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ENGINEERING MECHANICS BAA1113

Chapter 2: Force Vectors (Static)

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Chapter Description

- Aims
  - To review the Parallelogram Law and Trigonometry
  - To explain the Force Vectors
  - To explain the Vectors Operations (Parlaw & Cartesian)
  - To express force and position in Cartesian Vectors

- Expected Outcomes
  - Able to solve the problems of force vectors in the mechanics applications by using Cartesian Coordinate System

- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14^{th} Edition

Chapter Outline

2.1 Scalars and Vectors – part I

2.2 Vectors Operations – part I

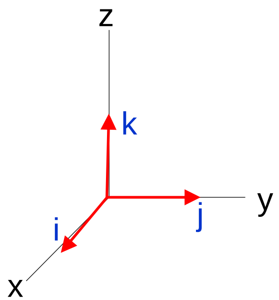
2.3 Vectors Addition of Forces – part I

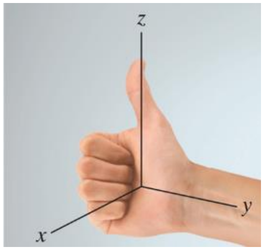
2.4 **Cartesian Vectors – part II**

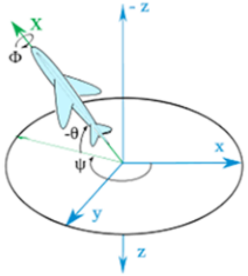
2.5 Force and Position Vectors – part III

2.4 Cartesian Vector

What is Cartesian Vector?



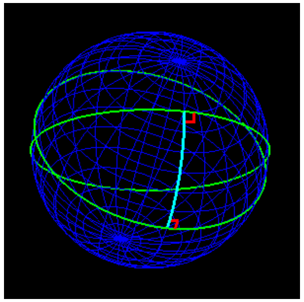




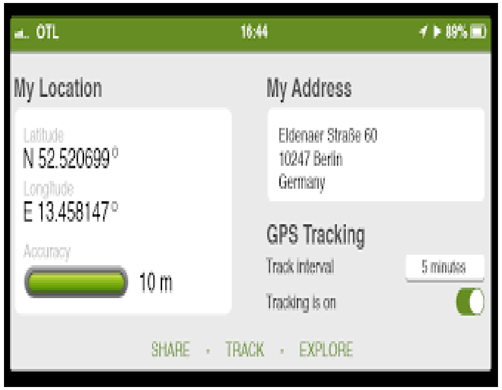
- It is a **coordinate system**
- Use to describe **position**
- Position can be defined by its **coordinate axis**
- It is a **unit vector** u_A = A / A
- Its magnitude is **1** and **dimensionless**
- It is denoted as **i,j,k**
- **i** is a unit vector pointing in the **x direction**
- **j** is a unit vector pointing in the **y direction**
- **k** is a unit vector pointing in the **z direction**
- **+ve** direction based on **right handed**
- It is important in **air transport**
- **Air Traffic controller** or **pilots** must know the **location** of every aircraft in the sky
- Without the coordinate system, the position or location of aircraft is **difficult** to know and may lead to **aircraft crashes**

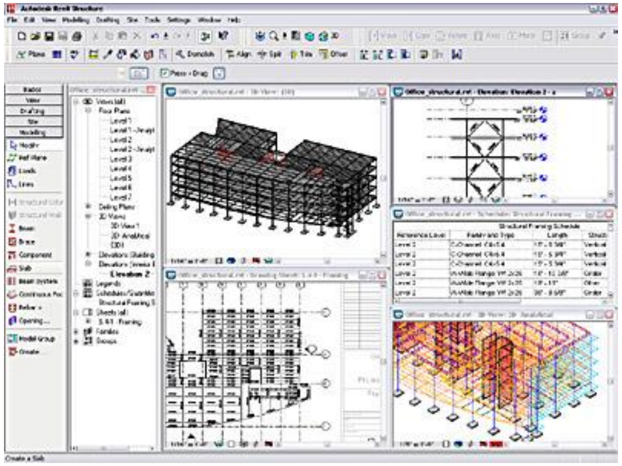
Source:https://en.wikipedia.org/wiki/Axes_conventions

Application of Cartesian Vector



- Military Service
- Position of any body in the real world

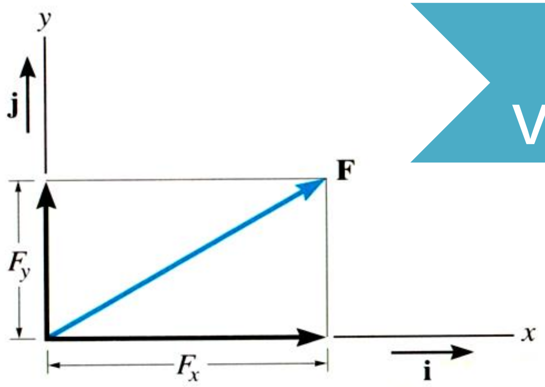




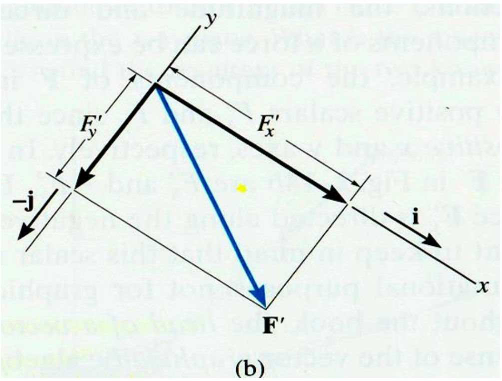
- Location / Geographic /Latitude/longitude
- Mapping Project

Application of Cartesian Vector

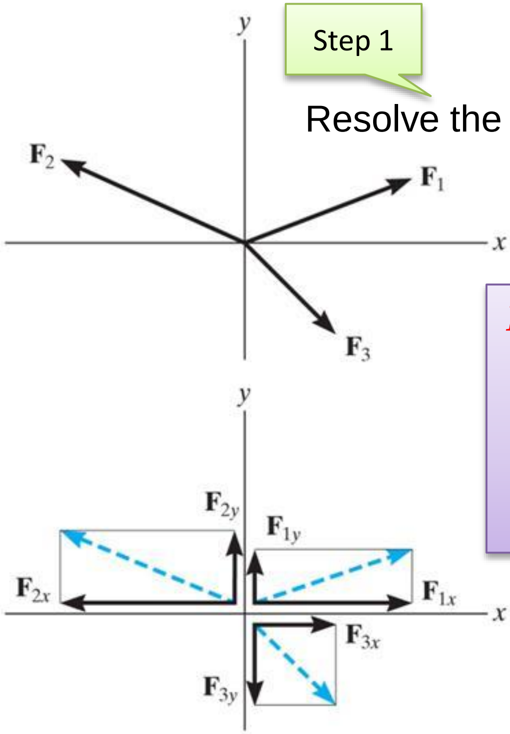
2-D vector → Resolve vector into Components → Addition vector



$$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j}$$



$$\boldsymbol{F'} = F'_x\,\boldsymbol{i} + (-\,F'_y)\,\boldsymbol{j}$$



Step 1

Resolve the vectors into X and Y components

Step 2

Then add them into respective components

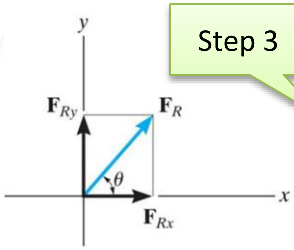
$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$$

$$= F_{1x}\,\boldsymbol{i} + F_{1y}\,\boldsymbol{j} - F_{2x}\,\boldsymbol{i} + F_{2y}\,\boldsymbol{j} + F_{3x}\,\boldsymbol{i} - F_{3y}\,\boldsymbol{j}$$

$$= (F_{1x} - F_{2x} + F_{3x})\,\boldsymbol{i} + (F_{1y} + F_{2y} - F_{3y})\,\boldsymbol{j}$$

$$= (F_{Rx})\,\boldsymbol{i} + (F_{Ry})\,\boldsymbol{j}$$

Step 3



$$F_R = \sqrt{F_{Rx}^{\,2} + F_{Ry}^{\,2}}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

