Computational Fluid Dynamics

Lecture 9

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Error Analysis

- Aims
  - The aim of this chapter is to introduce students the main stages of CFD analysis

- Expected Outcomes: At the end of this lecture, students should be able to understand
  - types of numerical errors,

- References

1) J. Tu, G.H. Yeoh, C. Liu, Computational Fluid Dynamics : A Practical Approach, Elsevier, 1st Edition, 2013.
2) C.T. Shaw, Using Computational Fluid Dynamics, Prentice Hall, 1992
3) AIAA, "Guide for the Verification and Validation of Computational Fluid Dynamics Simulations," AIAA G-077-1998, 1998

Accuracy of Numerical Solutions

- All CFD solutions are always approximate because
  - ✓ the discretized forms of the flow and thermal governing equations are always solved numerically on a finite grid layout and
  - ✓ the effect of turbulences are generally modeled through approximate theories,
- Hence, consideration of accuracy of CFD solutions is important.

- Truncation error is defined as the difference between the discretized equation and the exact one.
- It is used to evaluate the accuracy of the solution for the flow and thermal governing equations.
- The accuracy of high order approximations can be improved by applying sufficiently fine grid.
- Accuracy can also be assessed by obtaining solutions on highly refined meshes and checking further refinements will not bring significant changes on the solutions.

- Before we discuss the source of these errors in CFD in detail , we should understand the difference between error and uncertainty.
- Error in CFD can be defined as

  a recognizable deficiency in any phase or activity of modelling and simulation that is not due to lack of knowledge [3].
- uncertainty can be defined as

  a potential deficiency in any phase or activity of the modelling process that is due to the lack of knowledge [3].

Source of Solution Errors

- Some of the main sources of errors include:

  (a) discretization error

  (b) round-off error

  (c) iteration or convergence error

  (d) physical-modeling error

  (e) human error

i) Discretization Error

- It is caused due to the difference between the exact solution of the modeled equations and a numerical solution with a limited time and space resolution.
- Such type of error arises due to the fact that the solutions to the flow equations are numerically approximated.
- The discretization error may be:

  (a) local

  (b) global

- Consider the Taylor series expansion of derivatives of a transport variable *Φ* written in a finite-difference method at a specified mesh,

$$\left(\frac{\partial \phi}{\partial x}\right) = \frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta x} + \underbrace{\mathrm{O}(\Delta \mathrm{x})}_{\text{Truncation error}} \qquad \text{Spatial derivative}$$

$$\left(\frac{\partial \phi}{\partial \mathrm{t}}\right) = \frac{\phi_{i,j}^{n+1} - \phi_{i;j}^{n}}{\Delta t} + \underbrace{\mathrm{O}(\Delta \mathrm{t})}_{\text{Truncation error}} \qquad \text{Time derivative}$$

- Termination of the Taylor series expansion in the above equations results the so-called truncation errors involved in the approximation.
- The local error is the formulation associated with a single step and provides an idea about the accuracy of the method used. For this error, the accuracy of the numerical solution concerns mainly the approximation of the spatial derivative.
- The solution accuracy for a transient problem, however, focuses on the advancement of the transport variable Φ through time usually characterized by the global error.

ii) Round-Off Error

- It is the difference between the simulated approximate value of a number and its exact mathematical value due to rounding.
- Given that a = 6666666, b = -6666665. and c = 0.2222231, let us evaluate the operations of D = a + b+ c and E = a + c+ b. The arithmetic calculation for D proceeds as

$$\begin{aligned} D &= 6666666 - 6666665 + 0.2222231 \\ &= 1 + 0.2222231 \\ &= 1.2222231 \end{aligned}$$

- While E performs the following operations:

$$\begin{aligned} E &= 666666 + 0.2222231 - 6666665 \\ &= 6666666 - 6666665 \\ &= 1\,(\text{Rounded figure}) \end{aligned}$$

$$\begin{aligned} \text{Error} &= \frac{|\text{True Value-approximate value}|}{\text{True value}} * 100 \\ &= \frac{1.2222231 - 1}{1.2222231} \end{aligned}$$

$$\text{Error} = 18\%$$

Approximate Error in x_A = $E_a = x_A^{(k)} - x_A^{(k-1)}$

Approximate Relative Error in x_A = ε_a= $\dfrac{x_A^{(k)} - x_A^{(k-1)}}{x_A^{(k)}}$

Approximate Percentage Relative Error in x_A

$$= \frac{x_A^{(k)} - x_A^{(k-1)}}{x_A^{(k)}} \times 100\%$$

iii) Iteration or Convergence Error

- It is caused due to the difference between results of fully converged and that of partially converged solutions.
- Usually, such type of error occurs when the iteration is terminated prematurely.
- Convergence errors therefore can occur because of either being impatient to allow the solution algorithm to complete its progress to the final converged solution or applying too large convergence tolerances to halt the iteration process when the CFD solution may still be considerably far from its converged state.

iv) Physical-Modeling Error

- These errors are those due to uncertainty in the formulation of the mathematical models and deliberate simplifications of the models.
- The source of uncertainty in physical models are:
  - (a) the phenomenon is not thoroughly understood
  - (b) parameters employed in the model are known to possess some degree of uncertainty
  - (c) appropriate models are simplified thus uncertainty is introduced
  - (d) experimental confirmation of the models is not possible or is incomplete

v) Human Error

- These are essentially two categories of errors associated with human error:

  a) ***Computer-programming errors*** involve human mistakes made in programming, which are the direct responsibility of the programmers. These errors can be removed by systematically performing verification studies of subprograms of the computer code and the entire code, reviewing the details inserted into the code, and performing validation studies of the code.

  b) ***Usage errors*** are also due to application of the code in a less-than-accurate or improper manner.

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Research interest:

- Computational Fluid Dynamics,
- Thermo-fluids,
- Multidisciplinary Numerical Modelling and Simulation

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