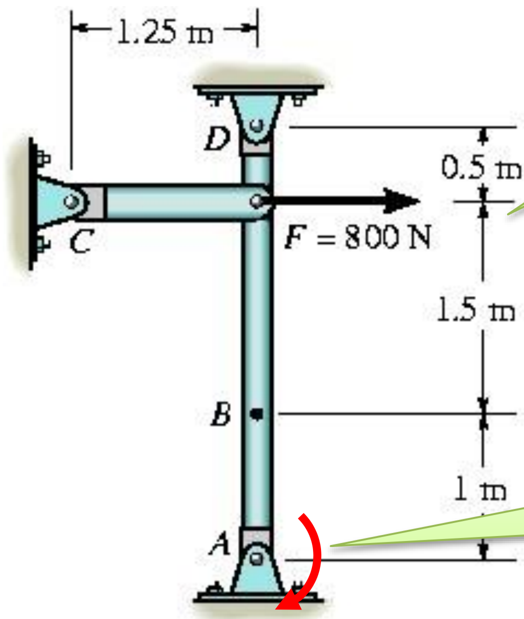
M_O = Fd = (7 kN) (4 m -1 m) = 21 kNm (CW)

Example 4.2

This is an example of a 2-D or coplanar force system. Determine the moments of the 800 N force acting on the frame about points **A**,B,C and D

Step 1: FBD (Sketch outline shape)

Step 2: det. The line of action/ moment arm (d)



Step 3: assume tendency to rotate/ moment

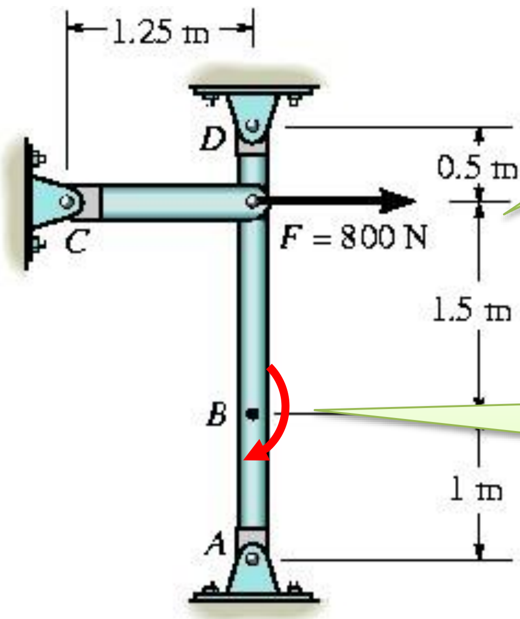
Step 4: use formula

$$M_A = Fd = (800\ \text{N})\ (1.5 + 1\ \text{m}) = 2000\ \text{Nm} \quad \text{(CW)}$$

Solution Example 4.2

This is an example of a 2-D or coplanar force system. Determine the moments of the 800 N force acting on the frame about points A,**B**,C and D

Step 1: FBD (Sketch outline shape)



Step 2: det. The line of action/ moment arm (d)

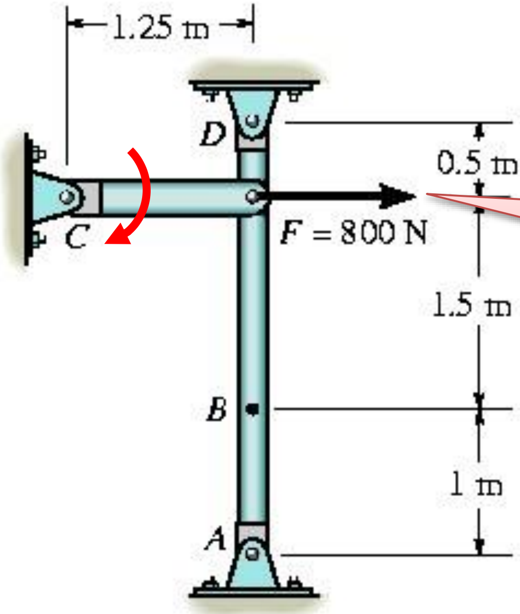
Step 3: assume tendency to rotate/ moment

Step 4: use formula

M_B = Fd = (800 N) (1.5 m) = 1200 Nm (CW)

Example 4.2

This is an example of a 2-D or coplanar force system. Determine the moments of the 800 N force acting on the frame about points A,B,**C** and D



Moment is zero

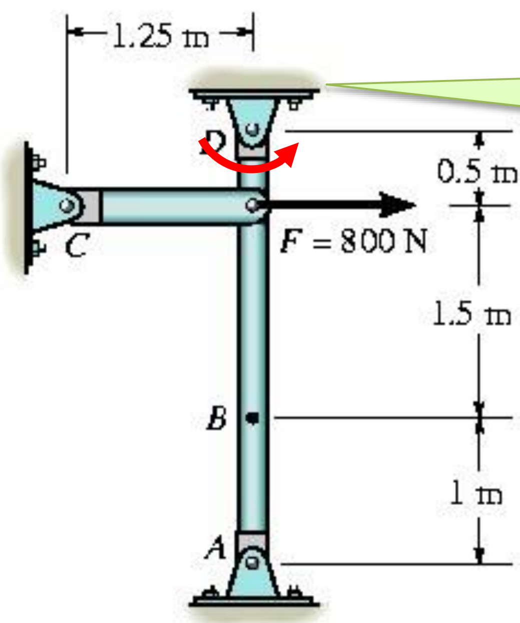
Step 2: line of action F passes through C

Step 4: use formula

$$M_C = Fd = (800\ N)\ (\ 0\ m) = 0\ Nm$$

Solution Example 4.2

This is an example of a 2-D or coplanar force system. Determine the moments of the 800 N force acting on the frame about points A,B,C and **D**