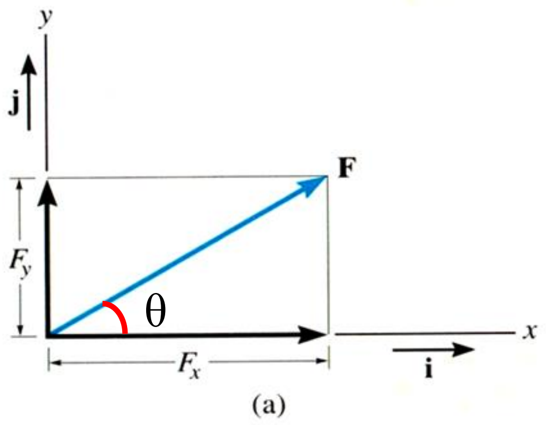
Application of Cartesian Vector

How to resolve into components Vector?

$$\boldsymbol{F} = F_x \boldsymbol{i} + F_y \boldsymbol{j}$$

$$|F_X| = F_X = F \cos\theta$$

$$|F_Y| = F_Y = F \sin\theta$$



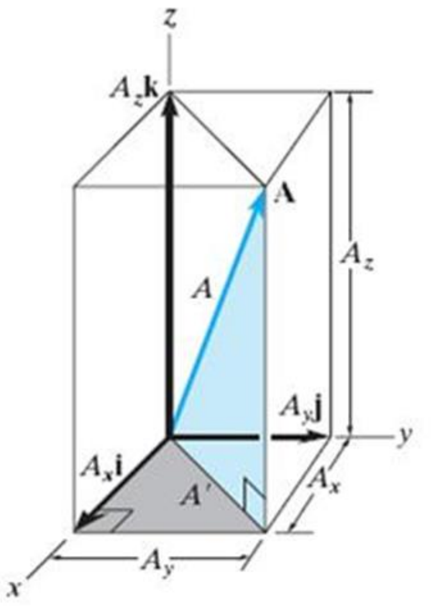
The magnitude of ***F***

$$F = \sqrt{F_X^2 + F_Y^2}$$

The direction of ***F***

$$\theta = \tan^{-1}\left|\frac{F_Y}{F_X}\right|$$

Application of Cartesian Vector



3-D vector → Resolve vector into components → Addition vector

The vector $\boldsymbol{A}$ can be resolved as

$$\boldsymbol{A} = A_X\,\boldsymbol{i} + A_Y\,\boldsymbol{j} + A_Z\,\boldsymbol{k}$$

Given

$$\boldsymbol{A} = A_X\,\boldsymbol{i} + A_Y\,\boldsymbol{j} + A_Z\,\boldsymbol{k}$$
$$\boldsymbol{B} = B_X\,\boldsymbol{i} + B_Y\,\boldsymbol{j} + B_Z\,\boldsymbol{k}$$

If $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{C}$

Sum of the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ can obtained vector $\boldsymbol{C}$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

Once the vectors are present in Cartesian term, it easy to add or subtract

$$\boldsymbol{C} = (A_X\,\boldsymbol{i} + A_Y\,\boldsymbol{j} + A_Z\,\boldsymbol{k}) + (B_X\,\boldsymbol{i} + B_Y\,\boldsymbol{j} + B_Z\,\boldsymbol{k})$$
$$= (A_X + B_X)\,\boldsymbol{i} + (A_Y + B_Y)\,\boldsymbol{j} + (A_Z + B_Z)\,\boldsymbol{k}$$
$$= C_X\,\boldsymbol{i} + C_Y\,\boldsymbol{j} + C_Z\,\boldsymbol{k}$$

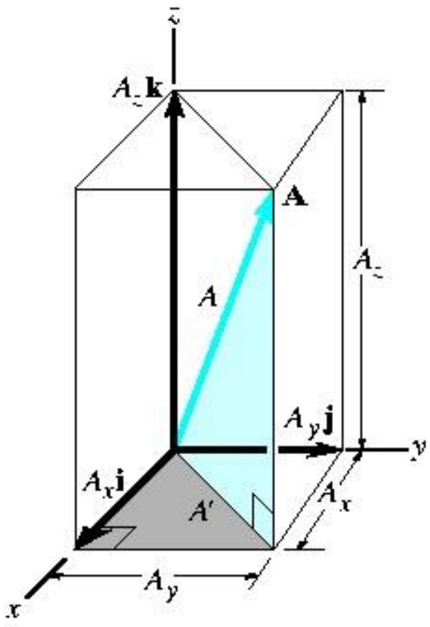
Application of Cartesian Vector

How to determine magnitude and direction angle in 3-D vector?

It should be noted that in 3-D vector information is given as:

- **Magnitude** and the **coordinate direction angles**, or
- **Magnitude** and **projection angles**

- Magnitude of a Cartesian Vector ***A*** in the x-y plane is A′



- From the colored triangle,

$$A = \sqrt{A'^2 + A_z^2}$$

- From the shaded triangle,

$$A' = \sqrt{A_x^2 + A_y^2}$$

- The magnitude of the position vector ***A***

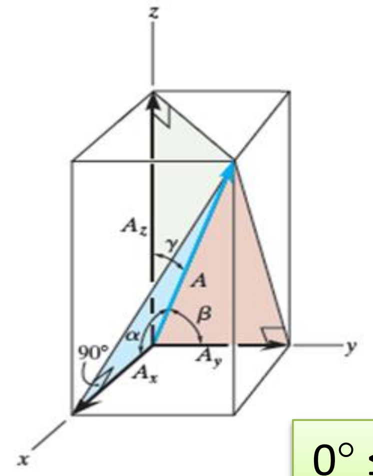
-Combining the equations gives magnitude of **A**

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Application of Cartesian Vector

- The direction or orientation of vector **A** is defined by the angles α, β, and γ
- These angles are **measured between the vector** and the positive X, Y and Z axes, respectively. Their range of values are from 0° to 180°
- Using trigonometry, "direction cosines" are found using

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$



$0° \le \alpha, \beta \text{ and } \gamma \le 180°$

- These angles are not independent. Its must satisfy the following equation

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

- It can be derived from the **coordinate direction angles** and the **unit vector**
- Unit vector of any position vector is

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

$$\boldsymbol{u}_A = \cos\alpha\, \boldsymbol{i} + \cos\beta\, \boldsymbol{j} + \cos\gamma\, \boldsymbol{k}$$

Application of Cartesian Vector

- $\mathbf{u}_A$ can also be expressed as

  $\mathbf{u}_A = \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$

How to determine direction angle of Cartesian Vector?

