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ENGINEERING MECHANICS BAA1113

Chapter 2: Force Vectors (Static)

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Chapter Description

- Aims
  - To review the Parallelogram Law and Trigonometry
  - To explain the Force Vectors
  - To explain the Vectors Operations
  - To express force and position in Cartesian Vectors

- Expected Outcomes
  - Able to solve the problems of force vectors in the mechanics applications by using Parallelogram law and Trigonometry

- References
  - Russel C. Hibbeler. Engineering Mechanics: Statics & Dynamics, 14^{th} Edition

Chapter Outline

2.1 Scalars and Vectors – part I

2.2 Vectors Operations – part I

2.3 Vectors Addition of Forces – part I

2.4 Cartesian Vectors – part II

2.5 Force and Position Vectors – part III

2.1 Scalars and Vectors

What is Scalars?

A quantity that has only a magnitude

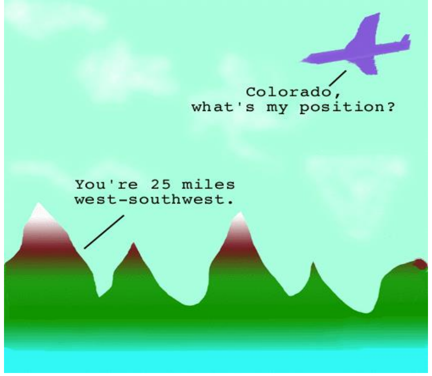
What is Vectors

A quantity that has both magnitude and direction

Source:http://www.automobiledimension.com/

Length of a mini car is **3.821** mm

↓ **quantity** ↓ **magnitude**



Source:http:http://www.shodor.org

Position of the plane is **25** miles from **west southwest**

↓ **quantity** ↓ **magnitude** ↓ **direction**

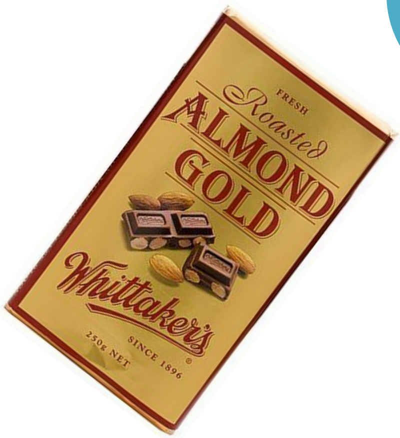
Identify scalars and vectors

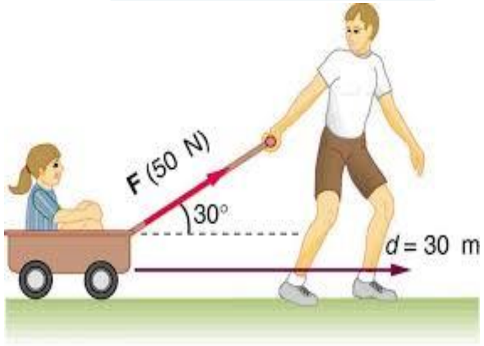
Scalars

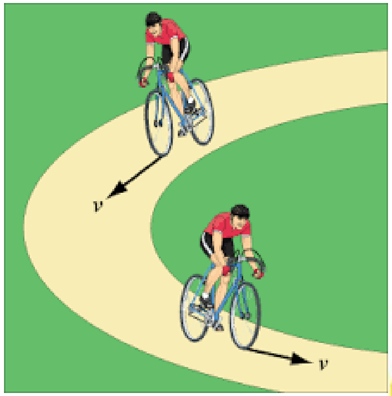
Identify example of scalars?

Vectors

Identify example of vectors?







Comparison of Scalars and Vectors

Scalars

A quantity that has only a magnitude

Mass, Length,
Time,
Temperature,
Volume,Density

Vectors

A quantity that has both magnitude and direction

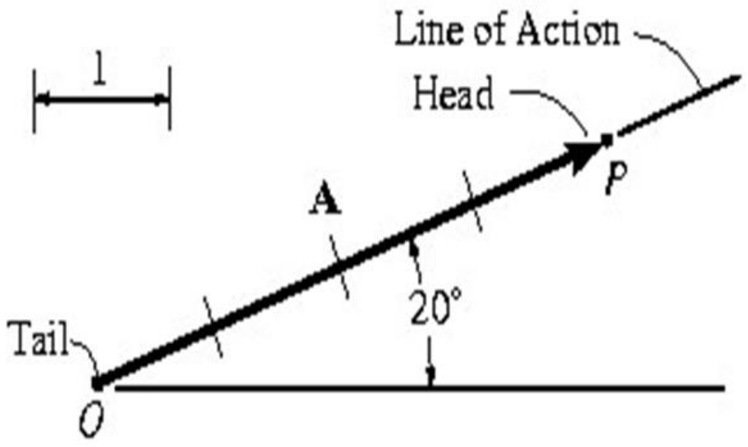
Position,
Displacement,
Velocity,
Acceleration,
Momentum,Force

Vectors

- Represent by a letter with an arrow over it such as $\vec{A}$ or **A**
- Magnitude is designated as $\left|\vec{A}\right|$ or simply A
- Commonly, vector is presented as **A** and its magnitude (positive quantity) as A

Characteristics of Vectors

- Represented graphically as an arrow
- Length of arrow = Magnitude of Vector
- Angle between the reference axis and arrow's line of action = Direction of Vector
- Arrowhead = Sense of Vector



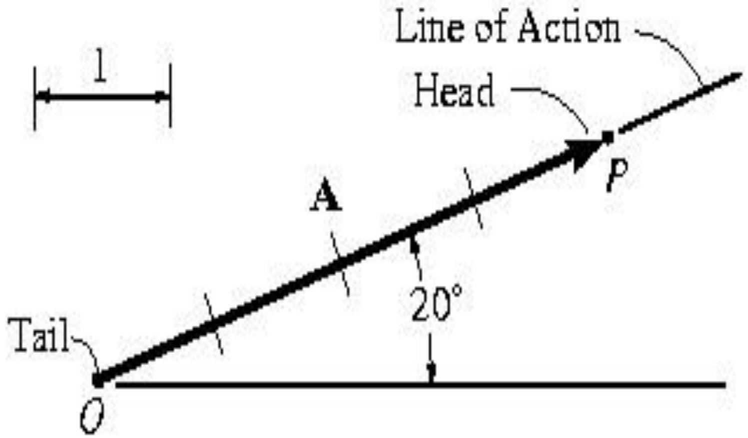
Example of Vectors

Magnitude of Vector = 4 units

Direction of Vector = 20° measured counterclockwise from the horizontal axis

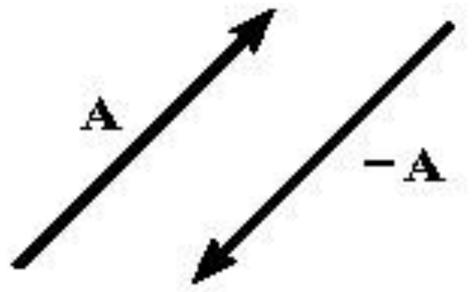
Sense of Vector = Upward and to the right

The point O is called ***tail*** of the vector and the point P is called the ***tip*** or ***head***