Step 3: assume tendency to rotate/ moment

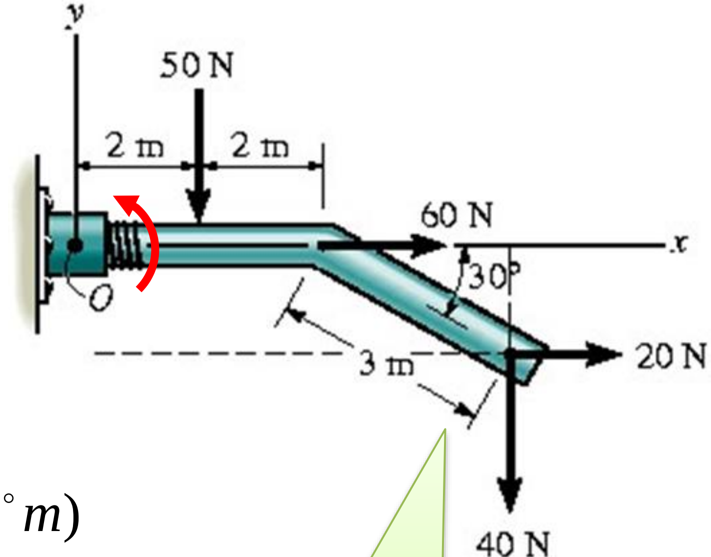
Step 2: det. The line of action/ moment arm (d)

Step 4: use formula

$M_D = Fd = (800\ N)\ (0.5\ m) = 400\ Nm$ (CCW)

Solution Example 4.3

This is an example of a 2-D or coplanar force system. Determine the moments of the four force acting on the rod about point O