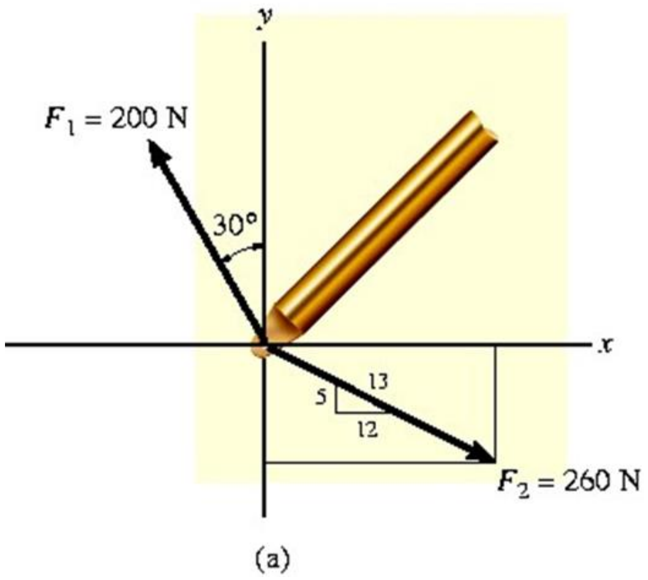
- $\mathbf{A}$ expressed in Cartesian vector form:

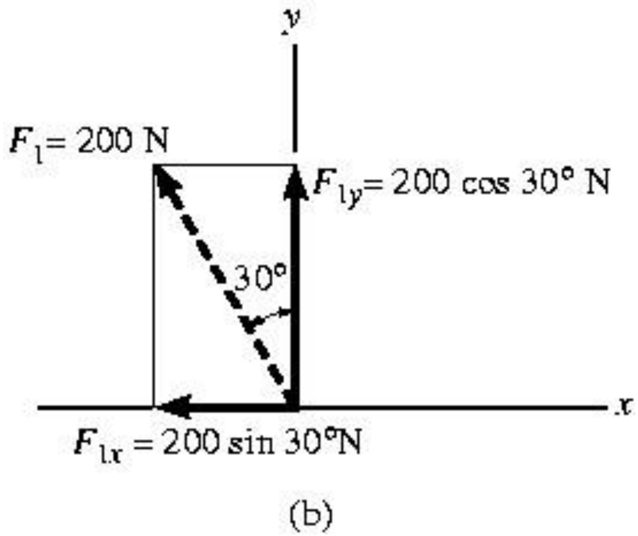
$$\mathbf{A} = A\mathbf{u}_A$$
$$= A\cos\alpha\mathbf{i} + A\cos\beta\mathbf{j} + A\cos\gamma\mathbf{k}$$
$$= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

Example 2.5

Determine the X and Y components of F_1 and F_2 in Cartesian vectors





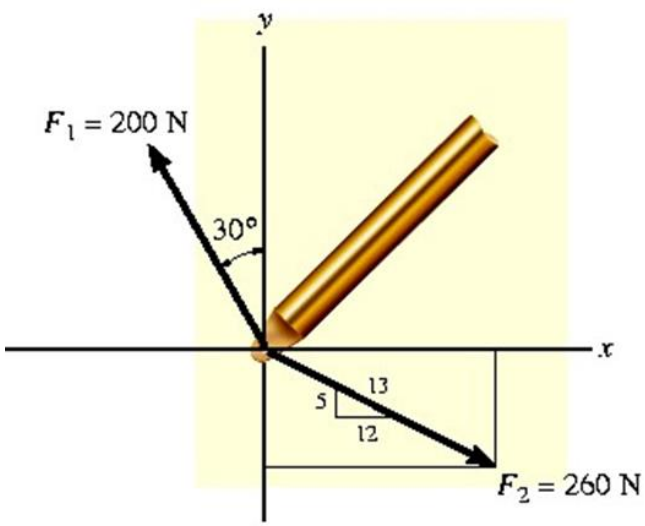
$$F_{1x} = -200 \sin 30^{\circ} N = -100 N = 100 N \leftarrow$$

$$F_{1y} = 200 \cos 30^{\circ} N = 173 N = 173 N \uparrow$$

Cartesian Vector Notation: $\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\}$N

Example 2.5

Determine the X and Y components of F_1 and F_2 in Cartesian vectors



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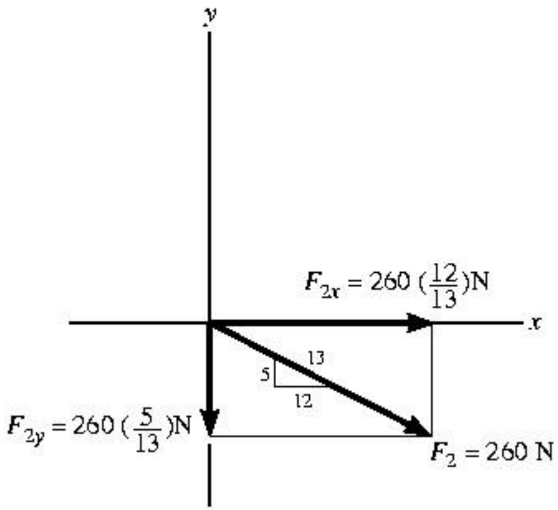


(c)

$$\frac{F_{2x}}{260N} = \frac{12}{13}$$

$$F_{2x} = 260N\left(\frac{12}{13}\right) = 240N \rightarrow \qquad F_{2y} = 260N\left(\frac{5}{13}\right) = 100N \downarrow$$

Cartesian Vector Notation: $\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\}$N

Example 2.6

Determine the magnitude and direction of the resultant force

Step 1

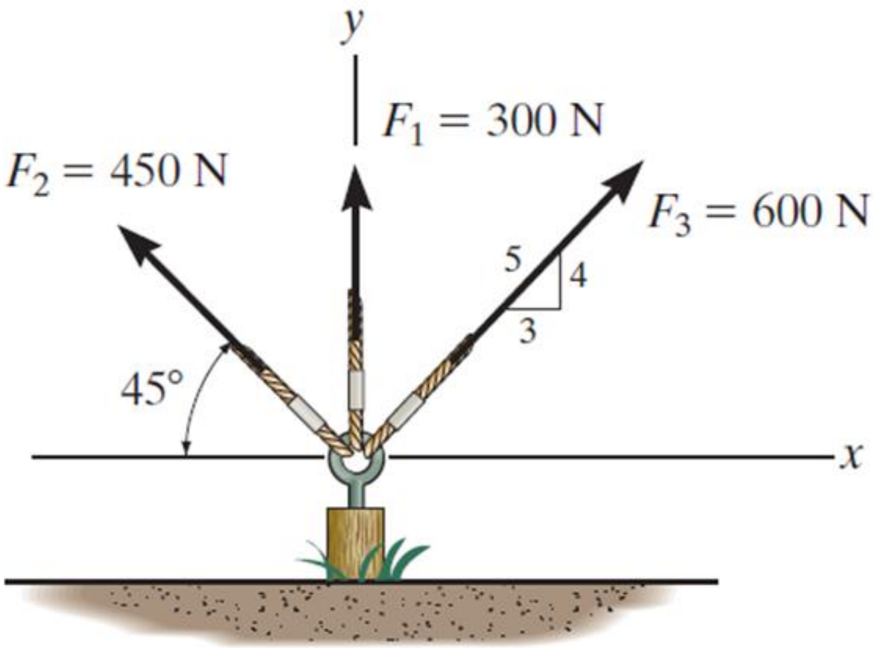
Resolve the forces into components x and y

