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ENGINEERING MECHANICS
BAA1113

Chapter 3: Equilibrium of a Particle (Static)

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Chapter Description

- Aims
  - To explain the Equilibrium Equation
  - To explain the Free Body Diagram
  - To apply the Equations of Equilibrium to solve particle equilibrium problems in Coplanar Force System (2-D &3-D)

- Expected Outcomes
  - Able to solve the problems of a particle or rigid body in the mechanics applications by using Equilibrium Equation

- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14^{th} Edition

Chapter Outline

3.1 Equilibrium Equation

3.2 Free Body Diagram

3.3 Coplanar Force Systems (2-D &3-D)

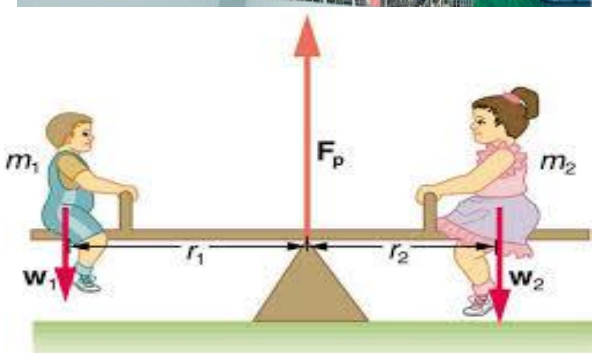
3.4 Example

3.1 Equilibrium Equation

What is Equilibrium?

- **Equilibrium** means the **forces are balanced** but not necessarily equal
- In physic, it means equal balance which the **opposing forces or tendencies neutralize each other**

How to know the body is in Equilibrium?



- A body at rest or in uniform motion (velocity) is in equilibrium

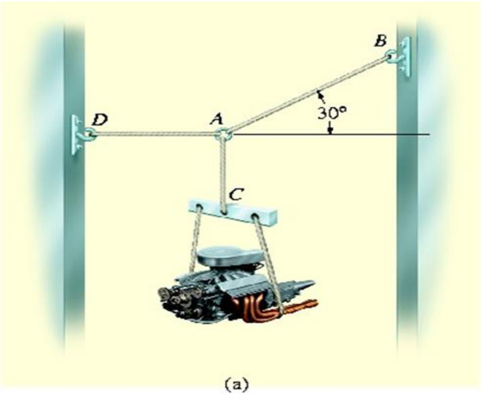
Condition for the Equilibrium of a Particle

How to know the body is in Equilibrium?

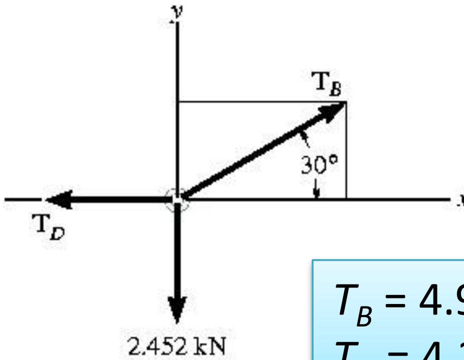
- Particle at *equilibrium* if
  - At rest
  - Moving at constant a constant velocity
- Newton's first law of motion

$$\sum \mathbf{F} = 0$$

where $\sum \mathbf{F}$ is the vector sum of all the forces acting on the particle



(a)



(b)

$T_B = 4.90\text{kN}$
$T_D = 4.25\text{kN}$

$+\rightarrow \quad \sum F_x = 0; \quad T_B\cos 30^\circ - T_D = 0$

$+\uparrow \quad \sum F_y = 0; \quad T_B\sin 30^\circ - 2.452\text{kN} = 0$

Condition for the Equilibrium of a Particle

- Newton's second law of motion

$$\sum \mathbf{F} = m\mathbf{a}$$

- When the force fulfill Newton's first law of motion,

$$m\mathbf{a} = 0$$

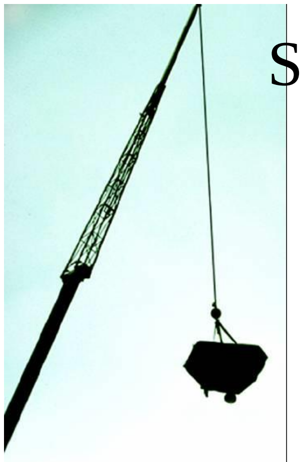
$$\mathbf{a} = 0$$

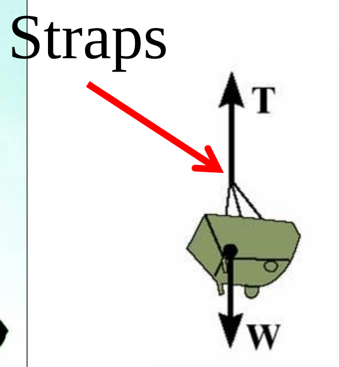
therefore, the particle is moving in constant velocity or at rest