Step 3: assume moment acts in + y direction

Step 4: use formula

Step 2: det. The line of action/ moment arm (d) for each forces

$$M_{Ro} = \sum Fd$$

$$M_{Ro} = (-50N)(2m) + (60N)(0m) + (20N)(3\sin 30^{\circ} m)$$

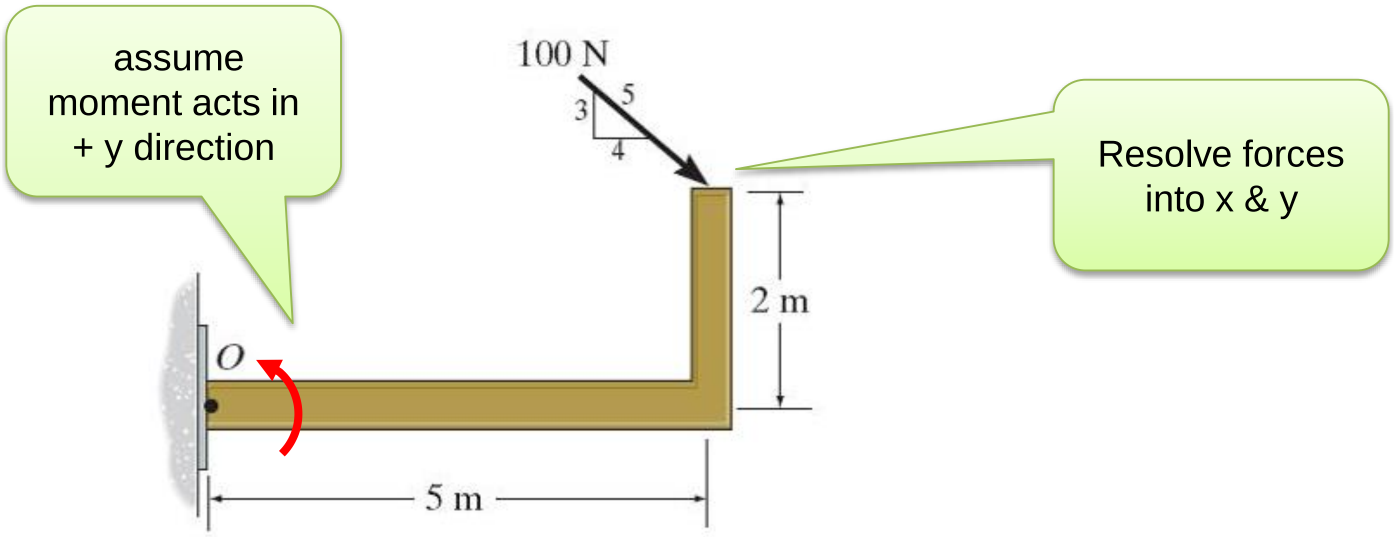
$$-(40N)(4m + 3\cos 30^{\circ} m)$$

$$= -334 N.m$$

$$= 334 N.m(CW)$$

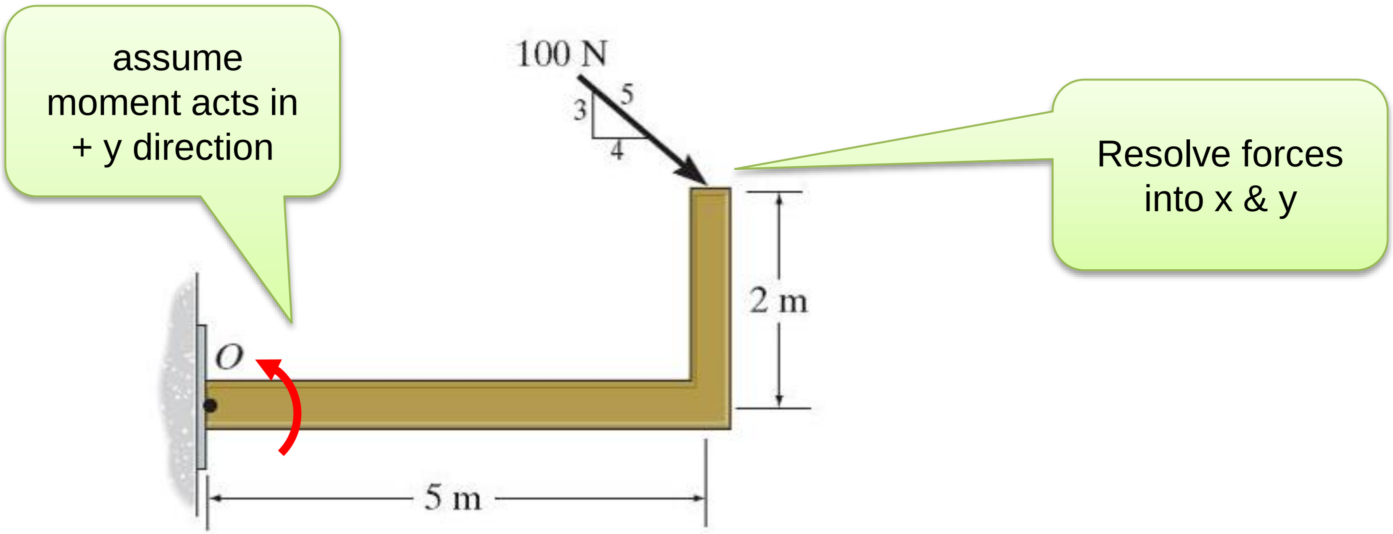
Example 4.4

Determine the moments of the 100 N force acting on the frame about point O



Solution Example 4.4

Determine the moments of the 100 N force acting on the frame about point O



$+\uparrow F_y = \underline{-100\ (3/5)\ \mathrm{N}}$

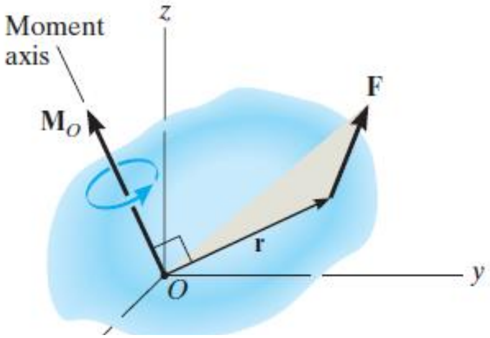
$+\rightarrow F_x = \underline{100\ (4/5)\ \mathrm{N}}$

$+\ M_O = \{-100\ (3/5)\mathrm{N}\ (5\ \mathrm{m}) - (100)(4/5)\mathrm{N}\ (2\ \mathrm{m})\}\ \mathrm{N{\cdot}m}$

$= \underline{-460\ \mathrm{N{\cdot}m}}$ or $\underline{460\ \mathrm{N{\cdot}m\ CW}}$

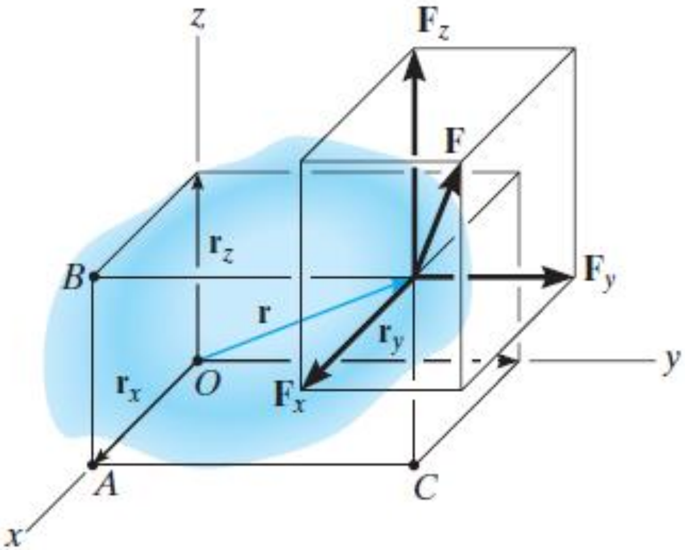
Moment of a force in 3-D (Vector formulation)

- Moments in 3-D can be calculated using scalar (2-D) approach, but it can be difficult (finding d when forces in 3-D) and time consuming



- It it easier to use vector cross product

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

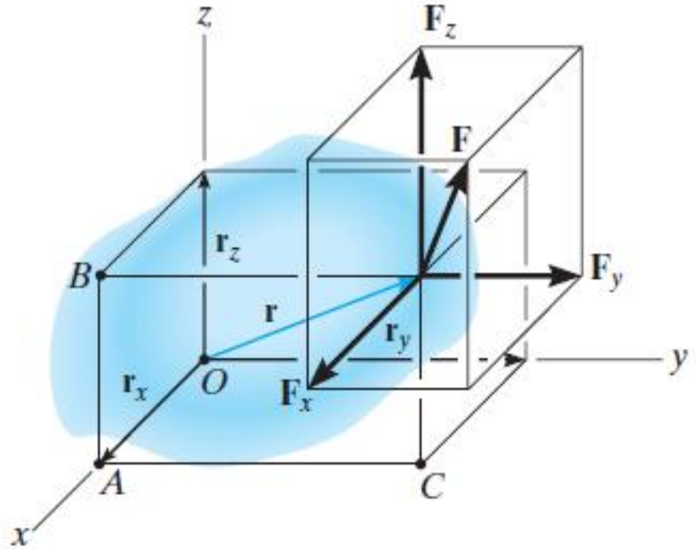


r is the position vector from point O to any point on the line of action of ***F***

Moment of a force in 3-D (Vector formulation)

Moment can be expressed as

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

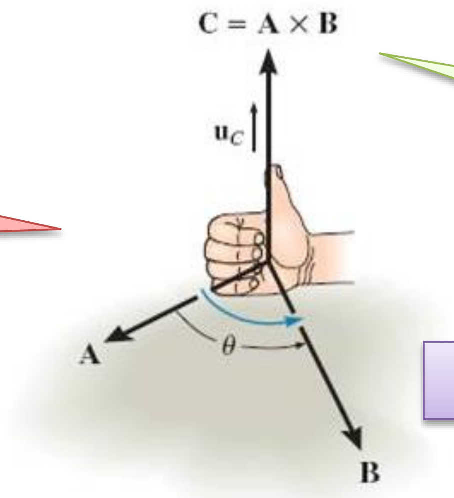


By expanding the above equation using 2 × 2 determinants

$$\boldsymbol{M_O} = (r_y\ F_z\ - r_z\ F_y)\ \boldsymbol{i}\ -\ (r_x\ F_z\ -\ r_z\ F_x)\ \boldsymbol{j}\ +\ (r_x\ F_y\ - r_y\ F_x)\ \boldsymbol{k}$$

Cross Product

What is vector cross product?