Step 2

Then add the respective components to get the resultant forces

Step 3

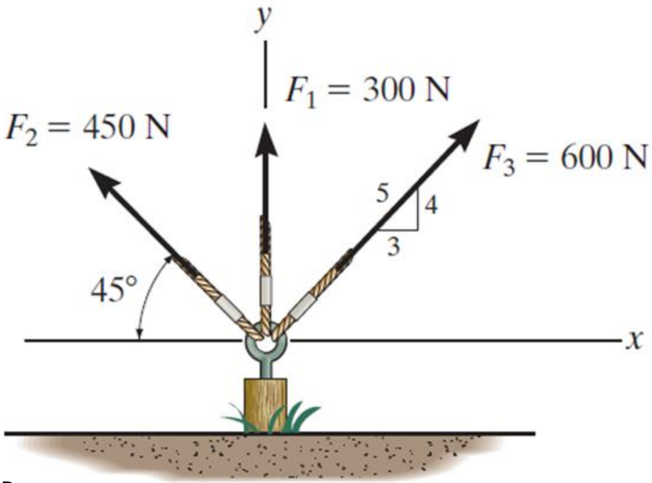
Calculate the magnitude and direction from the resultant forces



Solution Example 2.6

Step 1

Resolve the forces into components x and y



$$\boldsymbol{F_1} = \{0\ \boldsymbol{i} + 300\ \boldsymbol{j}\ \}\ \text{N}$$

$$\boldsymbol{F_2} = \{-\ 450 \cos(45°)\ \boldsymbol{i} + 450 \sin(45°)\ \boldsymbol{j}\ \}\ \text{N}$$

$$= \{-\ 318.2\ \boldsymbol{i} + 318.2\ \boldsymbol{j}\ \}\ \text{N}$$

$$\boldsymbol{F_3} = \{\ (3/5)\ 600\ \boldsymbol{i} + (4/5)\ 600\ \boldsymbol{j}\ \}\ \text{N}$$

$$= \{\ 360\ \boldsymbol{i} + 480\ \boldsymbol{j}\ \}\ \text{N}$$

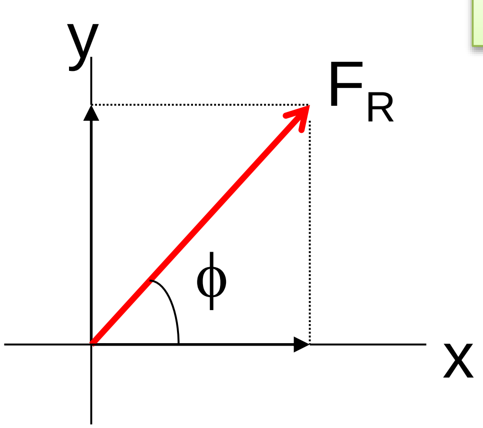
Solution Example 2.6

Step 2

- Summing up all the ***i*** and ***j*** components respectively:

$\boldsymbol{F_R}$ = { (0 – 318.2 + 360) ***i*** + (300 + 318.2 + 480) ***j*** } N

= { 41.80 ***i*** + 1098 ***j*** } N



Step 3

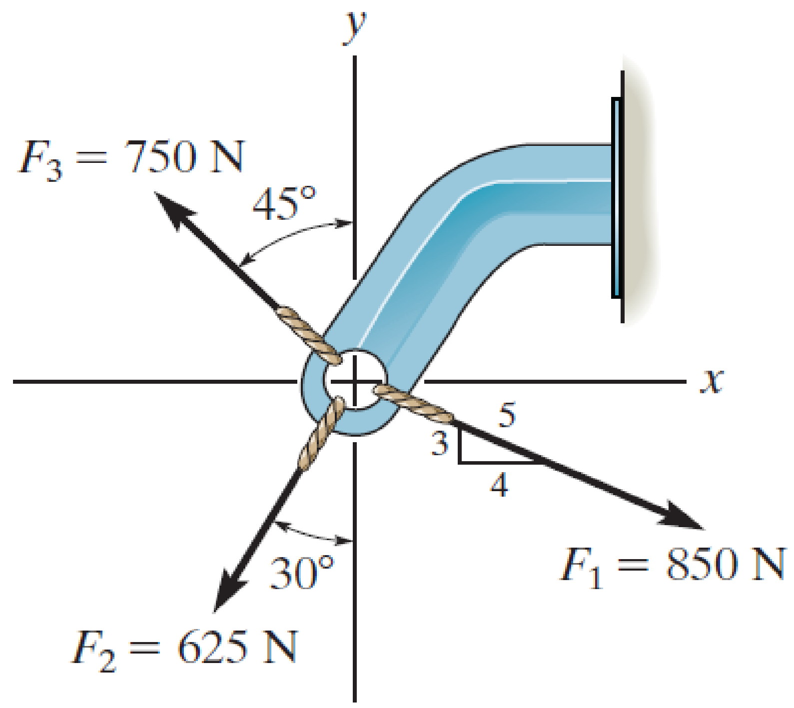
- Magnitude and direction:

$F_R = ((41.80)^2 + (1098)^2)^{1/2}$ = <u>1099 N</u>

$\phi = \tan^{-1}(1098/41.80)$ = <u>87.8°</u>

Example 2.7

Determine the magnitude and direction of the resultant force



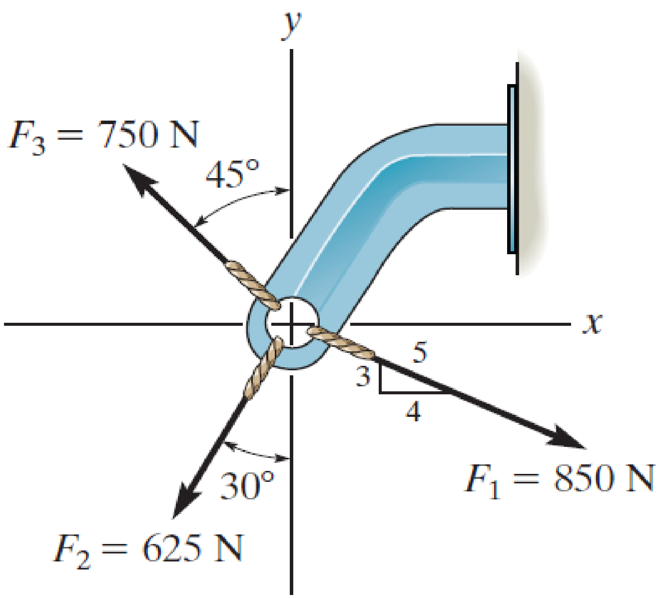
Solution Example 2.7

Step 1

Resolve the forces into components

$$\boldsymbol{F_3} = \{-750 \sin(45°)\, \boldsymbol{i} + 750 \cos(45°)\, \boldsymbol{j}\}\ \text{N}$$

$$\{-530.3\ \boldsymbol{i} + 530.3\ \boldsymbol{j}\}\ \text{N}$$



$$\boldsymbol{F_2} = \{-625 \sin(30°)\, \boldsymbol{i} - 625 \cos(30°)\, \boldsymbol{j}\}\ \text{N}$$

$$= \{-312.5\ \boldsymbol{i} - 541.3\ \boldsymbol{j}\}\ \text{N}$$

$$\boldsymbol{F_1} = \{850\,(4/5)\, \boldsymbol{i} - 850\,(3/5)\, \boldsymbol{j}\}\ \text{N}$$

$$= \{680\ \boldsymbol{i} - 510\ \boldsymbol{j}\}\ \text{N}$$

Solution Example 2.7

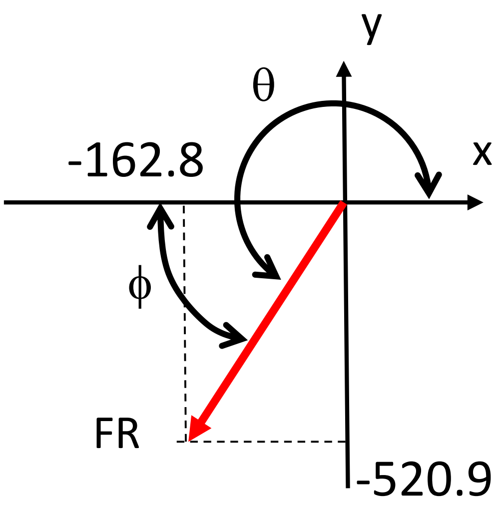
Step 2

- Summing all the ***i*** and ***j*** components, respectively:

$$F_R = \{ (680 - 312.5 - 530.3)\,\mathbf{i} + (-510 - 541.3 + 530.3)\,\mathbf{j} \}\text{N}$$

$$= \{-162.8\,\mathbf{i} - 520.9\,\mathbf{j}\}\ \text{N}$$

Step 3



- Magnitude and direction (angle):

$$F_R = ((-162.8)^2 + (-520.9)^2)^{½} = \underline{546\ \text{N}}$$

$$\phi = \tan^{-1}(520.9 / 162.8) = \underline{72.6°}$$

From the positive x-axis, $\theta = 253°$

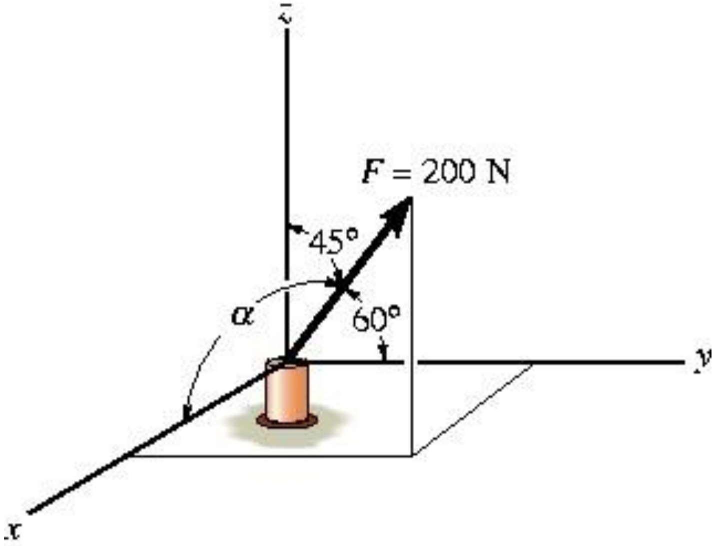
Example 2.8

Determine the force **F** as Cartesian Vector

Step 1

Resolve the forces into components

F = $F\cos\alpha\,\mathbf{i} + F\cos\beta\,\mathbf{j} + F\cos\gamma\,\mathbf{k}$



- Two angles are specified, the third angle is found by

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 60^\circ + \cos^2 45^\circ = 1$$

$$\cos\alpha = \sqrt{1-(0.5)^2-(0.707)^2} = \pm 0.5$$

- Two possibilities of α

$\alpha = \cos^{-1}(0.5) = 60^\circ$ or $\alpha = \cos^{-1}(-0.5) = 120^\circ$

Solution Example 2.8

By inspection, $\alpha = 60°$ since $\mathbf{F}_x$ is in the +x direction

Given F = 200N

$$\mathbf{F} = F\cos\alpha\,\mathbf{i} + F\cos\beta\,\mathbf{j} + F\cos\gamma\,\mathbf{k}$$
$$= (200\cos60°\text{N})\mathbf{i} + (200\cos60°\text{N})\mathbf{j} + (200\cos45°\text{N})\mathbf{k}$$

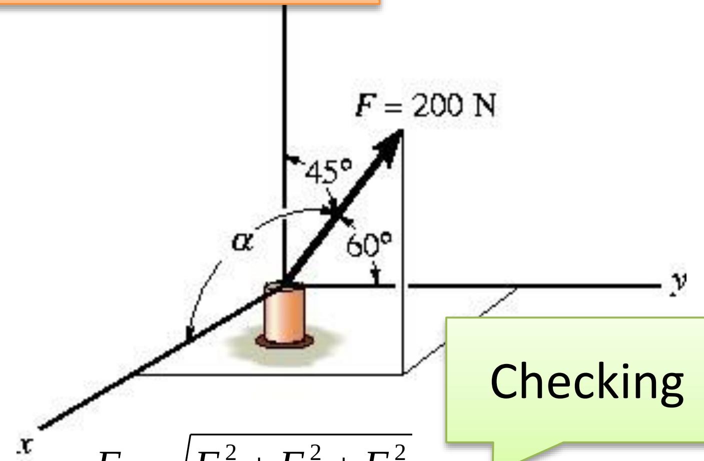
$F_x = 200 \cos 60° = 100$ N

$F_y = 200 \cos 60° = 100$ N

$F_z = 200 \cos 45° = 141.4$ N

Now ***F*** in Cartesian term :

$\boldsymbol{F} = \{100\,\boldsymbol{i} + 100\,\boldsymbol{j} + 141.4\,\boldsymbol{k}\}$ N

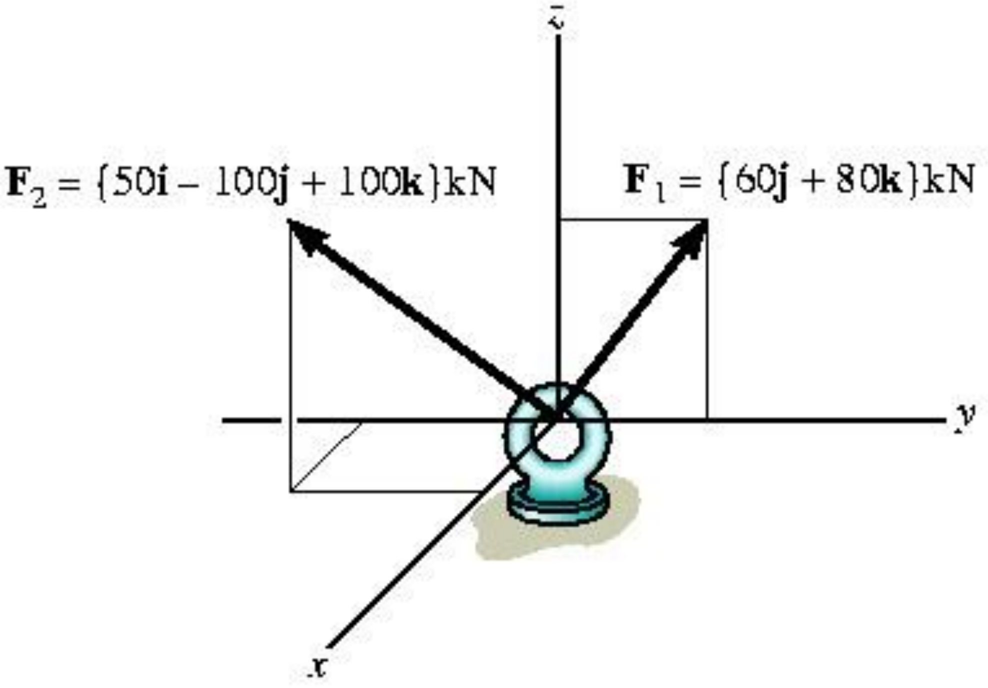


Checking

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{(100.0)^2 + (100.0)^2 + (141.4)^2} = 200\,N$$

Example 2.9

Determine the magnitude and coordinate direction of the resultant force acting on the ring