2.2 Vector Operations

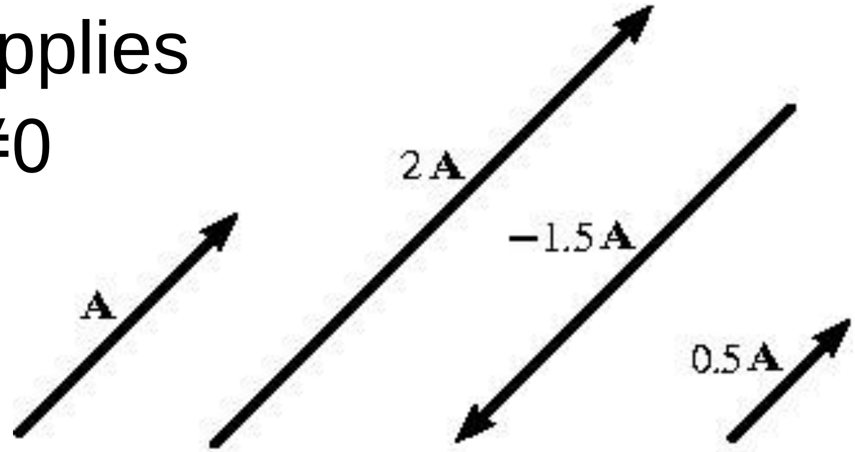
- Multiplication and Division of a Vector by a Scalar
- Product of vector **A** and scalar a = a**A**
- Magnitude = $|aA|$
- If a is positive, sense of a**A** is the same as sense of **A**
- If a is negative sense of
- a**A**, it is opposite to the
- sense of **A**



Vector **A** and its negative counterpart

2.2 Vector Operations

- Multiplication and Division of a Vector by a Scalar
- Negative of a vector is found by multiplying the vector by (-1)
- Law of multiplication applies
- Eg: **A**/a = (1/a) **A**, a≠0



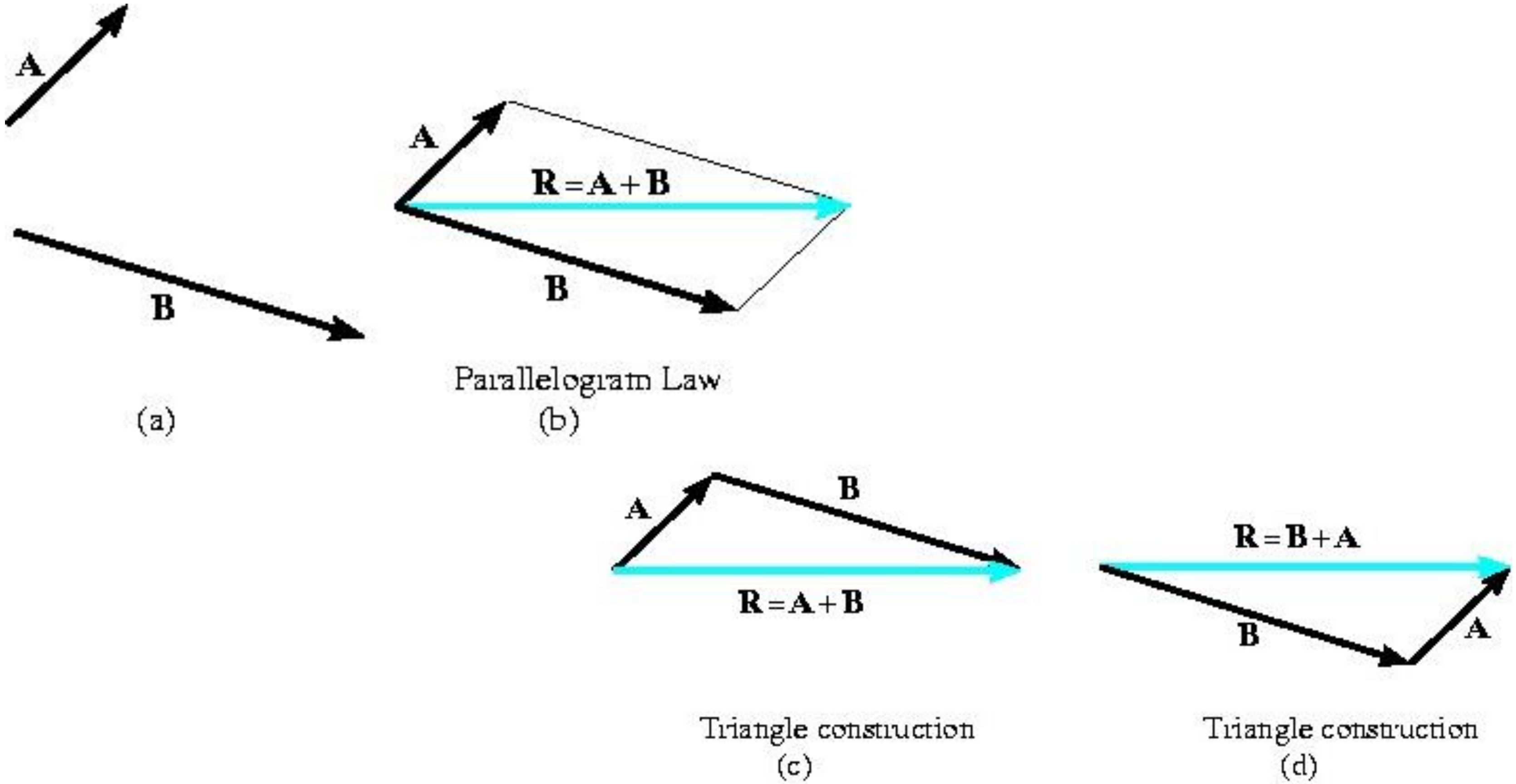
Scalar Multiplication and Division

2.2 Vector Operations

- Vector Addition
- Addition of two vectors **A** and **B** gives a resultant vector **R** by the ***parallelogram law***
- Result **R** can be found by ***triangle construction***
- Communicative
- Eg: **R** = **A** + **B** = **B** + **A**

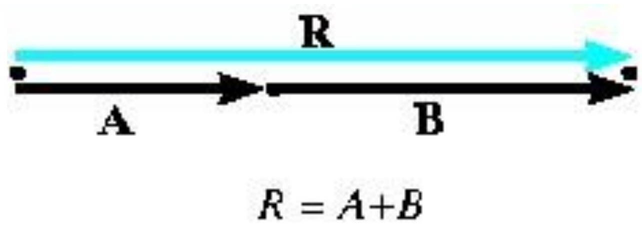
2.2 Vector Operations

- Vector Addition



2.2 Vector Operations

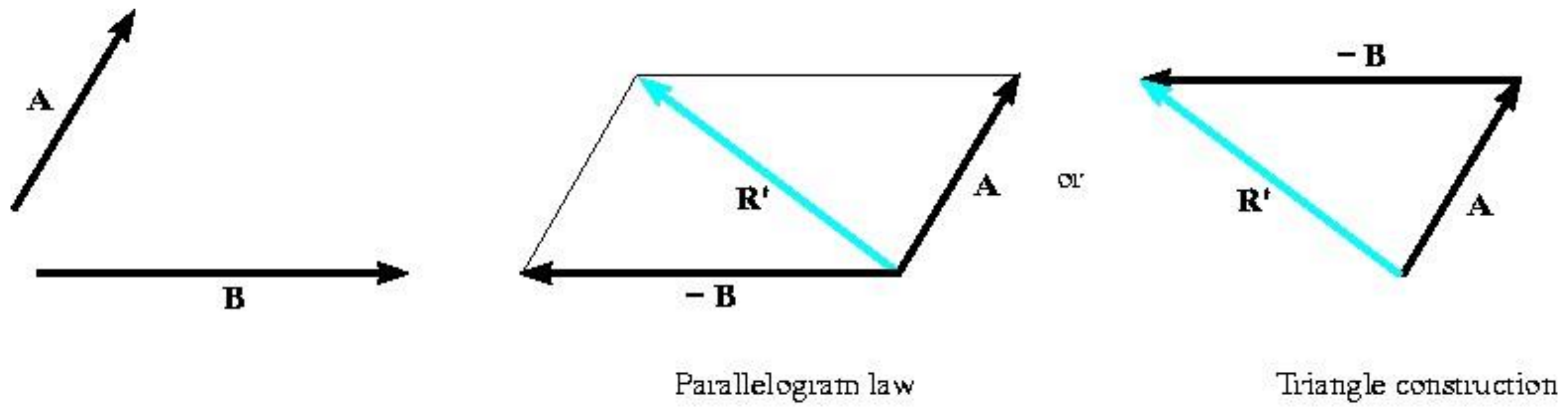
- Vector Addition
- Special case: Vectors **A** and **B** are *collinear* (both have the same line of action)