It is a vector operation

$0° \le \theta \le 180°$

- The cross product of two vectors ***A*** and ***B*** results in vector ***C***

$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$

- The **magnitude** and **direction** of the resulting vector can be written as

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = A\,B \sin\theta\, \boldsymbol{u_C}$$

Scalar $A\,B\sin\theta$ defines the magnitude of vector C

Unit vector $\boldsymbol{u_C}$ defines the direction of vector C

Cross Product

Laws of Operations

1. Commutative law is not valid

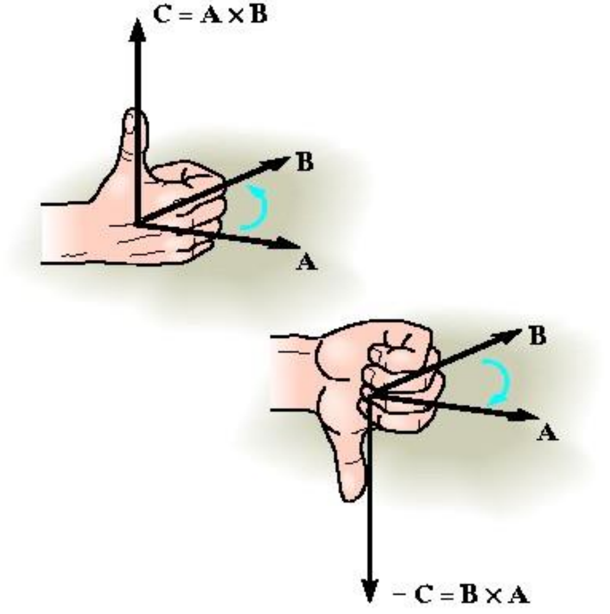
$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

Rather,

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

- Shown by the right hand rule
- Cross product **A** X **B** yields a vector opposite in direction to **C**

$$\mathbf{B} \times \mathbf{A} = -\mathbf{C}$$



Cross Product

Laws of Operations

2. Multiplication by a Scalar

$$a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$$

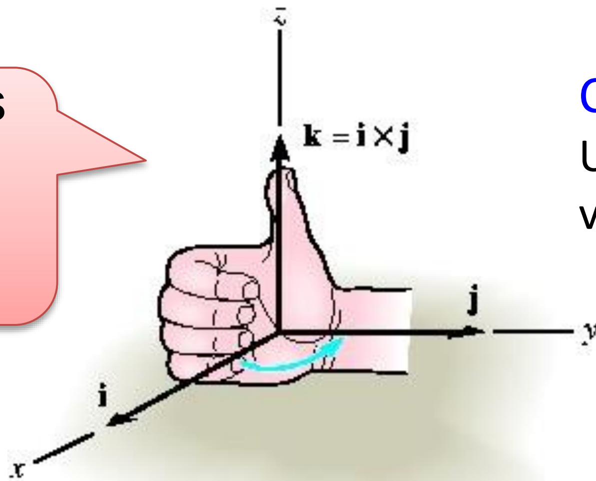
3. Distributive Law

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$$

- Proper order of the cross product must be maintained since they are not commutative

Cross Product

Direction is determine using right hand rule



Cartesian Vector Formulation

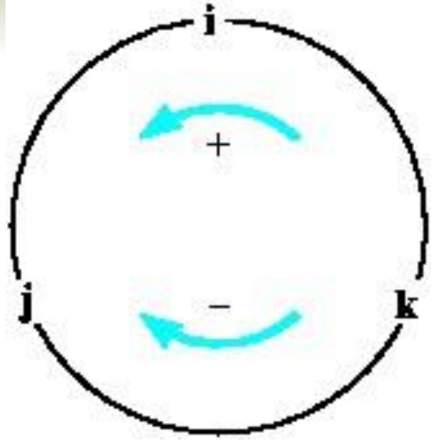
Use $C = AB \sin\theta$ on pair of Cartesian unit vectors

$$i \times j = k$$

For **i** X **j**, $(i)(j)(\sin 90°)$
$= (1)(1)(1) = 1$

Use the circle for the results

- Crossing CCW yield positive
- CW yields negative results



vector crossed into itself is zero

$i \times i = 0 \quad j \times j = 0 \quad k \times k = 0$

$i \times j = k$

$j \times k = i$

$k \times i = j$

$i \times k = -j$

$k \times j = -i$

$j \times i = -k$

Cross Product Rules

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cross product can be written as a determinant