If the Dynamic Equilibrium, the body move and continue to move

Static Equilibrium is when the body at rest

Application of Equilibrium Equation

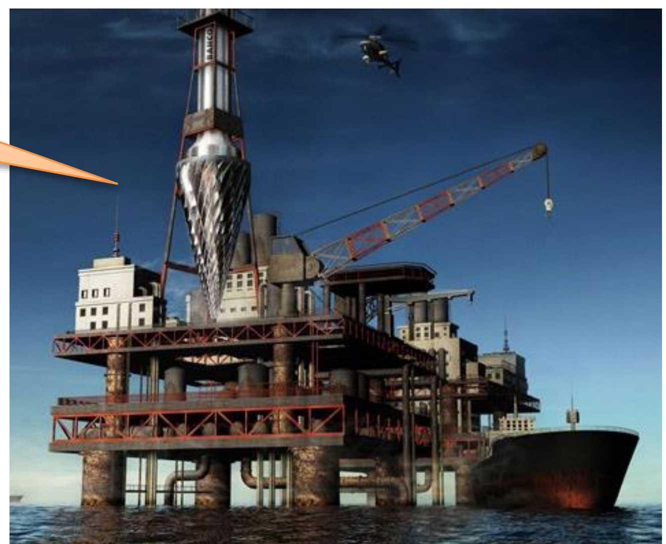




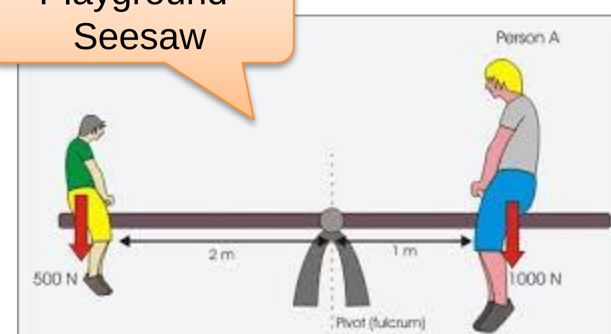
Construction-lifting the material

Day life activity-Paddle the boat

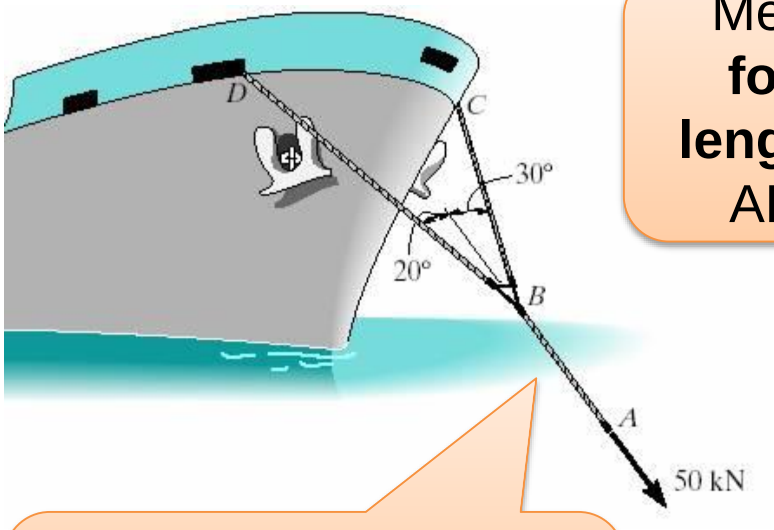
Support Oil Rig



Playground-Seesaw

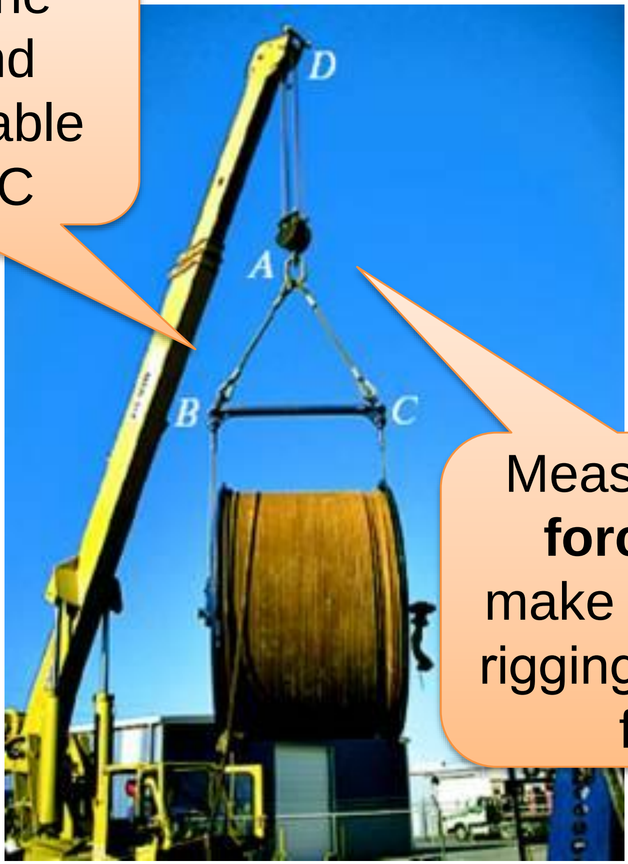


Application of Equilibrium Equation



Measure the **forces** and **length** of cable AB and AC

Measure the **forces, direction and size** of cable AB

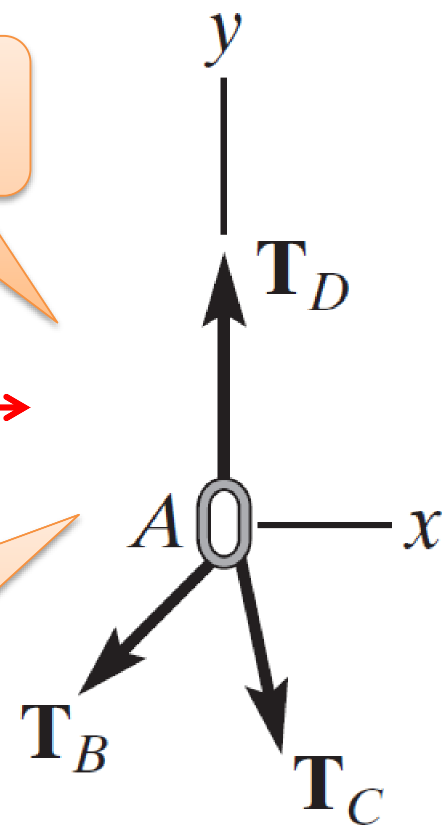


Measure the **forces** to make sure the rigging doesn't fail

3.2 Free Body Diagram (FBD)

What is FBD?

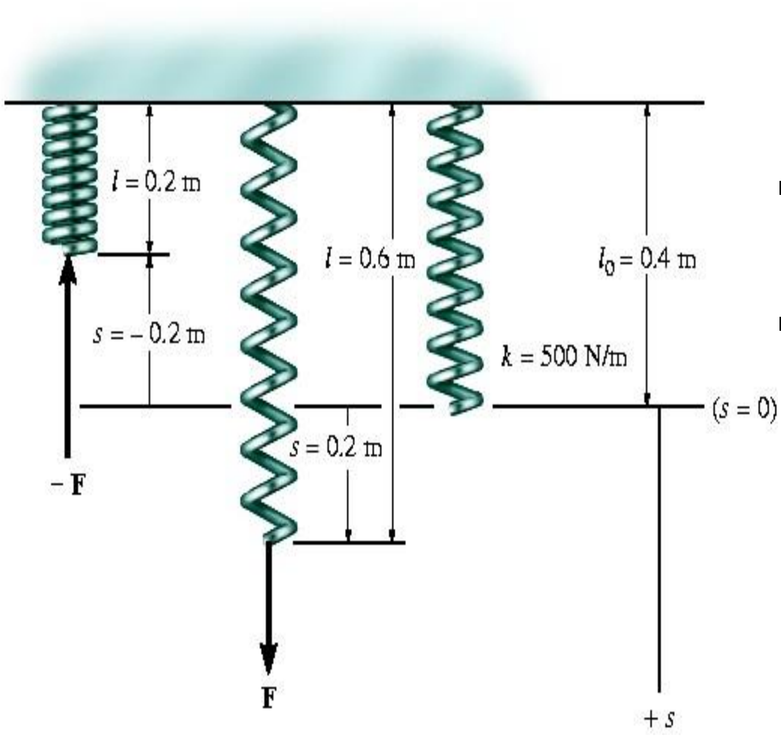
FBD is a sketch to show only the **forces** acting on **selected** body



3.2 Free Body Diagram (FBD)

- Best representation of all the unknown forces ($\sum \mathbf{F}$) which acts on a body
- A sketch showing the particle "free" from the surroundings with all the forces acting on it
- Consider two common connections in this subject – Spring
  – Cables and Pulleys

Spring



- Linear elastic spring: **change** in **length** is **directly proportional** to the **force** acting on it
- *spring constant or* ***stiffness k***: defines the ***elasticity*** of the spring
- **Magnitude of force** when spring is **elongated** or **compressed**

$$F = ks$$

- where ***s*** is determined from the difference in spring's deformed length ***l*** and its undeformed length $\boldsymbol{l_o}$