Addition of collinear vectors

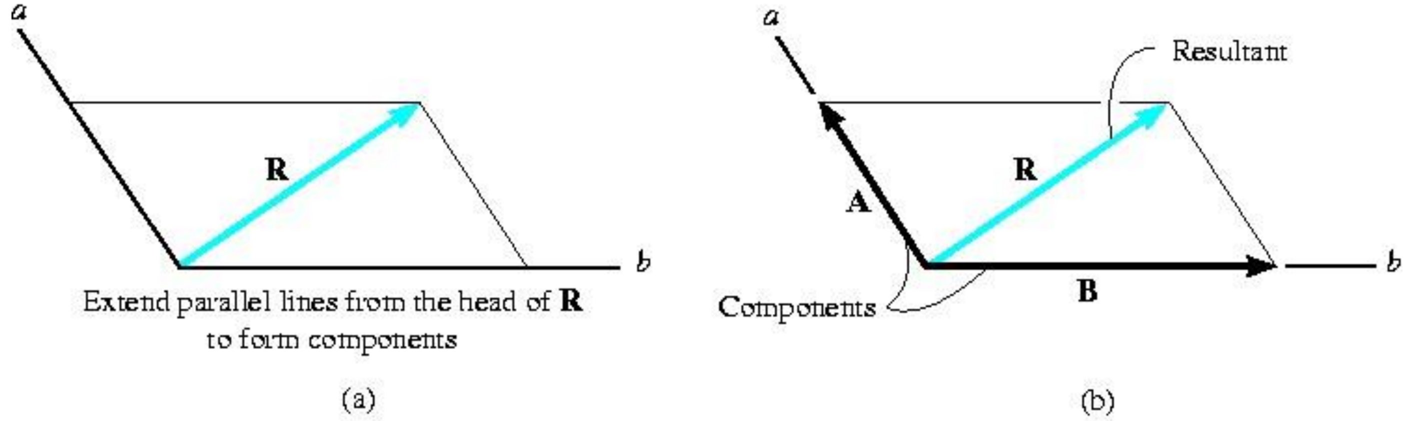
2.2 Vector Operations

- Vector Subtraction
- Special case of addition
- Eg: **R'** = **A** – **B** = **A** + (- **B**)
- Rules of Vector Addition Applies



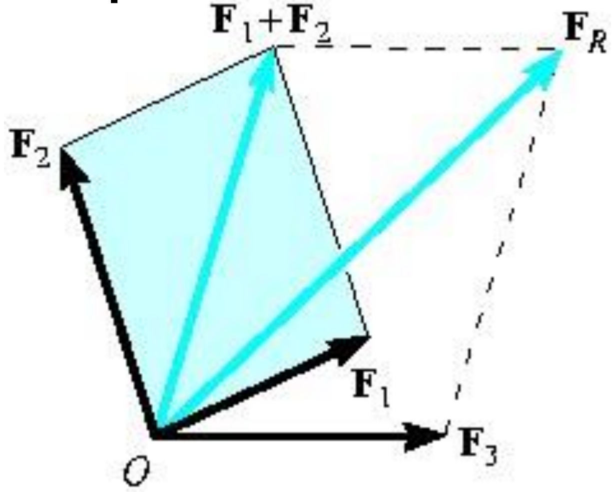
2.2 Vector Operations

- Resolution of Vector
- Any vector can be resolved into two components by the ***parallelogram law***
- The two components **A** and **B** are drawn such that they extend from the tail or **R** to points of intersection



2.3 Vector Addition of Forces

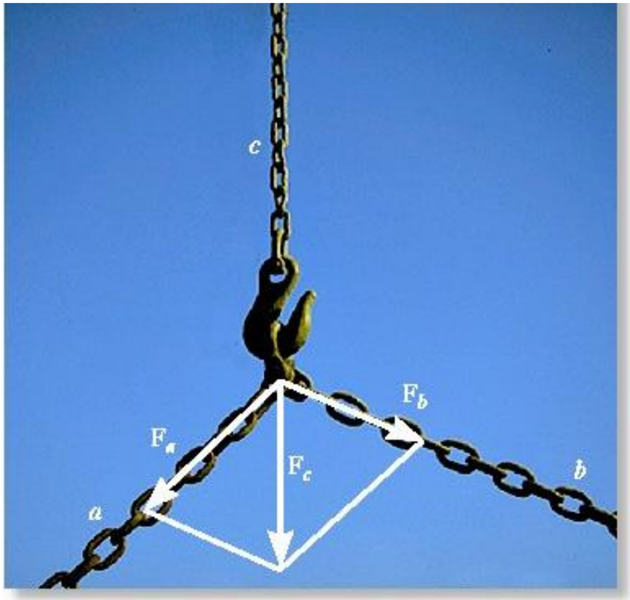
- When two or more forces are added, successive applications of the ***parallelogram law*** is carried out to find the resultant
- Eg: Forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acts at a point O
- First, find resultant of
- $\mathbf{F}_1 + \mathbf{F}_2$
- Resultant,
- $\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3$



Example of Vector Addition of Forces

- $\mathbf{F}_a$ and $\mathbf{F}_b$ are forces exerting on the hook.

- Resultant, $\mathbf{F}_c$ can be found using the *parallelogram law*
- Lines parallel to a and b
- from the heads of $\mathbf{F}_a$ and $\mathbf{F}_b$ are
- drawn to form a parallelogram
- Similarly, given $\mathbf{F}_c$, $\mathbf{F}_a$ and $\mathbf{F}_b$
- can be found



Steps to Solve the Vectors Operations

- Parallelogram Law
- Make a sketch using the *parallelogram law*
- Two components forces add to form the resultant force
- Resultant force is shown by the diagonal of the parallelogram
- The components is shown by the sides of the parallelogram

Steps to Solve the Vectors Operations

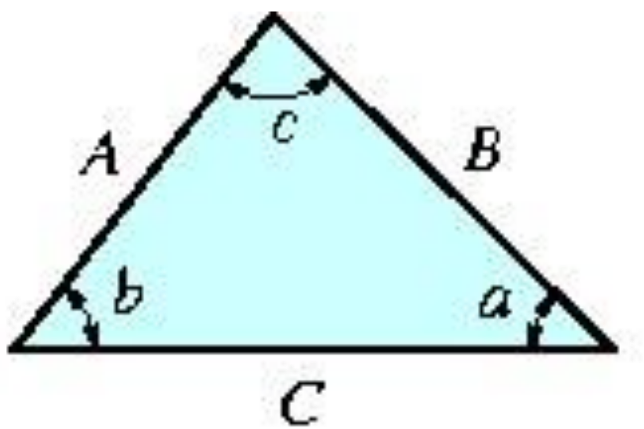
- Parallelogram Law
- To resolve a force into components along two axes directed from the tail of the force
- Start at the head, constructing lines parallel to the axes
- Label all the known and unknown force magnitudes and angles
- Identify the two unknown components