Each component can be determine using 2X2 determinants

For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

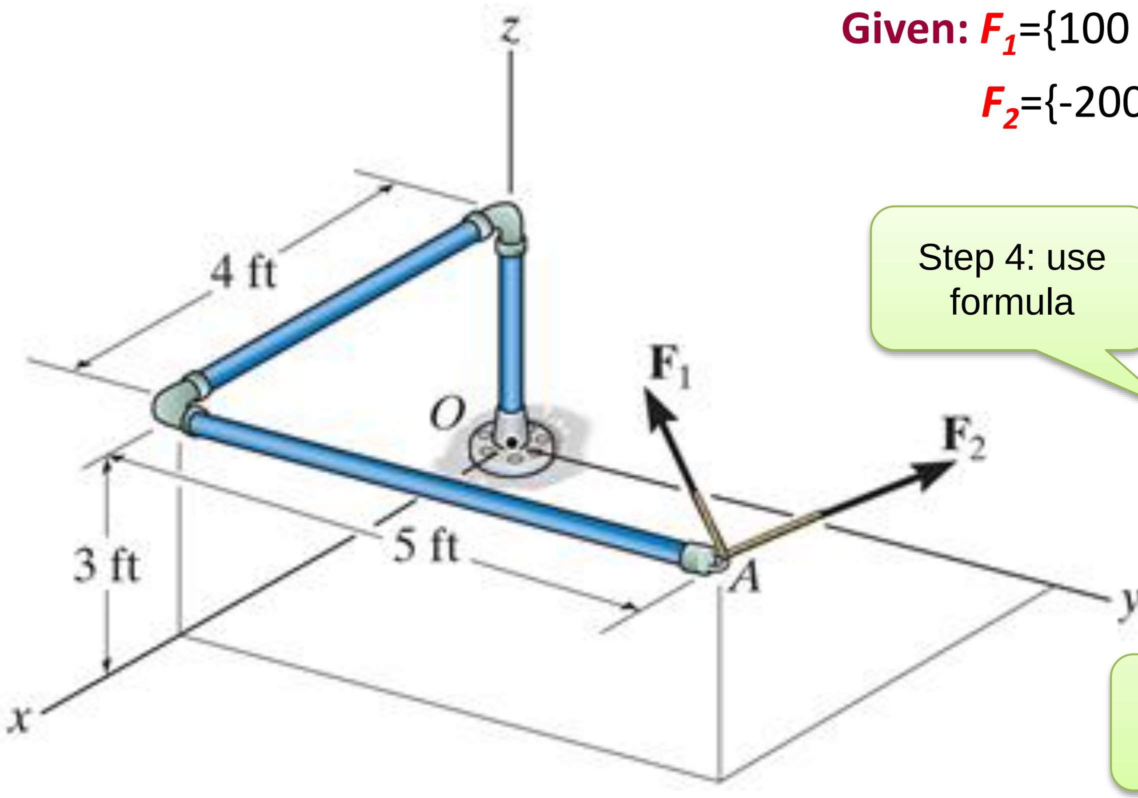
For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

Remember the negative sign

For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$

Example 4.5

Determine the resultant moment by forces about point O



Given: $\boldsymbol{F_1}$={100 $\boldsymbol{i}$ - 120 $\boldsymbol{j}$ + 75 $\boldsymbol{k}$}lb

$\boldsymbol{F_2}$={-200 $\boldsymbol{i}$ +250 $\boldsymbol{j}$ + 100 $\boldsymbol{k}$}lb

Step 4: use formula

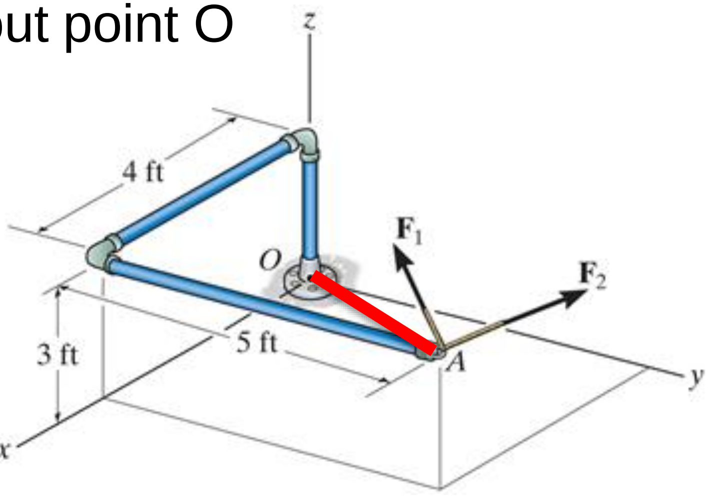
$$\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2}$$

$$\boldsymbol{M_O} = \boldsymbol{r_{OA}} \times \boldsymbol{F}$$

$\boldsymbol{r_{OA}}$?

Solution Example 4.5

Determine the resultant moment by forces about point O



$\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2}$

= { (100 - 200) $\boldsymbol{i}$ + (-120 + 250) $\boldsymbol{j}$ + (75 + 100) $\boldsymbol{k}$} lb

= {-100 $\boldsymbol{i}$ +130 $\boldsymbol{j}$ + 175 $\boldsymbol{k}$} lb

$\boldsymbol{r_{OA}}$

$\boldsymbol{r_{OA}}$ = {4 $\boldsymbol{i}$ + 5 $\boldsymbol{j}$ + 3 $\boldsymbol{k}$} ft

Use vector cross product $\boldsymbol{M_O} = \boldsymbol{r_{OA}} \times \boldsymbol{F}$

$$\boldsymbol{M_O} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 4 & 5 & 3 \\ -100 & 130 & 175 \end{vmatrix} = [\{5(175) - 3(130)\}\,\boldsymbol{i} - \{4(175) - 3(-100)\}\,\boldsymbol{j} + \{4(130) - 5(-100)\}\,\boldsymbol{k}]\ \text{ft·lb}$$

$$= \{485\,\boldsymbol{i} - 1000\,\boldsymbol{j} + 1020\,\boldsymbol{k}\}\ \text{ft·lb}$$

Example 4.6

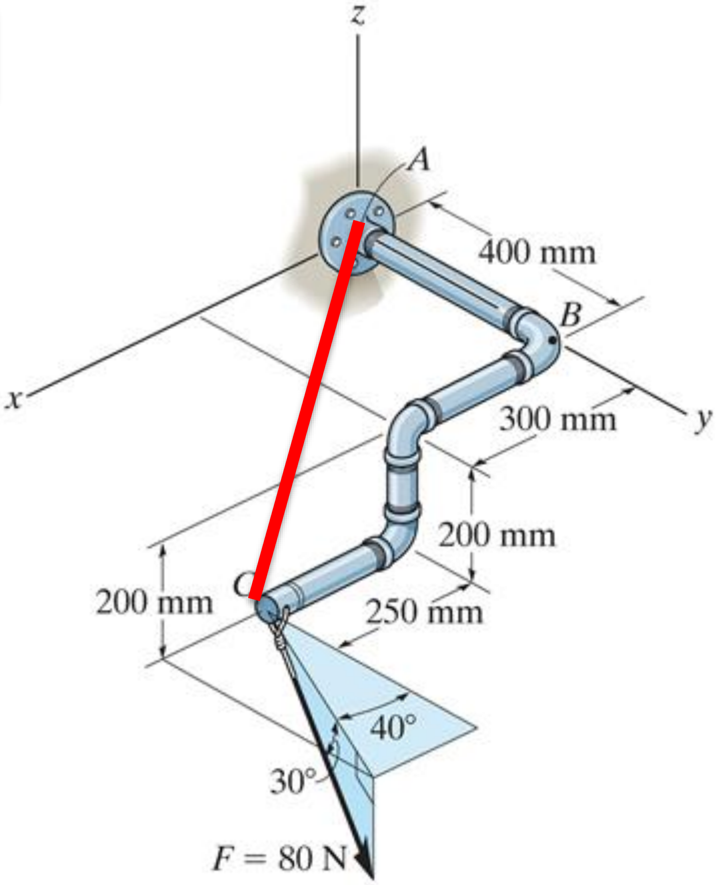
Determine the moment of F about point A

Step 4: use formula

$$M_O = r_{AC} \times F$$

F?

r_{AC}?