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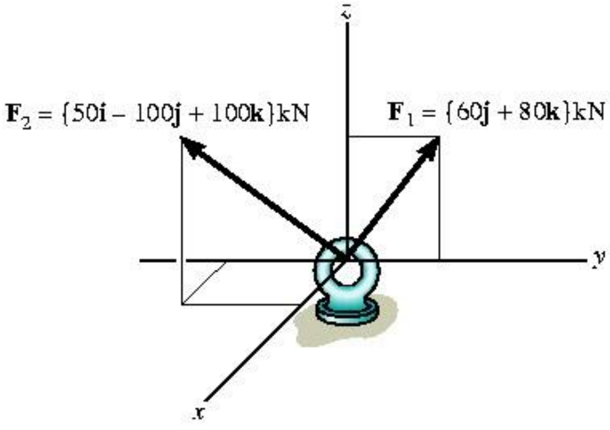
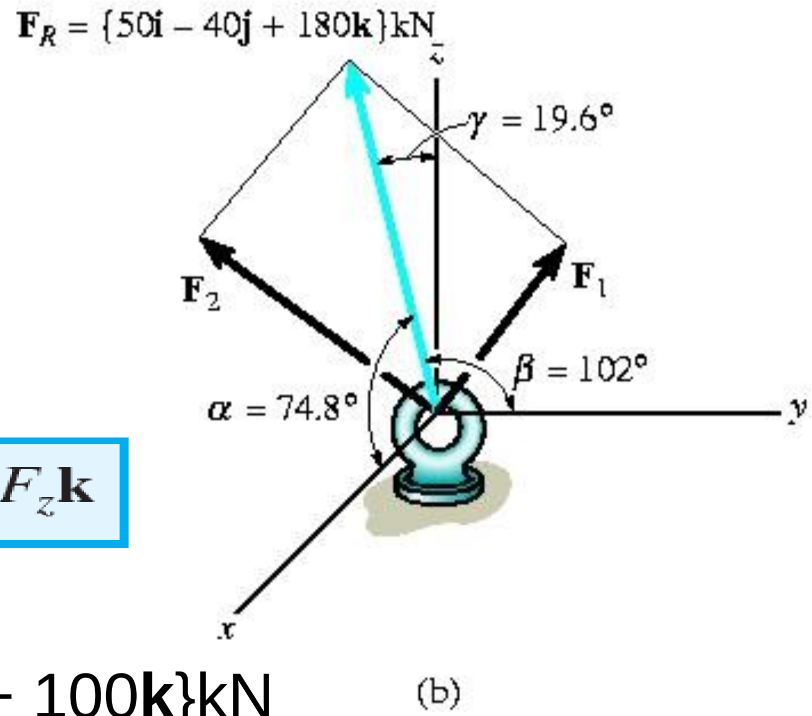
Solution Example 2.9





$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

$\mathbf{F}_R = \sum\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$= \{60\mathbf{j} + 80\mathbf{k}\}kN + \{50\mathbf{i} - 100\mathbf{j} + 100\mathbf{k}\}kN$

$= \{50\mathbf{i} - 40\mathbf{j} + 180\mathbf{k}\}kN$

- Magnitude of F_R :

$$F_R = \sqrt{(50)^2 + (-40)^2 + (180)^2}$$
$$= 191.0 = 191kN$$

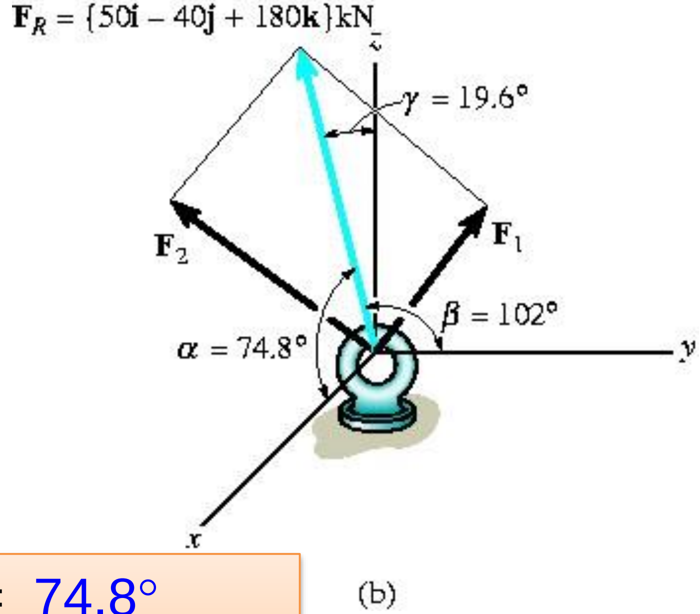
Solution Example 2.9

- Coordinate direction (angle) of F_R :

$\mathbf{F}_R = \{50\mathbf{i} -40\mathbf{j} + 180\mathbf{k}\}$kN

- **magnitude** $F_R = 191$ kN

- $\mathbf{u}_{FR} = \mathbf{F}_R / F_R$



So that

$\alpha = \cos^{-1}(F_{Rx} / F_R) = \cos^{-1}(50/191) = \underline{74.8°}$

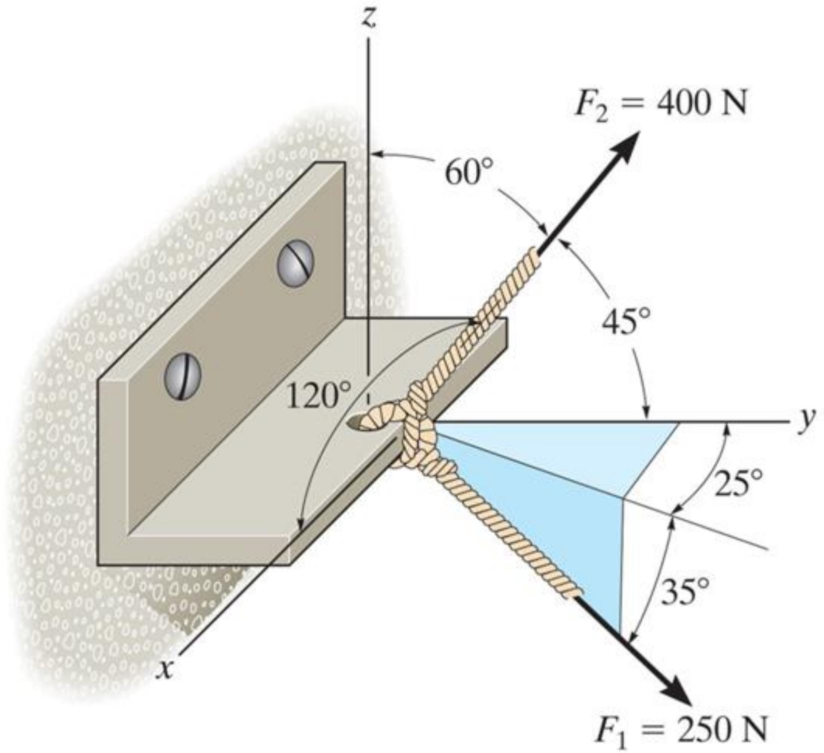
$\beta = \cos^{-1}(F_{Ry} / F_R) = \cos^{-1}(-40/191) = \underline{102°}$