Steps to Solve the Vectors Operations

- Trigonometry
- Redraw half portion of the parallelogram
- Magnitude of the resultant force can be determined by the ***law of cosines***
- Direction if the resultant force can be determined by the ***law of sines***

Steps to Solve the Vectors Operations

- Trigonometry
- Magnitude of the two components can be determined by the ***law of sines***



Sine law:

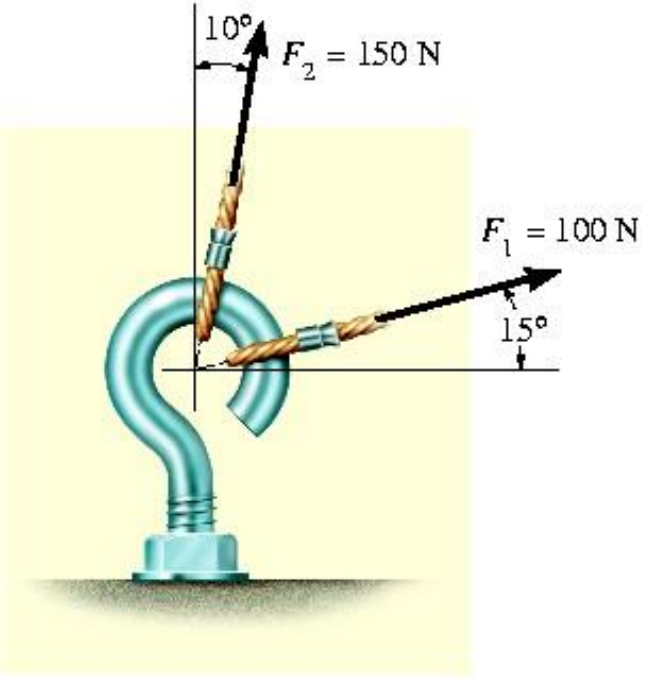
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Example 2.1

The screw eye is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.

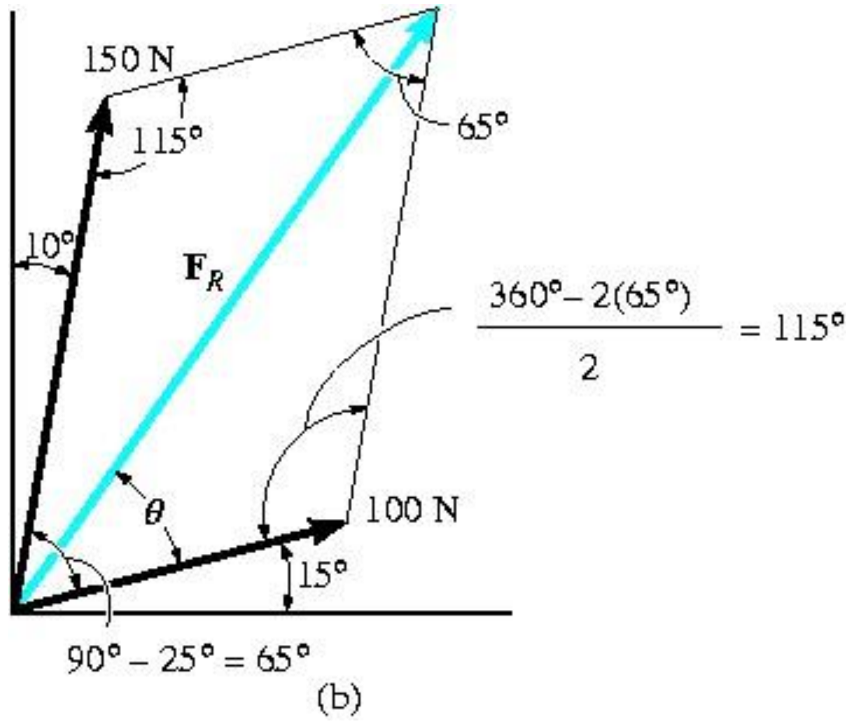


(a)

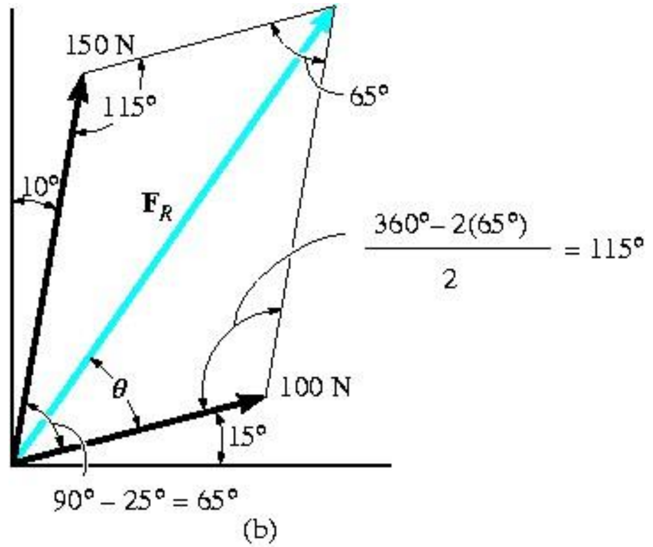
Solution Example 2.1

From Parallelogram Law

Unknown: magnitude of $\mathbf{F}_R$ and angle θ

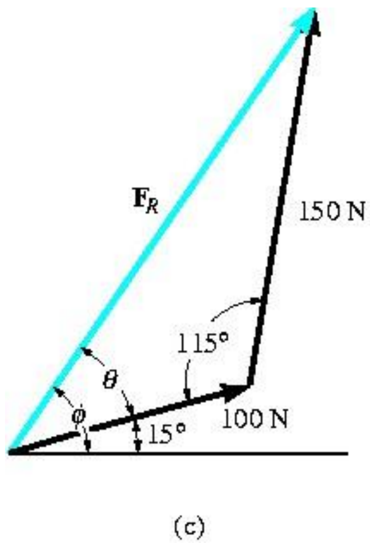


Solution Example 2.1



Trigonometry

Law of Cosines



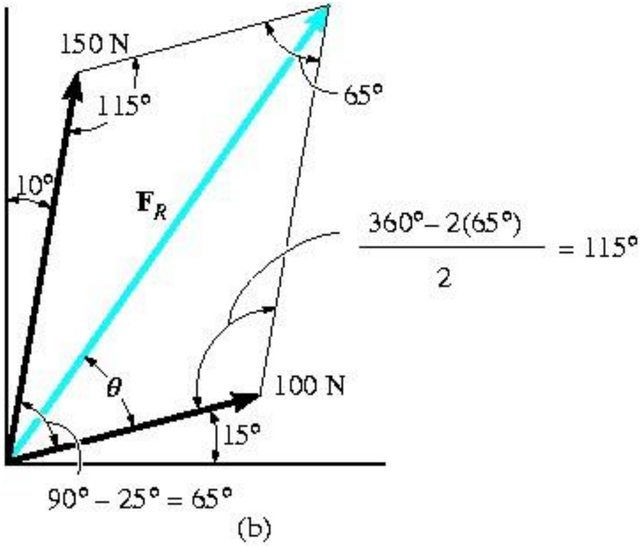
$$F_R = \sqrt{(100N)^2 + (150N)^2 - 2(100N)(150N)\cos 115^\circ}$$