Solution Example 4.6

Determine the moment of F about point A

$$\mathbf{F} = \{ (80\cos 30)\sin 40\,\mathbf{i} + (80\cos 30)\cos 40\,\mathbf{j} - 80\sin 30\,\mathbf{k} \}\ \text{N}$$

$$= \{44.53\,\mathbf{i} + 53.07\,\mathbf{j} - 40\,\mathbf{k}\}\ \text{N}$$

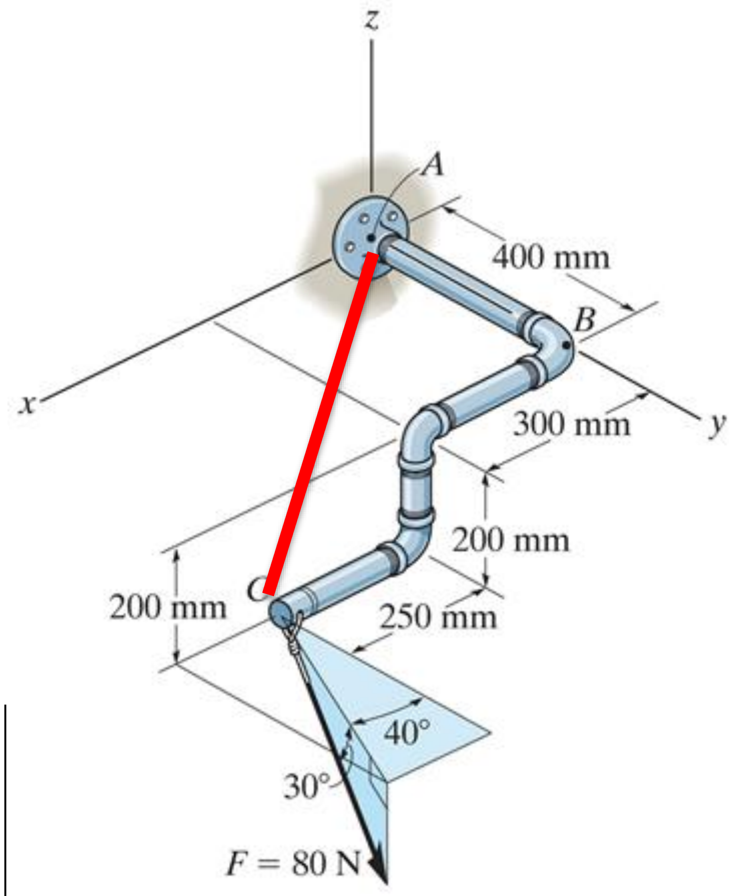
$$\mathbf{r}_{AC} = \{0.55\,\mathbf{i} + 0.4\,\mathbf{j} - 0.2\,\mathbf{k}\}\ \text{m}$$

$$\mathbf{M}_O = \mathbf{r}_{AC} \times \mathbf{F}$$

Det. moment by using cross product

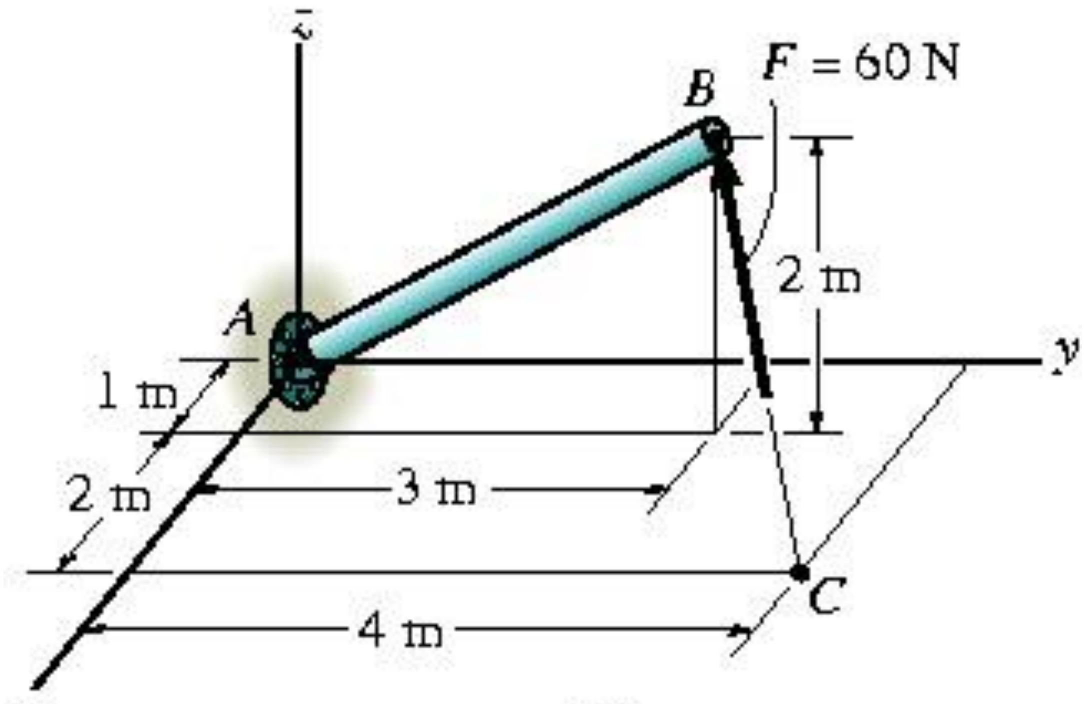
$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40 \end{vmatrix}$$

$$= \{ -5.39\,\mathbf{i} + 13.1\,\mathbf{j} + 11.4\,\mathbf{k} \}\ \text{N}\cdot\text{m}$$