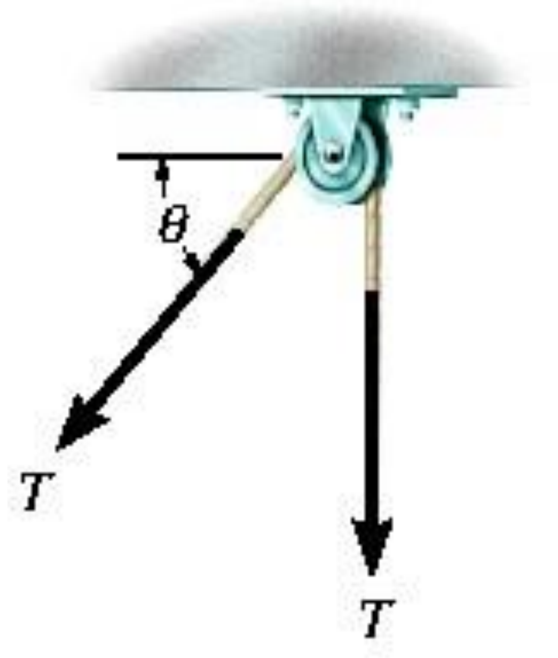
$$s = l - l_o$$

$$s = l - l_o$$

- If ***s*** is positive, **F** "pull" onto the spring
- If ***s*** is negative, **F** "push" onto the spring

Cables and Pulley

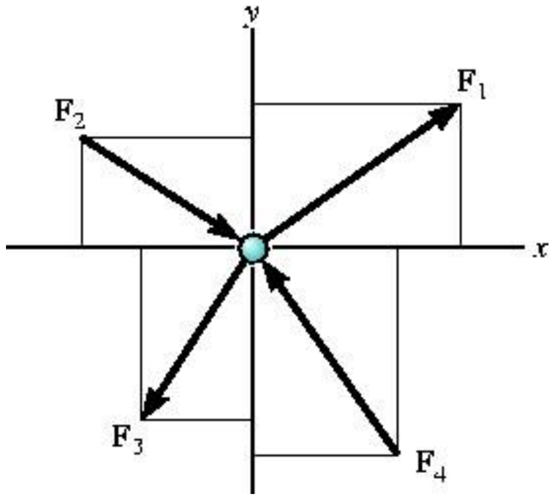
- Cables (or cords) are assumed to have negligible weight and they cannot stretch
- A cable only support tension or pulling force
- Tension always acts in the direction of the cable
- Tension force in a continuous cable must have a constant magnitude for equilibrium
- For any angle θ, the cable is subjected to a constant tension ***T*** throughout its length



Cable is in tension

With a frictionless pulley and cable $T_1 = T_2$.

3.3 Coplanar Systems 2-D



- A particle is subjected to coplanar forces in the x-y plane
- Resolve into **i** and **j** components for equilibrium

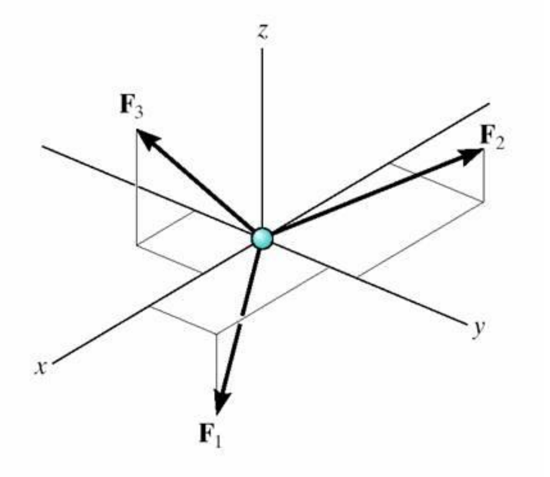
$$\sum \mathbf{F}_x = 0$$
$$\sum \mathbf{F}_y = 0$$

- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal o (zero)

Scalar Notation

- Sense of direction = an algebraic sign that corresponds to the arrowhead direction of the component along each axis
- For unknown magnitude, assume arrowhead sense of the force
- Since magnitude of the force is always positive, if the scalar is negative, the force is acting in the opposite direction

3.3 Coplanar Systems 3-D



- When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero
- (Σ ***F*** = 0)
- This equation can be written in terms of its x, y and z components.This form is written as follows

$$(\Sigma F_x)\,\boldsymbol{i} + (\Sigma F_y)\,\boldsymbol{j} + (\Sigma F_z)\,\boldsymbol{k} = 0$$

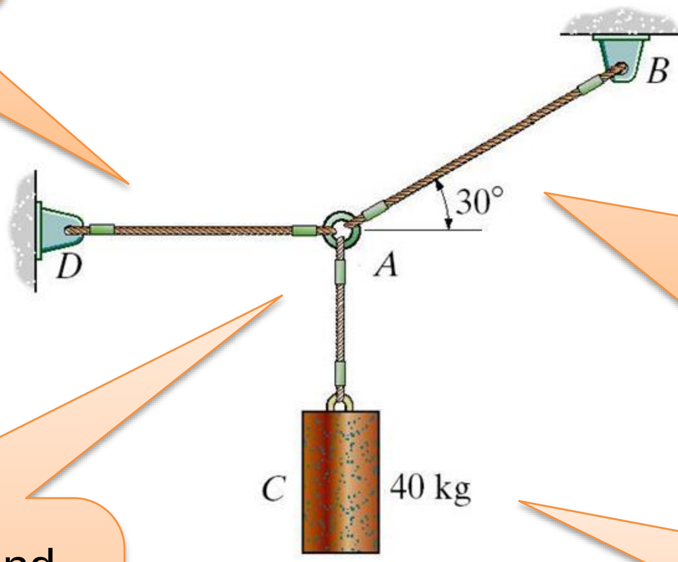
- This vector equation will be satisfied only when

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

- These equations are the three scalar equations of equilibrium. They are valid for any point in equilibrium and allow you to solve for up to three unknowns.

Step to draw FBD

Step 1: Sketch outline shape

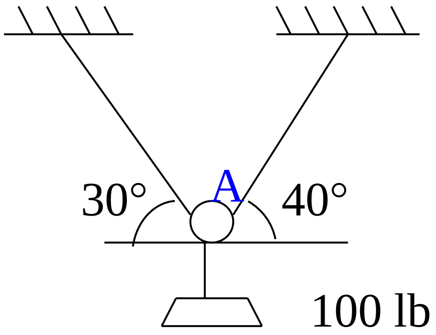


Step 2:Show all the forces that act on body and indicate the active (set the body in motion) or reactive forces (tend to resist the motion)

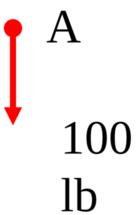
Step 3: Labeled the known forces (magnitude and direction)

Step 4: Apply EE and calculate the unknown forces(can be write in letters)

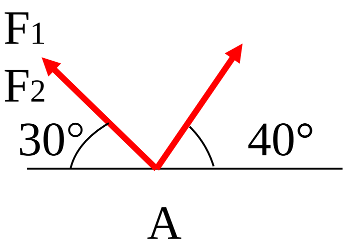
Select the correct FBD of Particle A



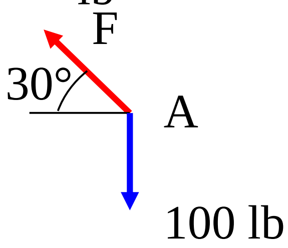
A)



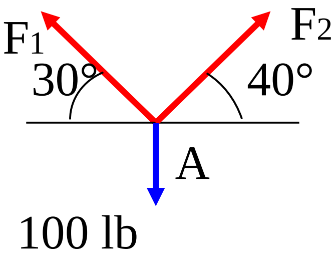
B)



C)

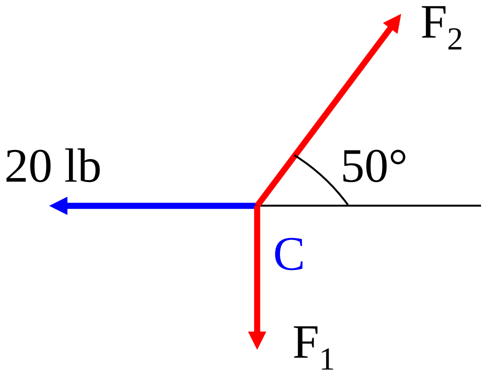


D)



FBD

Using this FBD of Point C, the sum of forces in the x-direction (ΣF_X) is ___ . Use a sign convention of $+ \rightarrow$.