$$= \sqrt{10000 + 22500 - 30000(-0.4226)}$$

$$= 212.6N$$

$$= 213N$$

Solution Example 2.1



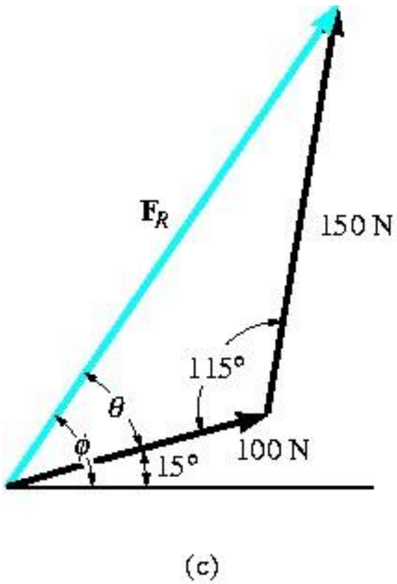
Trigonometry

Law of Sines

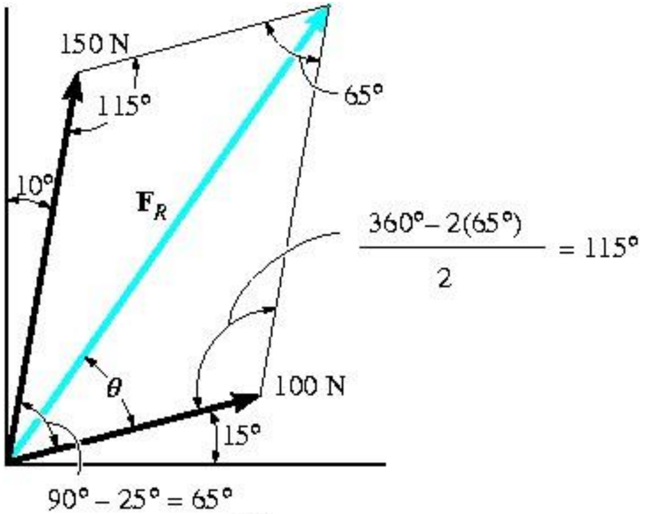
$$\frac{150N}{\sin\theta} = \frac{212.6N}{\sin 115^\circ}$$

$$\sin\theta = \frac{150N}{212.6N}(0.9063)$$

$$\sin\theta = 39.8^\circ$$



Solution Example 2.1



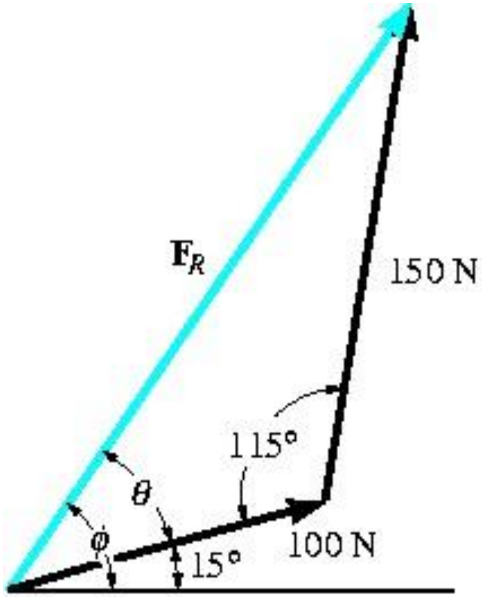
(b)

Trigonometry

Direction Φ of $\mathbf{F}_R$ measured from the horizontal

$$\phi = 39.8^\circ + 15^\circ$$

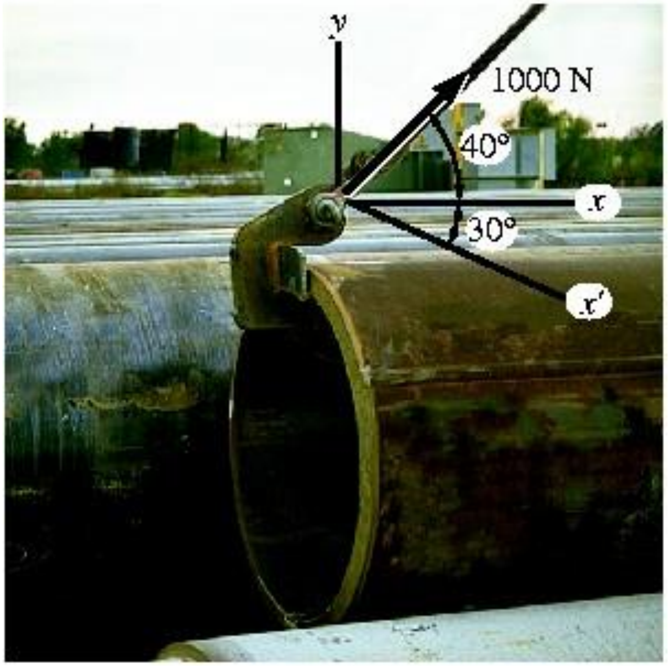
$$= 54.8^\circ \angle^{\phi}$$



(c)

Example 2.2

Resolve the 1000 N (≈ 100kg) force
acting on the pipe into the components in the
(a) x and y directions,
(b) and (b) x’ and y
directions.



(a)

Solution Example 2.2

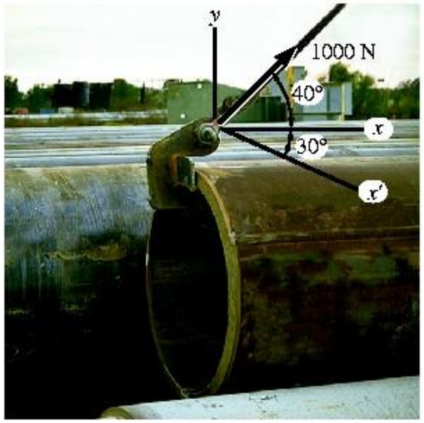
(a) Parallelogram Law

From the vector diagram,

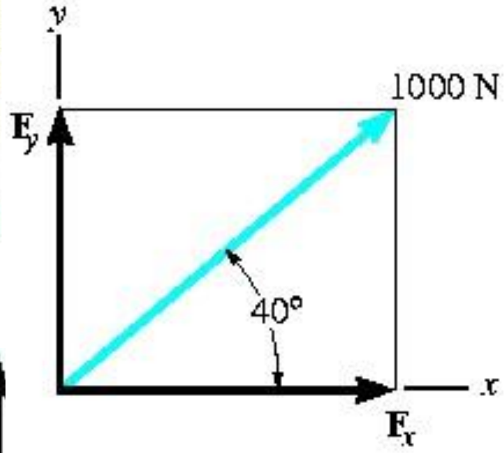
$$F = F_x + F_y$$

$$F_x = 1000\cos 40^\circ = 766\,N$$

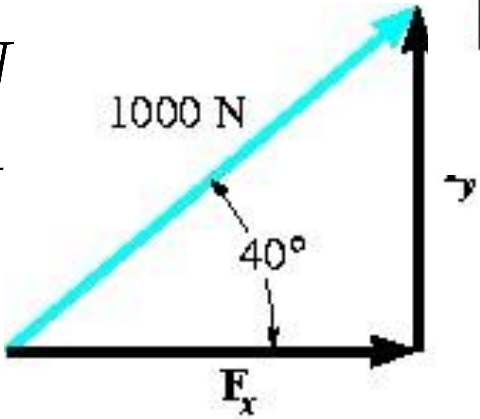
$$F_y = 1000\sin 40^\circ = 643\,N$$



(a)



(b)