Example 4.7

The pole is subjected to a 60N force that is directed from C to B. Determine the magnitude of the moment created by this force about the support at A



(a)

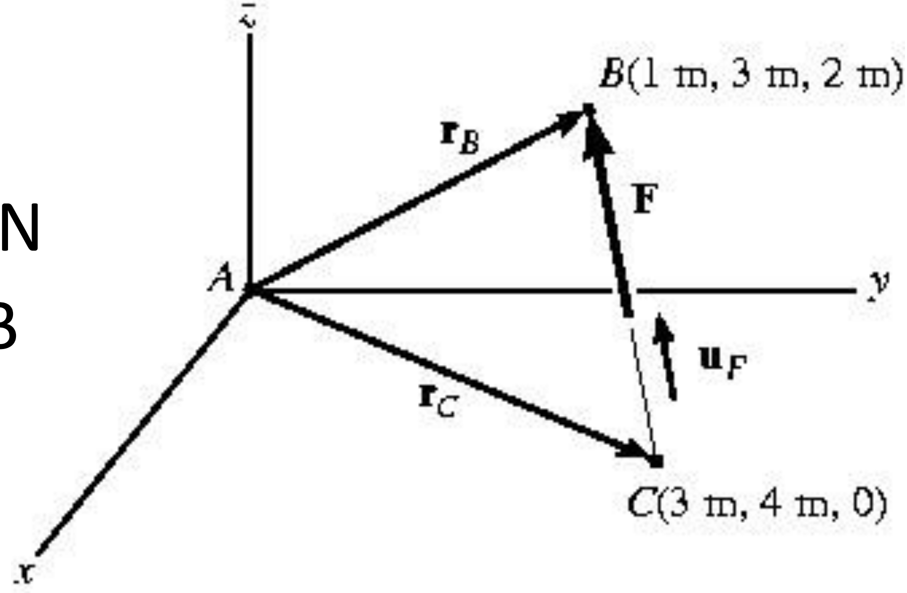
Solution example 4.7

- Either one of the two position vectors can be used for the solution, since $\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F}$ or $\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F}$
- Position vectors are represented as

$\mathbf{r}_B = \{1\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}\}$ m and

$\mathbf{r}_C = \{3\mathbf{i} + 4\mathbf{j}\}$ m

- Force **F** has magnitude 60N and is directed from C to B



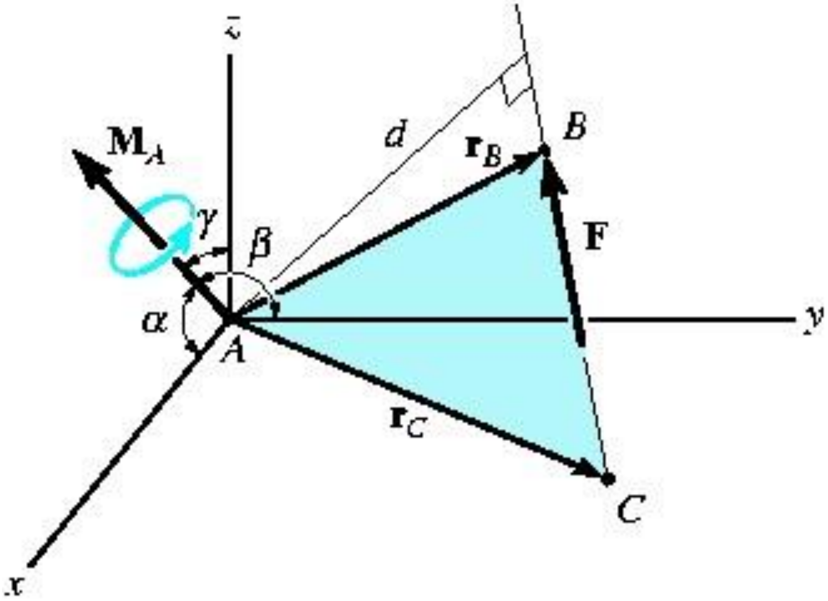
(b)

Solution example 4.7

$$\vec{F} = (60N)\vec{u}_F$$

$$= (60N)\left[\frac{(1-3)\vec{i} + 93-4)\vec{j} + 92-0)\vec{k}}{\sqrt{(-2)^2 + (-1)^2 + (2)^2}}\right]$$

$$= \left\{-40\vec{i} - 20\vec{j} + 40\vec{k}\right\}N$$



(c)

$$\vec{M}_A = \vec{r}_B X \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 2 \\ -40 & -20 & 40 \end{vmatrix}$$

$$= \left\{[3(40) - 2(-20)]\vec{i} - [1(40) - 2(-40)]\vec{j} + [1(-20) - 3(40)]\vec{k}\right\}$$

Solution example 4.7

$$\vec{M}_A = \vec{r}_C X \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & 0 \\ -40 & -20 & 40 \end{vmatrix}$$

$$= \{[4(40) - 0(-20)]\vec{i} - [3(40) - 0(-40)]\vec{j} + [3(-20) - 4(40)]\vec{k}\}$$