A) $F_2 \sin 50° - 20 = 0$

B) $F_2 \cos 50° - 20 = 0$

C) $F_2 \sin 50° - F_1 = 0$

D) $F_2 \cos 50° + 20 = 0$

FBD

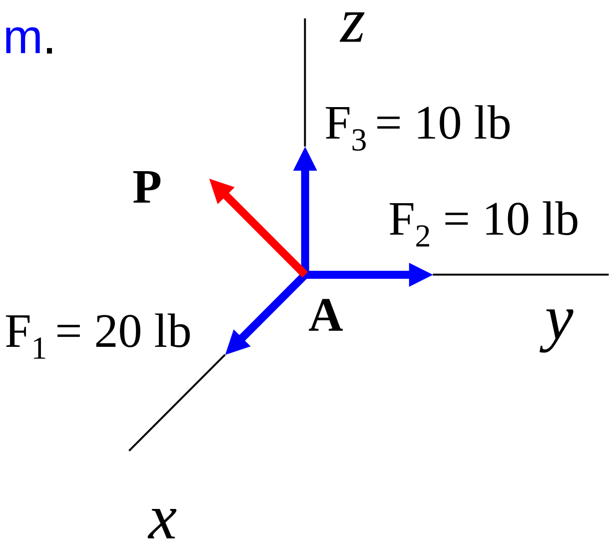
Four forces act at point A and point A is in equilibrium. Select the correct force vector ***P***.

A) {-20 ***i*** + 10 ***j*** – 10 ***k***}lb

B) {-10 ***i*** – 20 ***j*** – 10 ***k***} lb

C) {+ 20 ***i*** – 10 ***j*** – 10 ***k***}lb

D) None of the above.

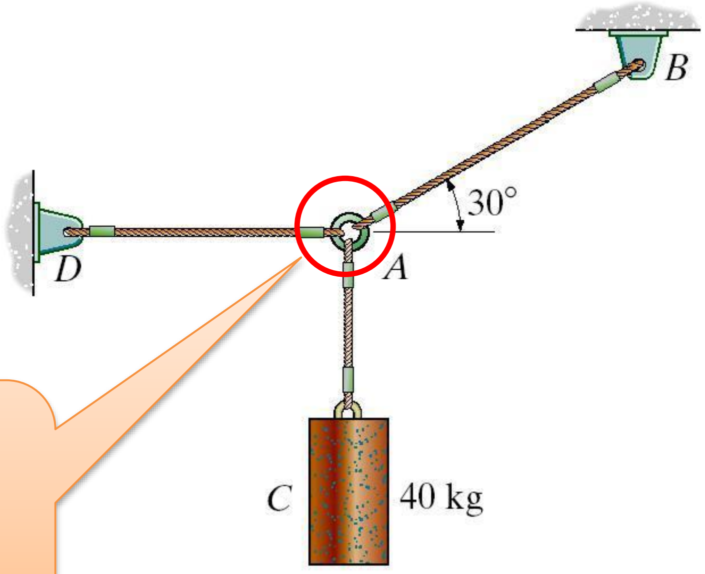


In 3-D, when you don't know the direction or the magnitude of a force, how many unknowns do you have corresponding to that force?

A) One B) Two C) Three D) Four

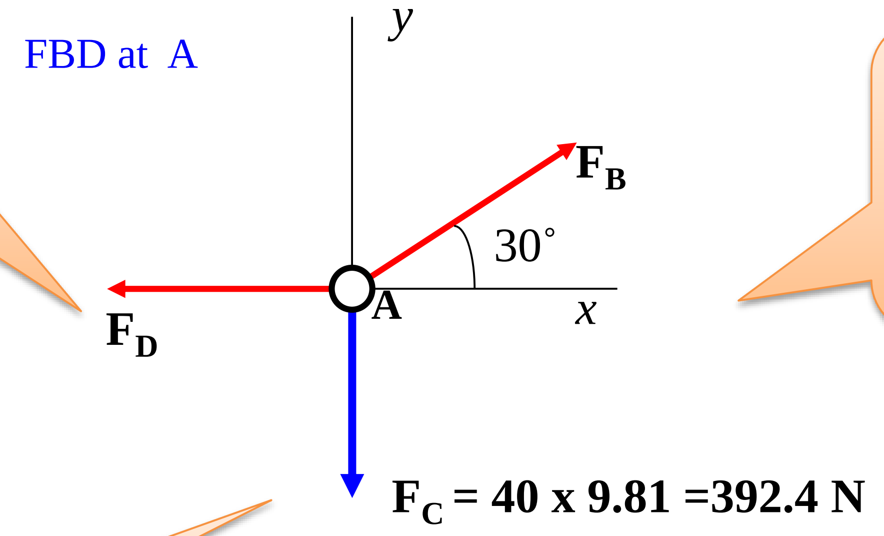
Example 3.1

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle A is also in equilibrium. Determine the tensions in the cables for a given weight of cylinder = 40kg



Step 1: FBD @A (Sketch outline shape)

Solution Example 3.1



Step 4: Apply EE and calculate the unknown forces(can be write in letters)

Step 2:Show all the forces that act on body and indicate the active (set the body in motion) or reactive forces (tend to resist the motion)

Step 3: Labeled the known forces (magnitude and direction)

Solution Example 3.1

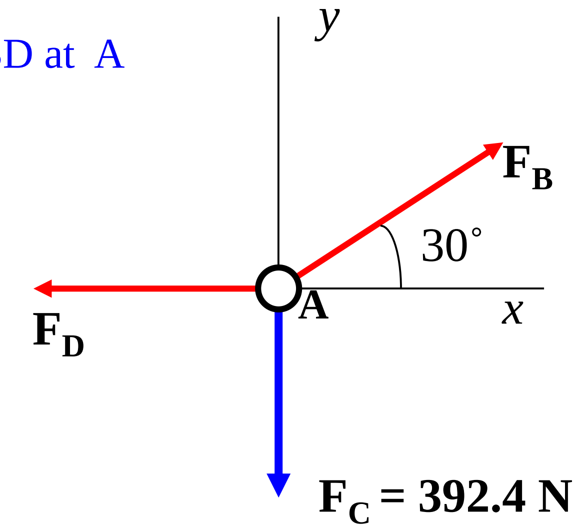
Step 4: Apply EE and calculate the unknown forces(can be write in letters)

Since particle A is in equilibrium, the net force at A is zero.