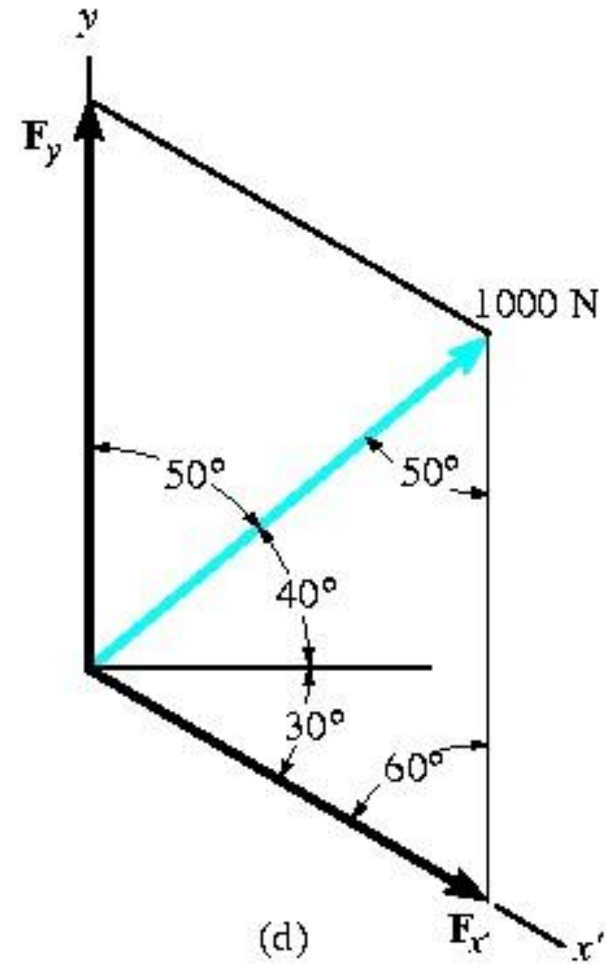


(c)

Solution Example 2.2

(b) Parallelogram Law

$$F = F_x + F_{y'}$$



Solution Example 2.2

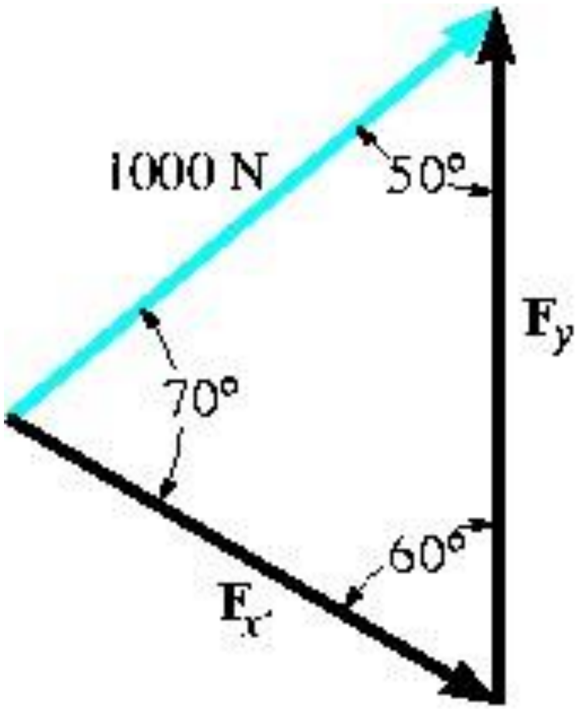
(b) Law of Sines

$$\frac{F_{x'}}{\sin 50^\circ} = \frac{1000\,N}{\sin 60^\circ}$$

$$F_{x'} = 1000\,N\left(\frac{\sin 50^\circ}{\sin 60^\circ}\right) = 884.6\,N$$

$$\frac{F_y}{\sin 70^\circ} = \frac{1000\,N}{\sin 60^\circ}$$

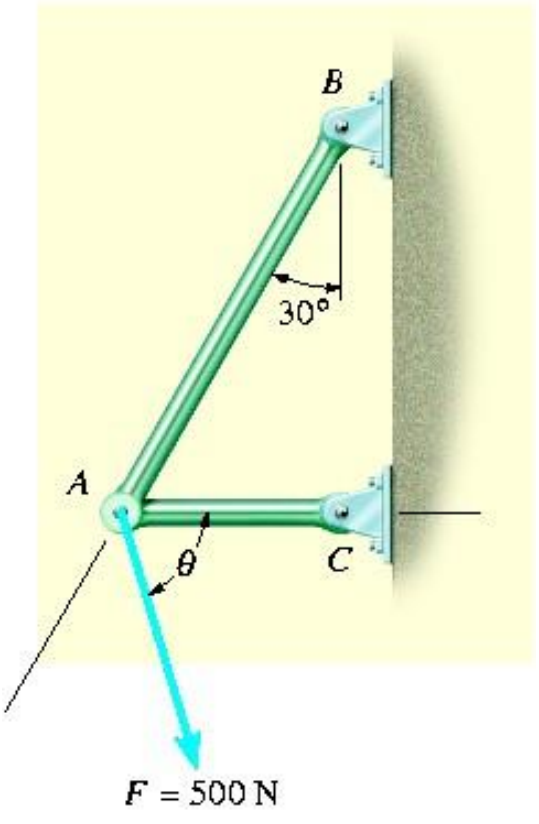
$$F_y = 1000\,N\left(\frac{\sin 70^\circ}{\sin 60^\circ}\right) = 1085\,N$$



(e)

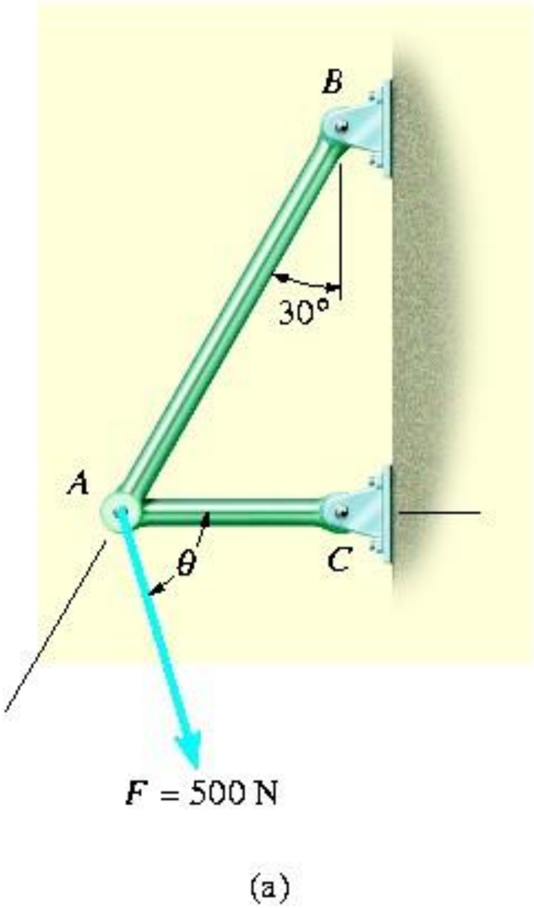
Example 2.3

The force **F** acting on the frame has a magnitude of 500N and is to be resolved into two components acting along the members AB and AC. Determine the angle θ, measured below the horizontal, so that components $\mathbf{F}_{AC}$ is directed from A towards C and has a magnitude of 400N.