$\gamma = \cos^{-1}(F_{Rz} / F_R) = \cos^{-1}(180/191) = \underline{19.5°}$

Note $\beta > 90°$ since ***j*** *component of* $\mathbf{u}_{FR}$ *is negative*

Example 2.10

Determine the magnitude and coordinate direction of the resultant force



Solution Example 2.10

Step 1

First resolve the force ***F₁***

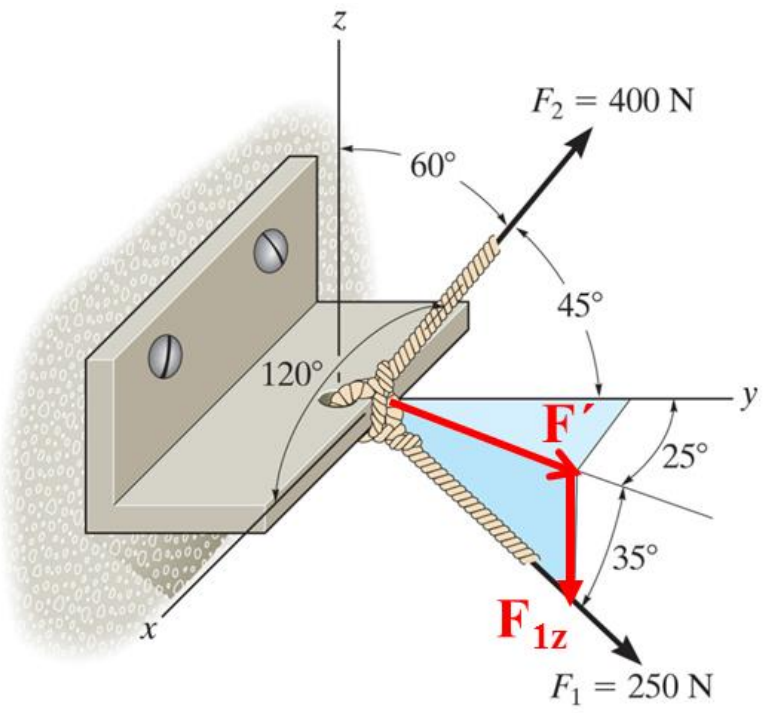
$F_{1z} = -250 \sin 35° = -143.4\ N$

$F' = 250 \cos 35° = 204.8\ N$

F′ can be further resolved as

$F_{1x} = 204.8 \sin 25° = 86.6\ N$

$F_{1y} = 204.8 \cos 25° = 185.6\ N$

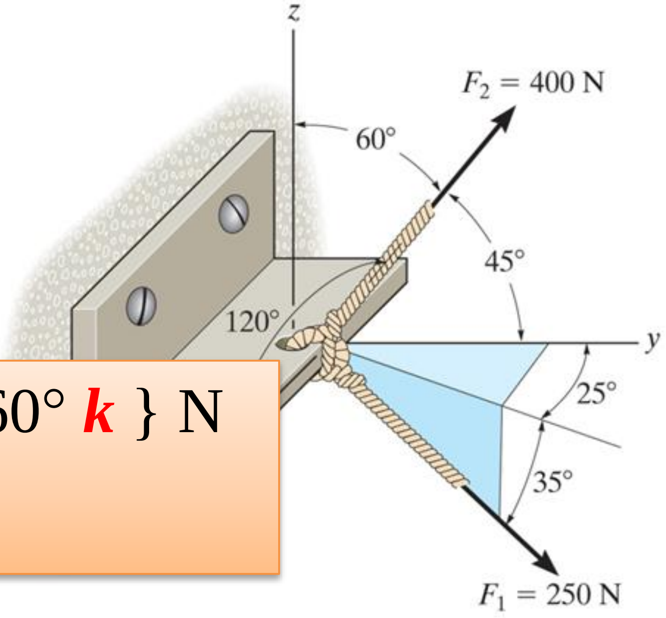


Now $\boldsymbol{F_1}$ in 3-D term :

$\boldsymbol{F_1} = \{86.6\,\boldsymbol{i} + 185.6\,\boldsymbol{j} - 143.4\,\boldsymbol{k}\}\ N$

Solution Example 2.10

Then resolve the force $\boldsymbol{F_2}$



$$\boldsymbol{F_2} = 400\{ \cos 120° \, \boldsymbol{i} + \cos 45° \, \boldsymbol{j} + \cos 60° \, \boldsymbol{k} \} \text{ N}$$

$$= \{ -200 \, \boldsymbol{i} + 282.8 \, \boldsymbol{j} + 200 \, \boldsymbol{k} \} \text{ N}$$

$$\boldsymbol{F_2} = \{ -200 \, \boldsymbol{i} + 282.8 \, \boldsymbol{j} + 200 \, \boldsymbol{k} \} \text{ N}$$

Solution Example 2.10

Step 2

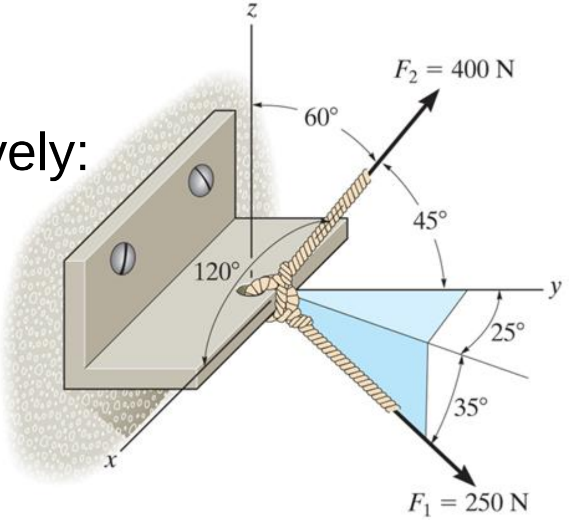
- Summing all the ***I, j*** and ***k*** components, respectively:

$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$$

$$\boldsymbol{F_1} = \{\ 86.6\ \boldsymbol{i} + 185.6\ \boldsymbol{j} - 143.4\ \boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F_2} = \{\ -200\ \boldsymbol{i} + 282.8\ \boldsymbol{j} + 200\ \boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F_R} = \{\ -113.4\ \boldsymbol{i} + 468.4\ \boldsymbol{j} + 56.6\ \boldsymbol{k}\}\ \text{N}$$



Step 3

- Magnitude and coordinate direction (angle):

$$F_R = \{(-113.4)^2 + 468.4^2 + 56.6^2\}^{1/2} = 485.2 = \underline{485\ \text{N}}$$

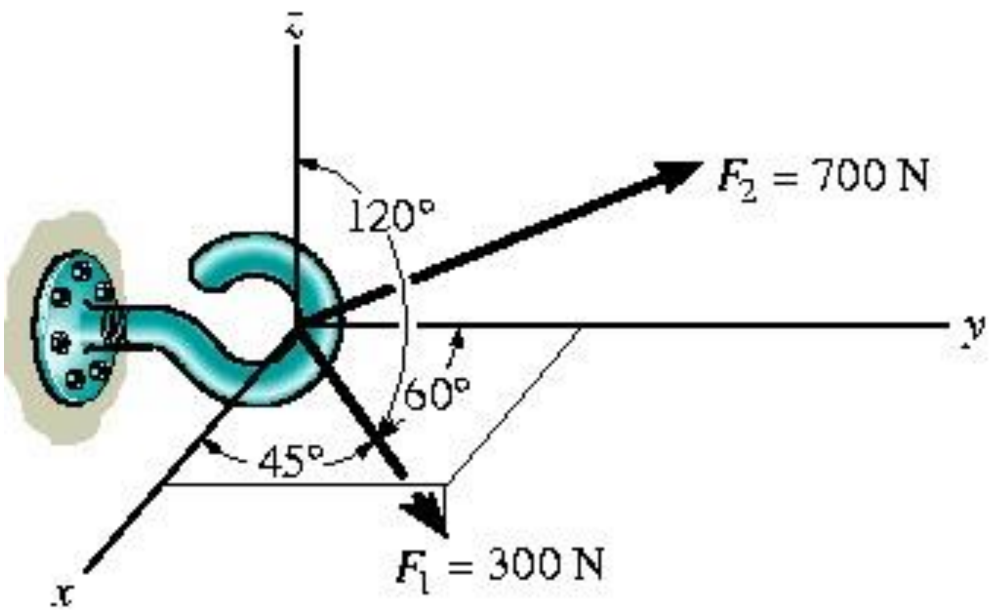
$$\alpha = \cos^{-1}(F_{Rx} / F_R) = \cos^{-1}(-113.4 / 485.2) = \underline{104^\circ}$$

$$\beta = \cos^{-1}(F_{Ry} / F_R) = \cos^{-1}(468.4 / 485.2) = \underline{15.1^\circ}$$

$$\gamma = \cos^{-1}(F_{Rz} / F_R) = \cos^{-1}(56.6 / 485.2) = \underline{83.3^\circ}$$