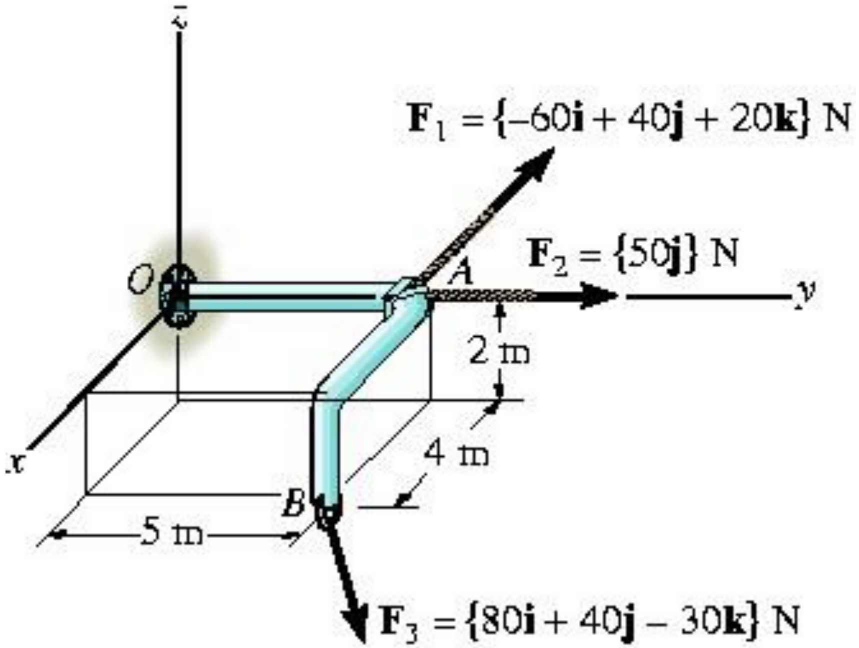
$$\vec{M}_A = \{160\vec{i} - 120\vec{j} + 100\vec{k}\} N.m$$

$$\left|\vec{M}_A\right| = \sqrt{(160)^2 + (120)^2 + (100)^2}$$

$$= 224 N.m$$

Example 4.8

Three forces act on the rod. Determine the resultant moment they create about the flange at O and determine the coordinate direction angles of the moment axis



(a)

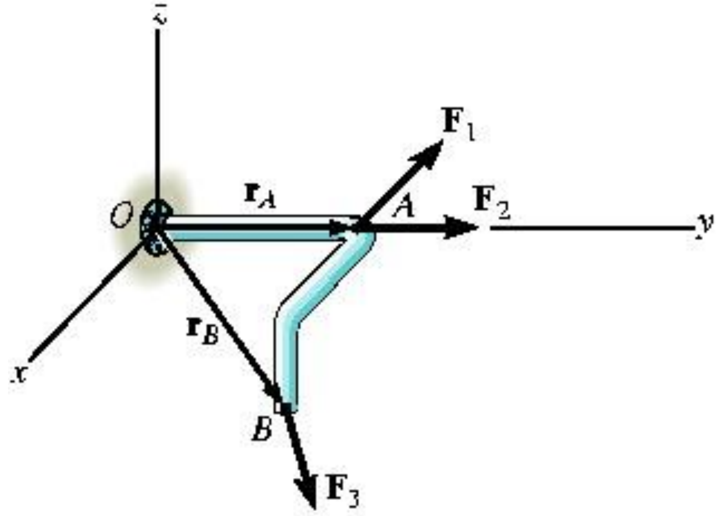
Solution example 4.8

Position vectors are directed from point O to each force

r$_A$ = {5**j**} m and

r$_B$ = {4**i** + 5**j** - 2**k**} m

For resultant moment about O,



(b)

$$\vec{M}_{Ro} = \Sigma(\vec{r} X \vec{F}) = \vec{r}_A X \vec{F}_1 + \vec{r}_B X F_2 + \vec{r}_C X \vec{F}_3$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 5 & 0 \\ 0 & 50 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix} = \{30\vec{i} - 40\vec{j} + 60\vec{k}\} N.m$$

Solution example 4.8

For **magnitude**

$$\left|\vec{M}_{Ro}\right| = \sqrt{(30)^2 + (-40)^2 + (60)^2} = 78.10 N.m$$

For unit vector defining the **direction** of moment axis,

$$\vec{u} = \frac{\vec{M}_{Ro}}{\left|\vec{M}_{Ro}\right|} = \frac{30\vec{i} - 40\vec{j} + 60\vec{k}}{78.10}$$

$$= 0.3941\vec{i} - 0.5121\vec{j} + 0.76852\vec{k}$$

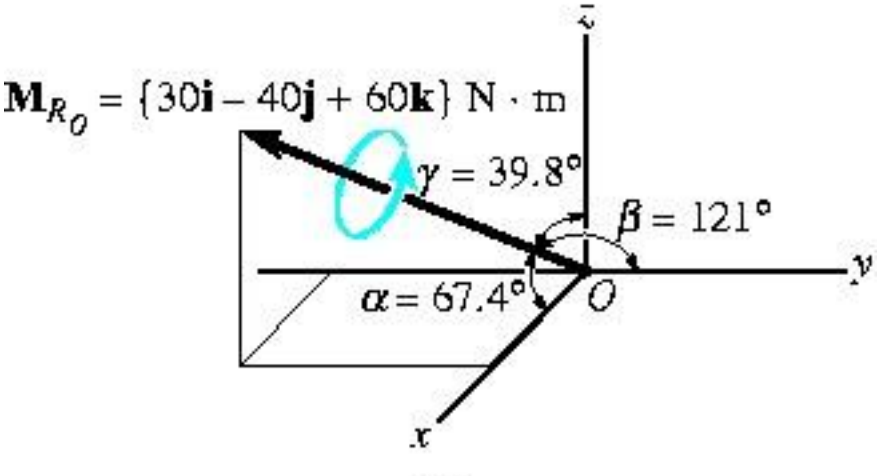
Solution example 4.8

For the coordinate angles of the moment axis,

$$\cos\alpha = 0.3841; \alpha = 67.4^\circ$$

$$\cos\beta = -0.5121; \beta = 121^\circ$$

$$\cos\gamma = 0.7682; \gamma = 39.8^\circ$$



(c)

Conclusion of The Chapter 4

- Conclusions
  - The Moment of a Force been identified
  - The Vector cross product have been implemented to solve Moment problems in Coplanar Forces Systems

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