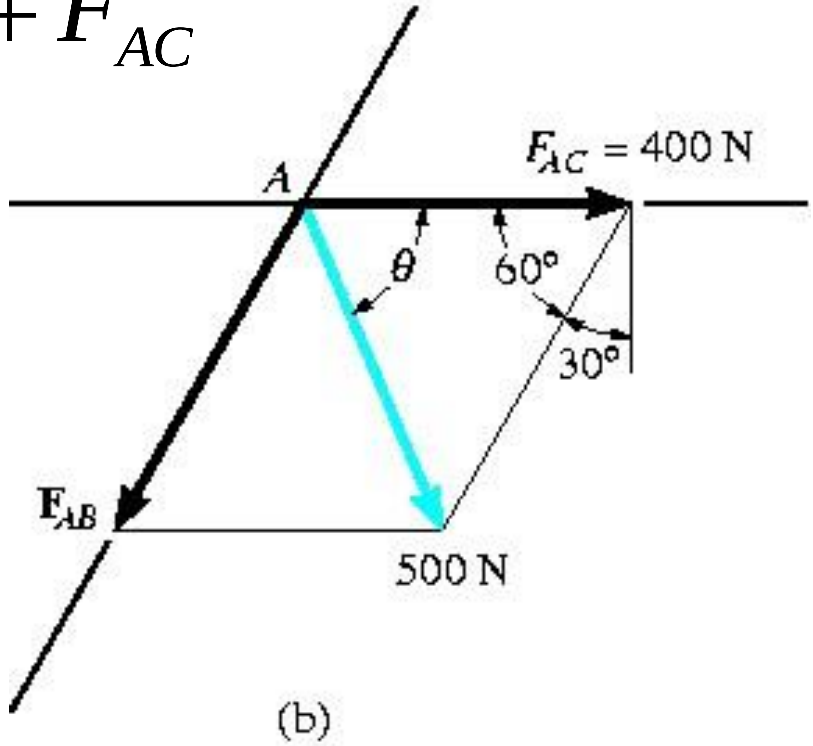
(a)

Solution Example 2.3



Parallelogram Law

$$500\,N = F_{AB} + F_{AC}$$



Solution Example 2.3

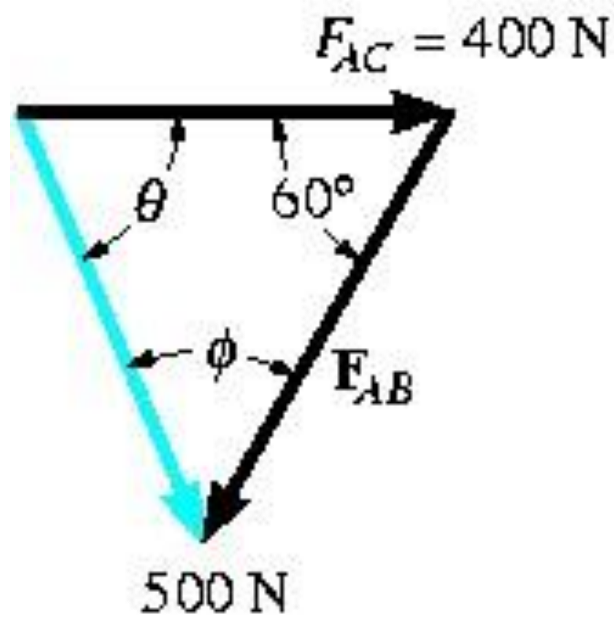
Law of Sines

$$\frac{400\,N}{\sin\phi} = \frac{500\,N}{\sin 60^\circ}$$

$$\sin\phi = \left(\frac{400\,N}{500\,N}\right)\sin 60^\circ$$

$$\sin\phi = 0.6928$$

$$\phi = 43.9^\circ$$



(c)

Solution Example 2.3

Hence,

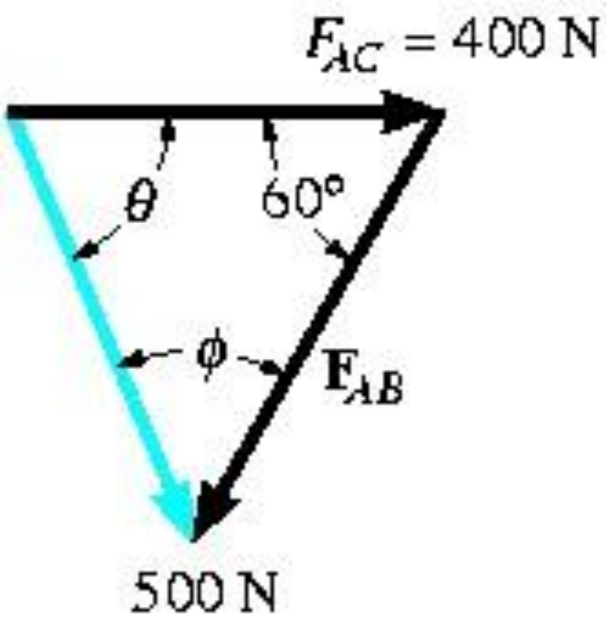
$$\theta = 180^\circ - 60^\circ - 43.9^\circ = 76.1^\circ \angle^{\theta}$$

By Law of Cosines or

Law of Sines

Hence, show that $\mathbf{F}_{AB}$

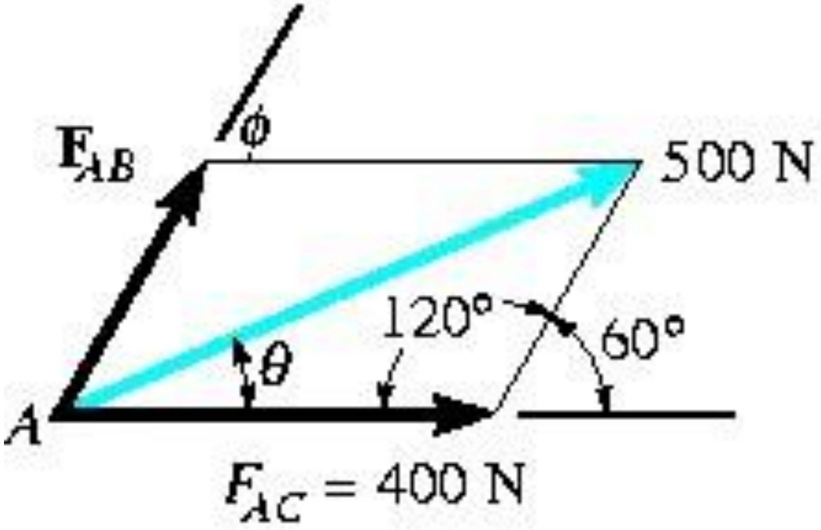
has a magnitude of 561N



(c)

Solution Example 2.3

F can be directed at an angle θ above the horizontal to produce the component $\mathbf{F}_{AC}$. Hence, show that θ = 16.1° and $\mathbf{F}_{AB}$ = 161N



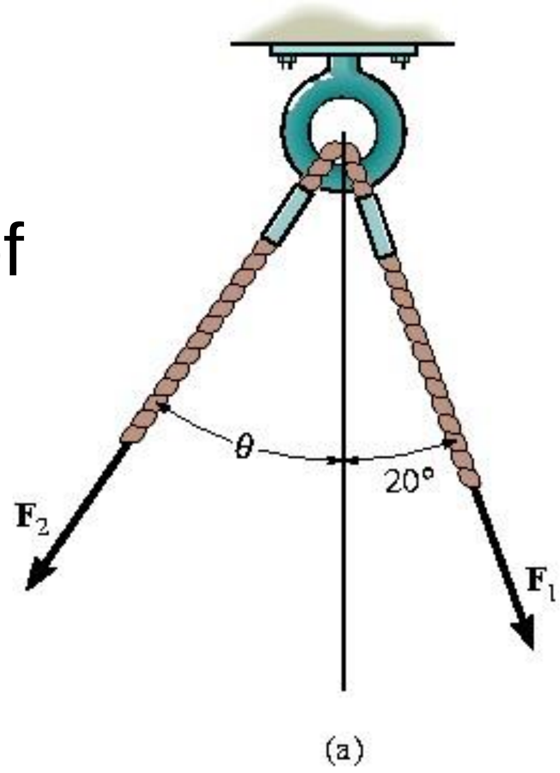
(d)