So $\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} = 0$

or $\Sigma \boldsymbol{F} = 0$

FBD at A



In general, for a particle in equilibrium,

$\Sigma \boldsymbol{F} = 0$ or $\Sigma F_x \boldsymbol{i} + \Sigma F_y \boldsymbol{j} = 0 = 0\boldsymbol{i} + 0\boldsymbol{j}$ (a vector equation)

Or, written in a scalar form,

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

- Two scalar equations of equilibrium (E-of-E)
- Used to solve for up to two unknowns

Solution Example 3.1

Write the scalar E-of-E:

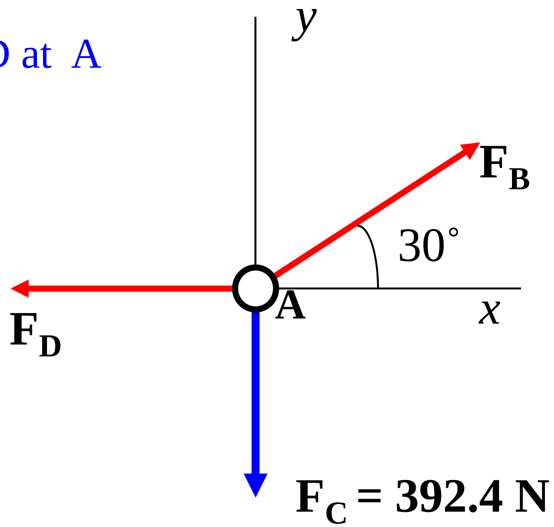
$+ \rightarrow \Sigma F_x = F_B \cos 30^o - F_D = 0$

$+ \uparrow \Sigma F_y = F_B \sin 30^o - 392.4 \text{ N} = 0$

Solving the second equation, $F_B = 785 \text{ N} \rightarrow$

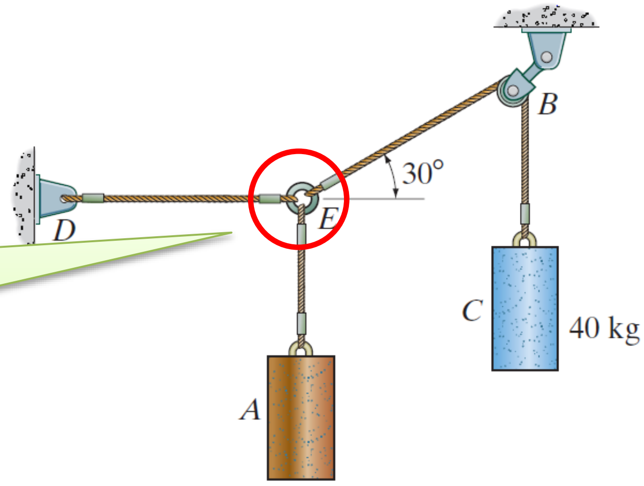
From the first equation, $F_D = 680 \text{ N} \leftarrow$

FBD at A



Example 3.2

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle E is also in equilibrium. Determine the tensions in the cables DE,EA and EB for a given weight of cylinder = 40kg



Step 1: Draw FBD @ point E

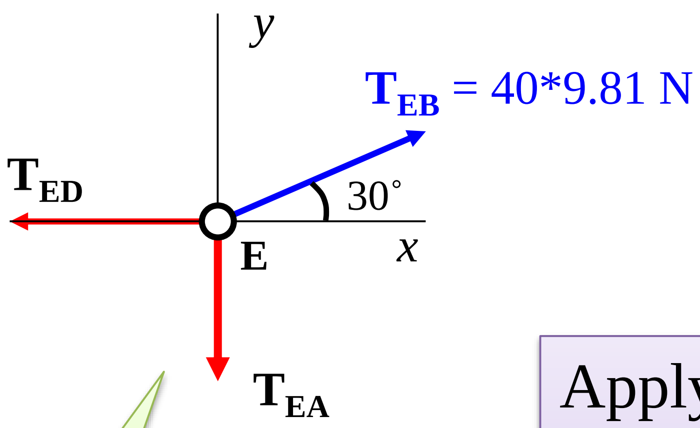
Step 2

Step 3

Solution Example 3.2

FBD at point E

Step 2



Step 3

Step 4

Applying the scalar E-of-E at E,

$$+\rightarrow \sum F_x = -T_{ED} + (40*9.81)\cos 30° = 0$$

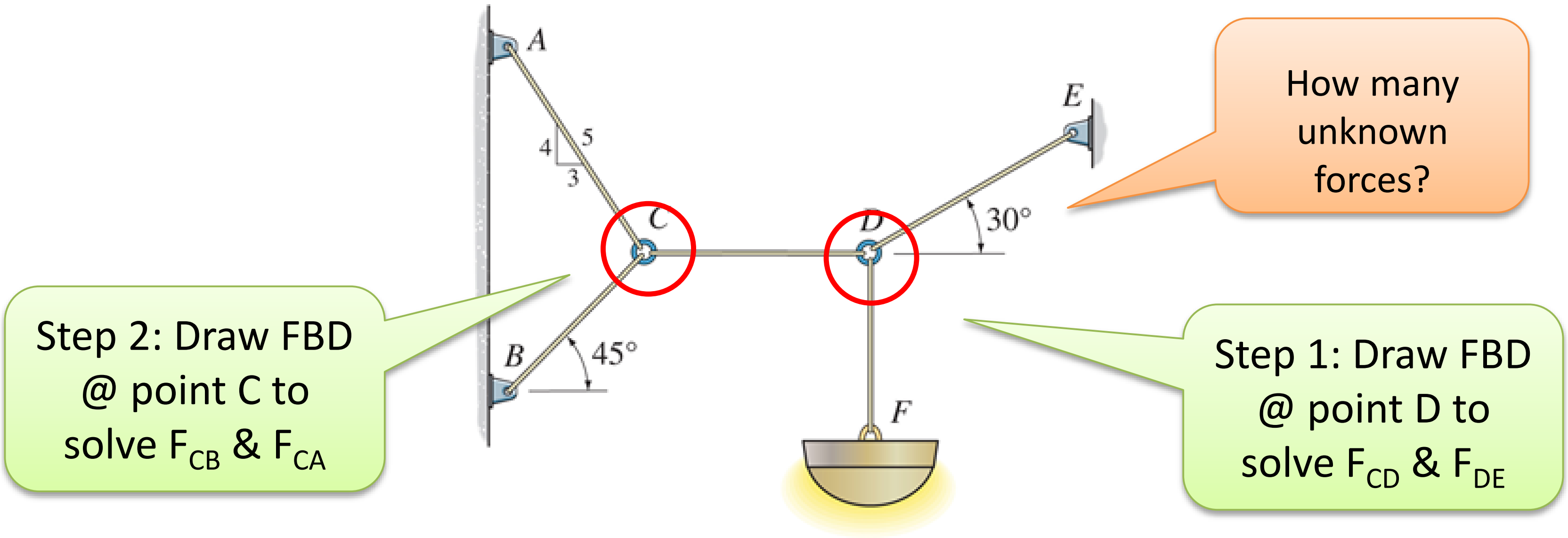
$$+\uparrow \sum F_y = (40*9.81)\sin 30° - T_{EA} = 0$$

Solving the above equations,

$\underline{T_{ED} = 340\ N} \leftarrow$ and $\underline{T_{EA} = 196\ N} \downarrow$