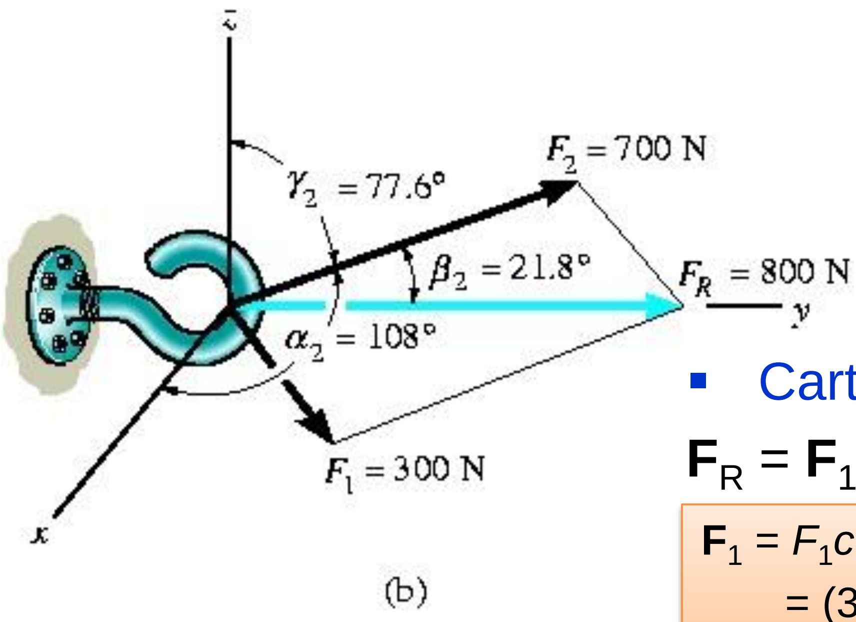
Example 2.11

Determine the coordinate direction angle of F_2, so that the resultant force F_R acts along the positive Y axis and has a magnitude of 800 N



(a)

Solution Example 2.11



Step 1

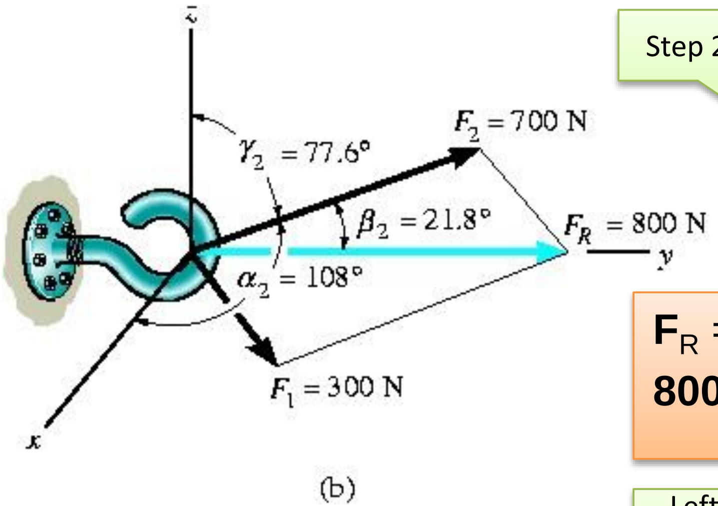
- Cartesian vector form:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_1 = F_1 cos\alpha_1 \mathbf{i} + F_1 cos\beta_1 \mathbf{j} + F_1 cos\gamma_1 \mathbf{k}$$
$$= (300cos45^\circ N)\mathbf{i} + (300cos60^\circ N)\mathbf{j} + (300cos120^\circ N)\mathbf{k}$$
$$= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}N$$

$$\mathbf{F}_2 = F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$

Solution Example 2.11



Step 2

- Magnitude of $\mathbf{F}_R$ is **800N** acts in **+j** direction:

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = 800$

$\mathbf{F}_R = \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}\text{N} + (F_{2X}\mathbf{i} + F_{2Y}\mathbf{j} + F_{2Z}\mathbf{k})$ N

$\mathbf{800\ j} = (212.1 + F_{2X})\mathbf{i} + (150 + F_{2Y})\mathbf{j} + (-150 + F_{2Z})\mathbf{k}$

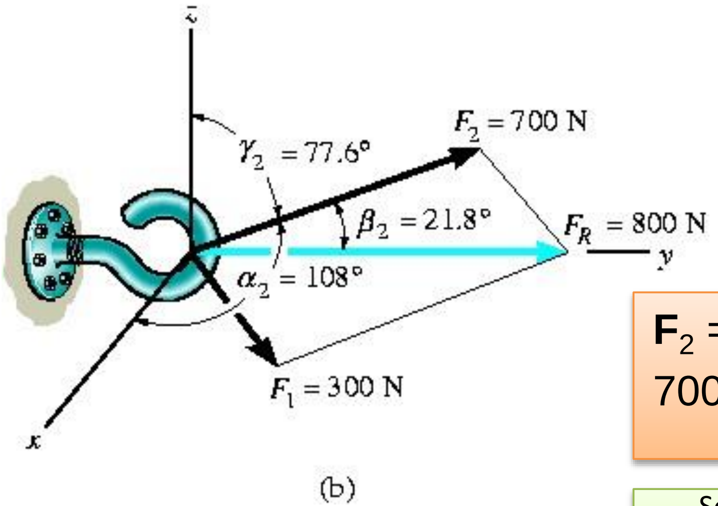
Left and right sides must be equal

$\mathbf{0} = 212.1 + F_{2X}$ hence $F_{2X} = -212.1$ N

$\mathbf{800} = 150 + F_{2Y}$ hence $F_{2Y} = 650$ N

$\mathbf{0} = -50 + F_{2Z}$ hence $F_{2Z} = 150$ N

Solution Example 2.11



Given $F_2 = 700N$

$\mathbf{F}_2 = (F_{2X}\mathbf{i} + F_{2Y}\mathbf{j} + F_{2Z}\mathbf{k})$ N

$700 = (-212.1\mathbf{i} + 650\mathbf{j} + 150\mathbf{k})$ N

So, coordiante direction of F_2

$\alpha = \cos^{-1}(F_{2x} / F_2) = \cos^{-1}(-212.1 / 700) = \underline{108°}$

$\beta = \cos^{-1}(F_{2y} / F_2) = \cos^{-1}(650 / 700) = \underline{21.8°}$

$\gamma = \cos^{-1}(F_{2Z} / F_2) = \cos^{-1}(150 / 700) = \underline{77.6°}$

Conclusion of The Chapter 2 part II

- Conclusions

  - The Cartesian vector have been identified and determined in the mechanics
  - The 2-D and 3-D vector has been represent in a Cartesian coordinate system
  - The magnitude and direction of 2-D and 3-D of resultant forces have been determined in a Cartesian coordinate system

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