Example 2.4

The ring is subjected to two forces **F_1** and **F_2**. If it is required that the resultant force have a magnitude of 1kN and be directed vertically downward, determine

(a) magnitude of **F_1** and **F_2** provided $\theta = 30°$, and

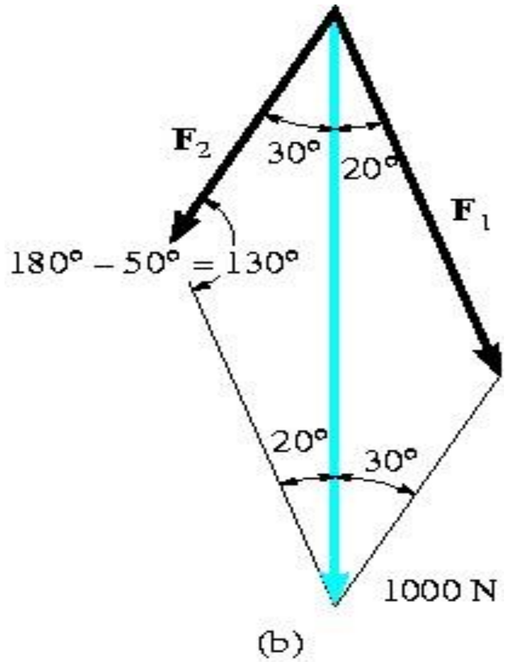
(b) the magnitudes of **F_1** and **F_2** if F2 is to be a minimum.

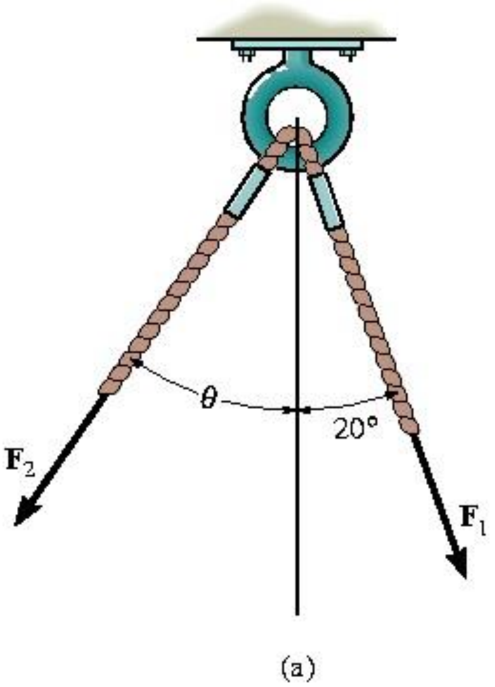


Solution Example 2.4

(a) Parallelogram Law

Unknown: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$





Solution Example 2.4

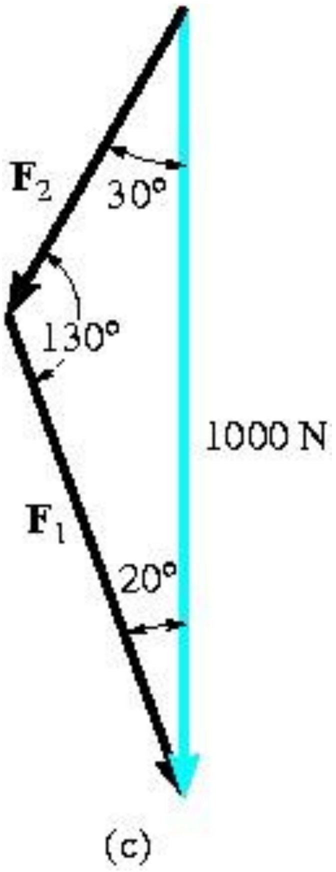
Law of Sines

$$\frac{F_1}{\sin 30^\circ} = \frac{1000\,N}{\sin 130^\circ}$$

$$F_1 = 653\,N$$

$$\frac{F_2}{\sin 20^\circ} = \frac{1000\,N}{\sin 130^\circ}$$

$$F_2 = 446\,N$$

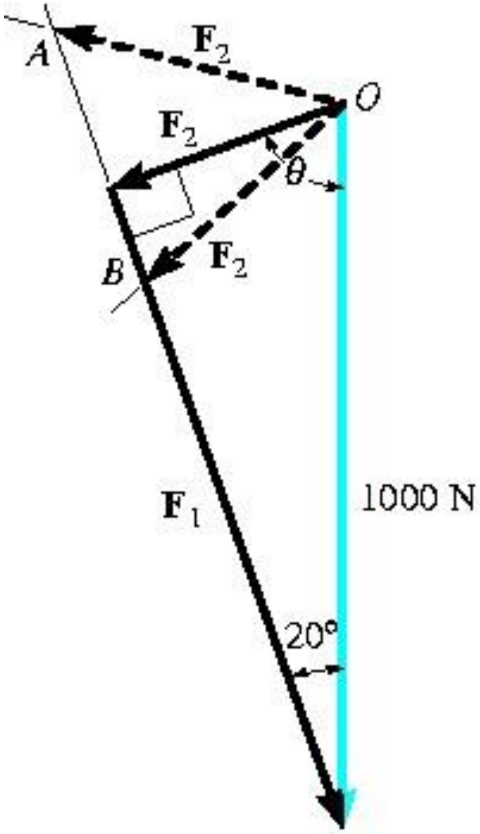


Solution Example 2.4

(b) Minimum length of $\mathbf{F}_2$ occur when its line of action is perpendicular to $\mathbf{F}_1$. Hence when

$$\theta = 90^\circ - 20^\circ = 70^\circ$$

$\mathbf{F}_2$ is a minimum



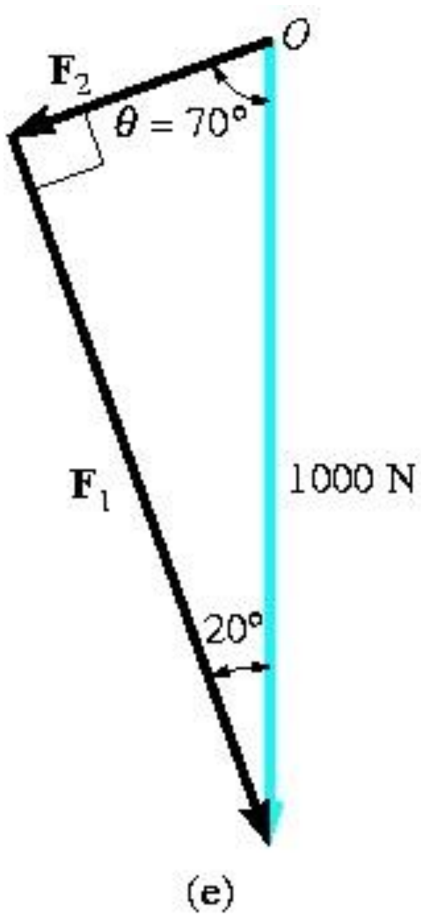
(d)

Solution Example 2.4

(b) From the vector diagram

$$F_1 = 1000 \sin 70^\circ \, N = 940 \, N$$

$$F_2 = 1000 \cos 70^\circ \, N = 342 \, N$$



Conclusion of The Chapter 2 part I

- Conclusions
  - The scalars and vectors have been identified and implemented in the mechanics
  - The vector operations have been identified and implemented in the mechanics

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