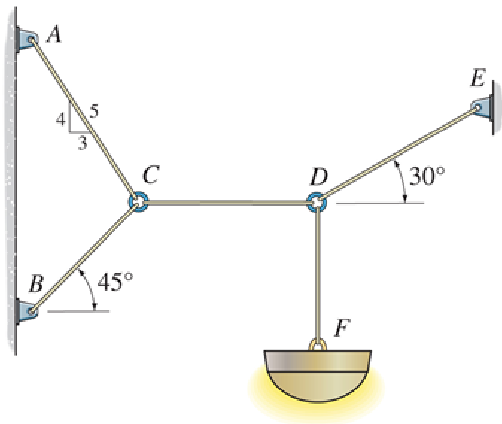
Example 3.3

This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle C and D are in equilibrium. Determine the force in each cables for a given weight of lamp = 20kg

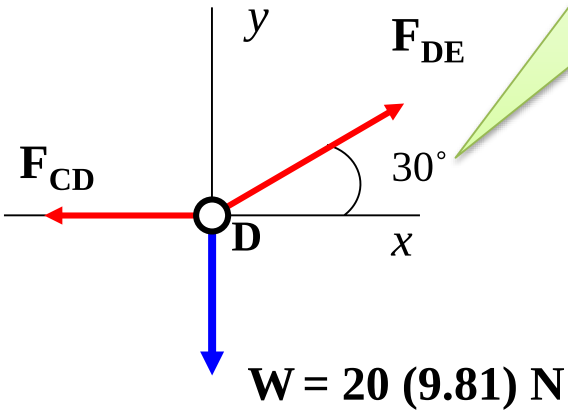


Solution Example 3.3

Step 1: Draw FBD @ point D to solve F_{CD} & F_{DE}



FBD at point D



Applying the scalar E-of-E at D,

$$+\uparrow \sum F_y = F_{DE} \sin 30^\circ - 20\,(9.81) = 0$$

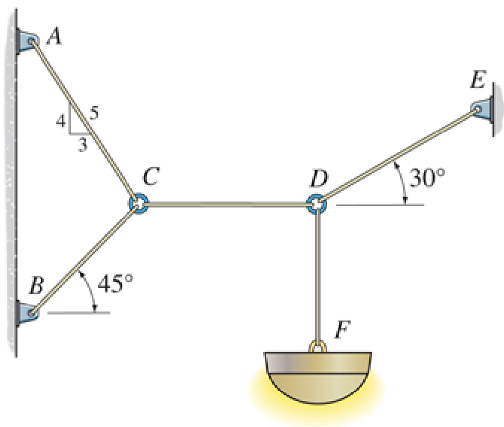
$$+\rightarrow \sum F_x = F_{DE} \cos 30^\circ - F_{CD} = 0$$

Solving the above equations,

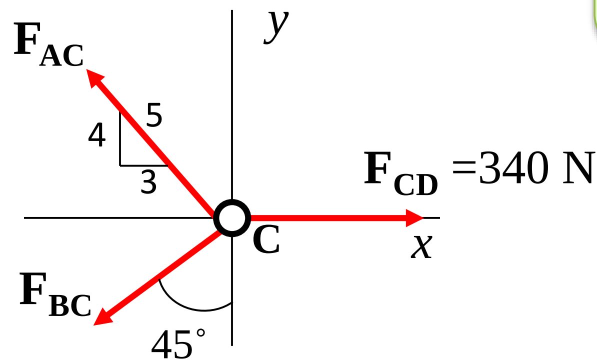
$F_{DE} = 392\ \text{N}$ and $F_{CD} = 340\ \text{N}$

Solution Example 3.3

Step 2: Draw FBD @ point C to solve F_{CB} & F_{CA}



FBD at point C



Applying the scalar E-of-E at C,

$+\rightarrow \sum F_x = 340 - F_{BC} \sin 45° - F_{AC} (3/5) = 0$

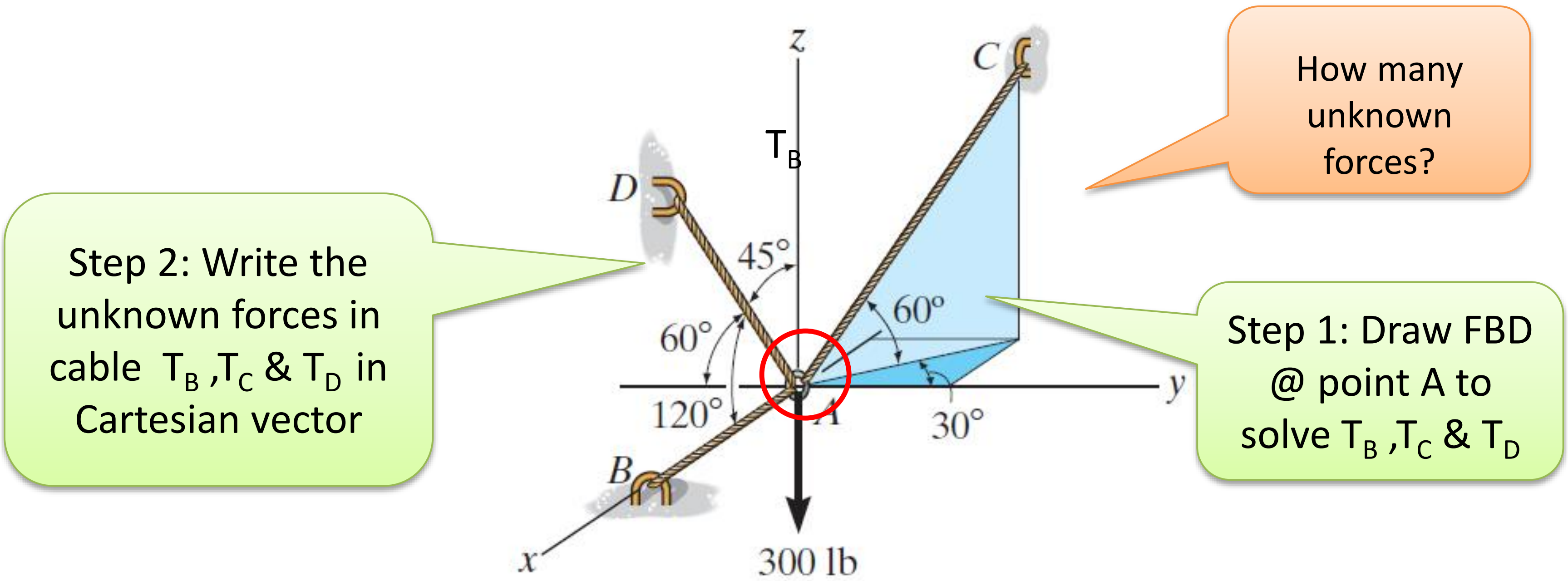
$+\uparrow \sum F_y = F_{AC} (4/5) - F_{BC} \cos 45° = 0$

Solving the above equations,

$F_{BC} = 275$ N ↙ and $F_{AC} = 243$ N ↖

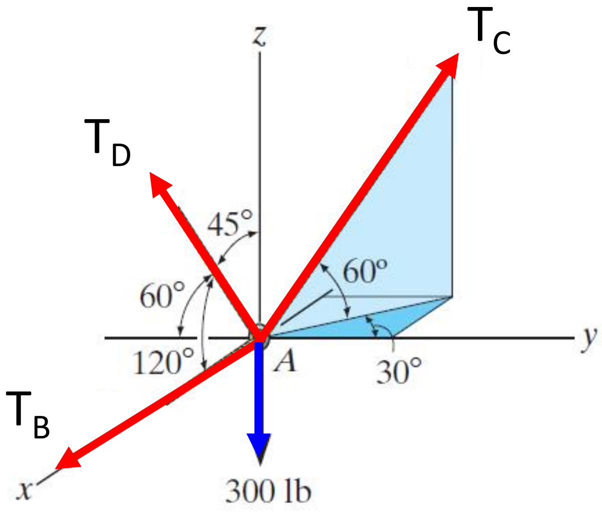
Example 3.4

This is an example of a 3-D or coplanar force system. If the whole assembly is in equilibrium, determine the tension developed in each cables



Solution Example 3.4

FBD at A



$$\boldsymbol{T_B} = T_B\ \boldsymbol{i}$$

$$\boldsymbol{T_C} = -(T_C \cos 60°) \sin 30°\ \boldsymbol{i} + (T_C \cos 60°) \cos 30°\ \boldsymbol{j} + T_C \sin 60°\ \boldsymbol{k}$$

$$\boldsymbol{T_C} = T_C\,(-0.25\ \boldsymbol{i}\ +0.433\ \boldsymbol{j}\ +0.866\ \boldsymbol{k}\ )$$

$$\boldsymbol{T_D} = T_D \cos 120°\ \boldsymbol{i} + T_D \cos 120°\ \boldsymbol{j} + T_D \cos 45°\ \boldsymbol{k}$$

$$\boldsymbol{T_D} = T_D\,(-0.5\ \boldsymbol{i} - 0.5\ \boldsymbol{j} + 0.7071\ \boldsymbol{k}\ )$$

$$\boldsymbol{W} = -300\ \boldsymbol{k}$$

Solution Example 3.4

- Applying equilibrium equations:

$\Sigma \mathbf{F}_R = 0 = T_B\,\mathbf{i}$
$+ T_C(-0.25\,\mathbf{i} + 0.433\,\mathbf{j} + 0.866\,\mathbf{k})$
$+ T_D(-0.5\,\mathbf{i} - 0.5\,\mathbf{j} + 0.7071\,\mathbf{k})$
$- 300\,\mathbf{k}$

Equating the respective ***i***, ***j***, ***k*** components to zero,

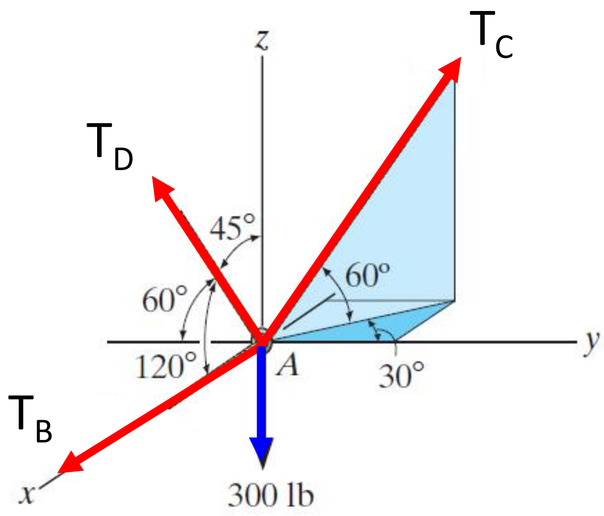
$\Sigma F_x = T_B - 0.25\,T_C - 0.5\,T_D = 0$ (1)

$\Sigma F_y = 0.433\,T_C - 0.5\,T_D = 0$ (2)

$\Sigma F_z = 0.866\,T_C + 0.7071\,T_D - 300 = 0$ (3)

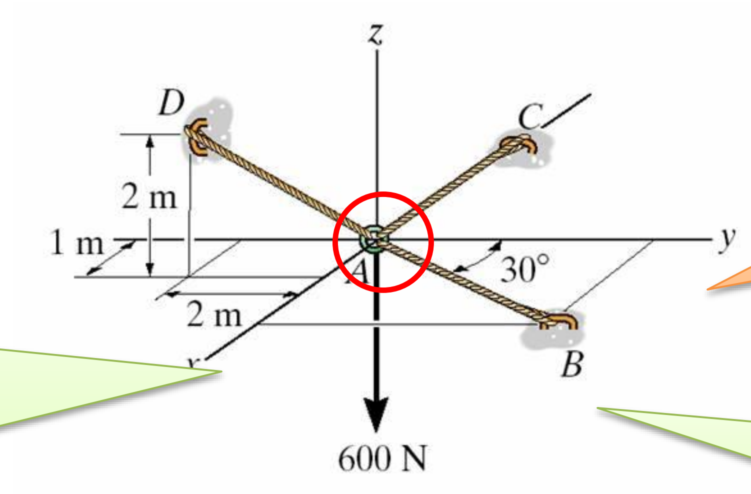
Using (2) and (3), $T_C = 203$ lb, $T_D = 176$ lb
Substituting T_C and T_D into (1), $T_B = 139$ lb

FBD at A



Example 3.5

This is an example of a 3-D or coplanar force system. If the whole assembly is in equilibrium, determine the tension developed in each cables

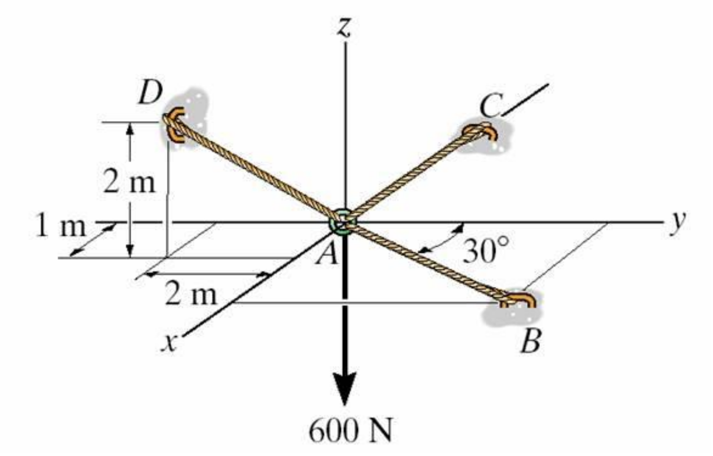


How many unknown forces?

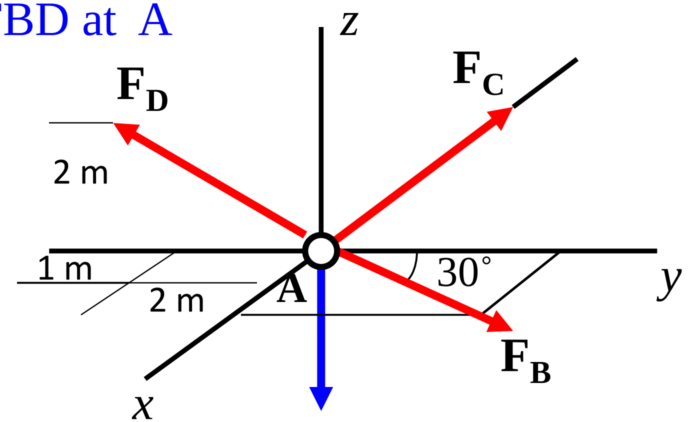
Step 2: Write the unknown forces in cable T_B ,T_C & T_D in Cartesian vector

Step 1: Draw FBD @ point A to solve T_B ,T_C & T_D

Solution Example 3.5



FBD at A



$\boldsymbol{F_B} = F_B (\sin 30° \, \boldsymbol{i} + \cos 30° \, \boldsymbol{j})$ N

$= \{0.5 F_B \, \boldsymbol{i} + 0.866 F_B \, \boldsymbol{j}\}$ N

$\boldsymbol{F_C} = -F_C \, \boldsymbol{i}$ N

$\boldsymbol{F_D} = F_D (\boldsymbol{r_{AD}} / r_{AD})$

$= F_D \{ (1 \, \boldsymbol{i} - 2 \, \boldsymbol{j} + 2 \, \boldsymbol{k}) / (1^2 + 2^2 + 2^2)^{½} \}$ N

$= \{ 0.333 F_D \, \boldsymbol{i} - 0.667 F_D \, \boldsymbol{j} + 0.667 F_D \, \boldsymbol{k} \}$ N

Solution Example 3.5

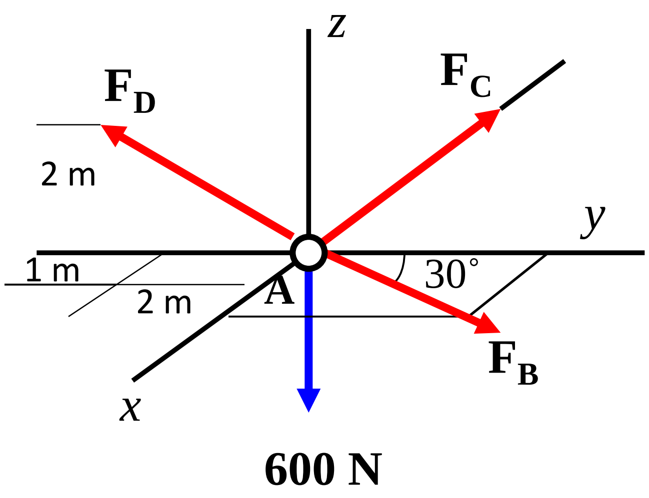
Now equate the respective ***i***, ***j***, and ***k*** components to zero

$\sum F_x = 0.5\ F_B - F_C + 0.333\ F_D = 0$

$\sum F_y = 0.866\ F_B - 0.667\ F_D = 0$

$\sum F_z = 0.667\ F_D - 600 = 0$

FBD at A



Solving the three simultaneous equations yields

F_C = 646 N (since it is positive, it is as assumed, e.g., in tension)

F_D = 900 N

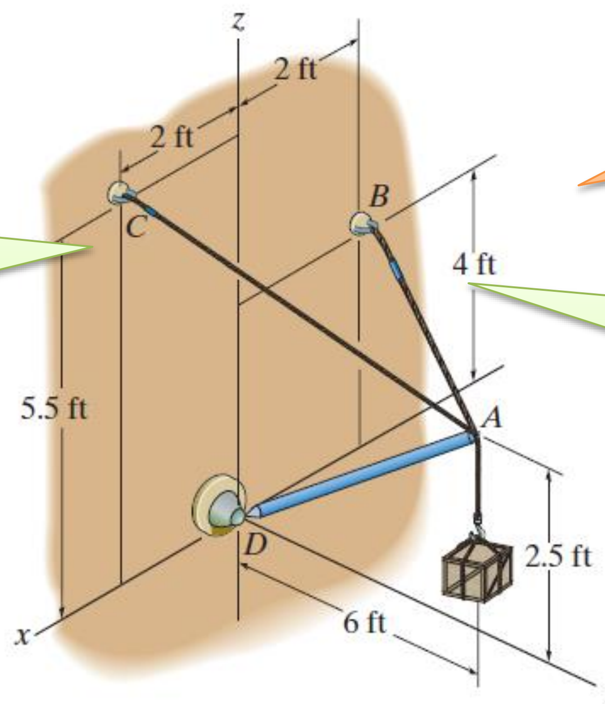
F_B = 693 N

Example 3.6

This is an example of a 3-D or coplanar force system. If the whole assembly is in equilibrium, and supported by two cables and strut AD. Given 400 lb crate, determine the magnitude of the tension developed in each cables and the force developed along strut ADs

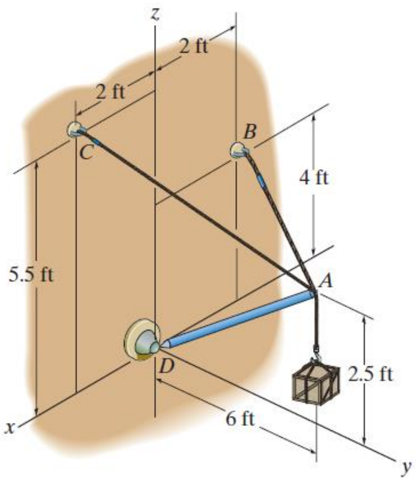
How many unknown forces?

Step 2: Write the unknown forces in cable F_B ,F_C & F_D in Cartesian vector

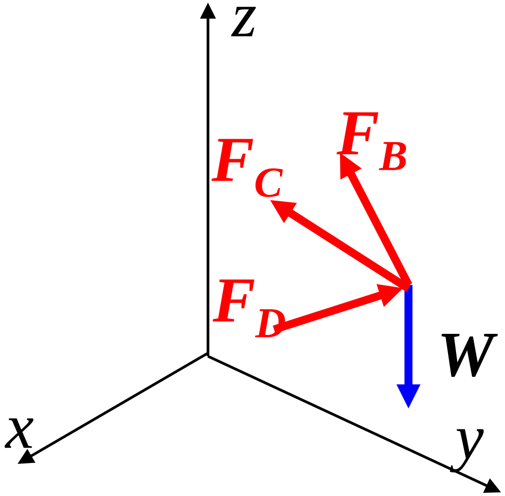


Step 1: Draw FBD @ point A to solve F_B ,F_C & F_D

Solution Example 3.6



FBD of Point A



W = weight of crate = - 400 ***k*** lb

$\boldsymbol{F_B}$ = $F_B(\boldsymbol{r_{AB}}/r_{AB})$ = $F_B\{(-4\,\boldsymbol{i} - 12\,\boldsymbol{j} + 3\,\boldsymbol{k}) / (13)\}$ lb

$\boldsymbol{F_C}$ = $F_C(\boldsymbol{r_{AC}}/r_{AC})$ = $F_C\{(2\,\boldsymbol{i} - 6\,\boldsymbol{j} + 3\,\boldsymbol{k}) / (7)\}$lb

$\boldsymbol{F_D}$ = $F_D(\boldsymbol{r_{AD}}/r_{AD})$ = $F_D\{(12\,\boldsymbol{j} + 5\,\boldsymbol{k}) / (13)\}$lb

Solution Example 3.6

The particle A is in equilibrium, hence

FB + FC + FD + W = 0

Now equate the respective i, j, k components to zero (i.e., apply the three scalar equations of equilibrium).

$\sum F_x = -(4/13)\,F_B + (2/7)\,F_C = 0$ (1)

$\sum F_y = -(12/13)\,F_B - (6/7)\,F_C + (12/13)\,F_D = 0$ (2)

$\sum F_z = (3/13)\,F_B + (3/7)\,F_C + (5/13)\,F_D - 400 = 0$ (3)

Solving the three simultaneous equations gives the forces

FB = 274 lb

FC = 295 lb

FD = 547 lb

Conclusion of The Chapter 3

- Conclusions
  - The Equilibrium and FBD have been identified
  - The Equilibrium Equation have been implemented to solve a particle problems in Coplanar Forces Systems

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