Computational Fluid Dynamics

Lecture 5

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Numerical Discretization

- Aims
  - The aim of this chapter is to identify and understand the common methods used in process of discretization
- Expected Outcomes: At the end of this chapter, students should be able to understand
  - types of numerical discretization techniques
  - numerical solutions to algebraic equations
  - How to apply common discretization techniques to typical flow equations
- References

1) J. Tu, G.H. Yeoh, C. Liu, Computational Fluid Dynamics : A Practical Approach, Elsevier, 1st Edition, 2013.
2) C.T. Shaw, Using Computational Fluid Dynamics, Prentice Hall, 1992

Contents

The Finite Volume Discretization Method

- Advantages and disadvantages of FVM
- Integral approximation
- Location of the solution in the finite-volume cell

The Finite-Volume Discretization Technique

- The finite-volume method (FVM) discretized the integral form of the governing equations directly in the physical space.
- In this case, the computational domain is partitioned into a finite number of cell.
- The flow values are calculated at the centroid of each finite-volume cells.

Disadvantages of this method:

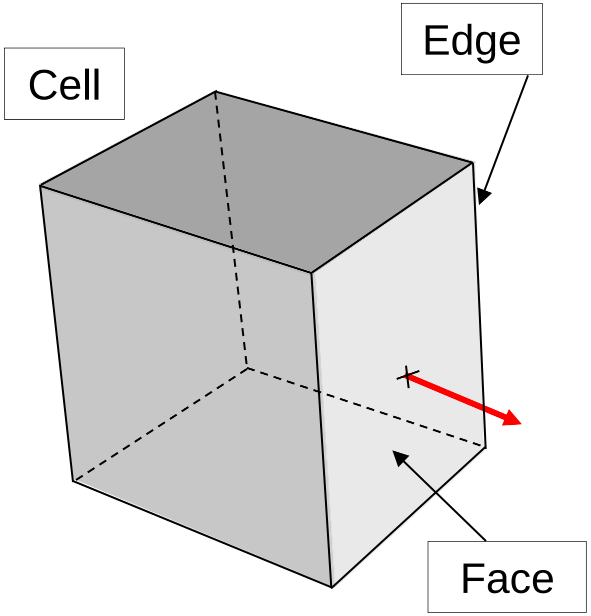
- The FVM can approximate higher order differencing (i.e., higher than the 2nd order), however it is difficult to develop in three dimension.

Advantages of this method:

- The grids can be formed by combining triangular and quadrilaterals meshes for two dimensional or tetrahedral and hexahedral in three dimension
- Unstructured grids can be used or complex geometries due to its greater flexibility.
- Unlike the FDM, it doesn't require the equations to be transformed in terms of body-fitted coordinate system.

Finite-Volume Cell:

- A cell contains a finite (positive) volume.
- Integral equation will be approximated on the cell to form an algebraic relation.
- Size of cell indicates the level of computational resolution of the CFD analysis.
- Control surface is *faceted* into a finite number of faces.
- Face is bounded by edges.
- Edges are usually straight lines.



Hexahedral Cell

6 Quadrilateral Faces

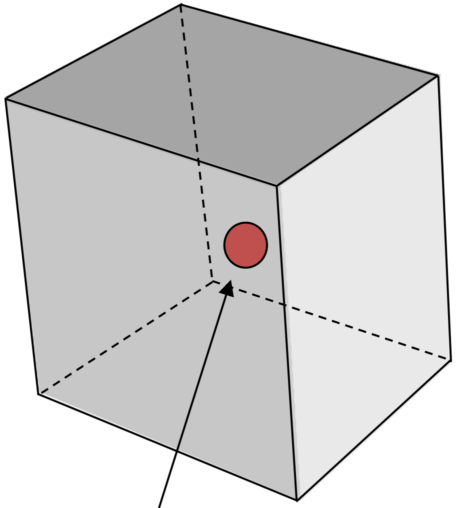
12 Linear Edges

Assume flow variable ψ *is uniform within the finite-volume shown below.*

Thus:

$$\int_V \psi \, dV \approx V \psi$$

Where *V* is the volume of the finite-volume cell.



Cell *i*
(*i* is an index for the cell)

- Applying Gauss's divergence theorem to a volume integral, a first order derivative of any flow variable ψ , the differential equation along the x direction can be represented as:

$$\left(\frac{\partial\psi}{\partial x}\right)=\frac{1}{\Delta V}\int_V\frac{\partial\psi}{\partial x}dV=\frac{1}{\Delta V}\int_A\psi dA^x\approx\frac{1}{\Delta V}\sum_{i=1}^{N}\psi_i A_i^x$$

where;

- ψ_i = flow values at the finite-volume cell surfaces
- N = number of enclosing surfaces on the finite-volume cell
- A = surface area in the finite-volume cell.

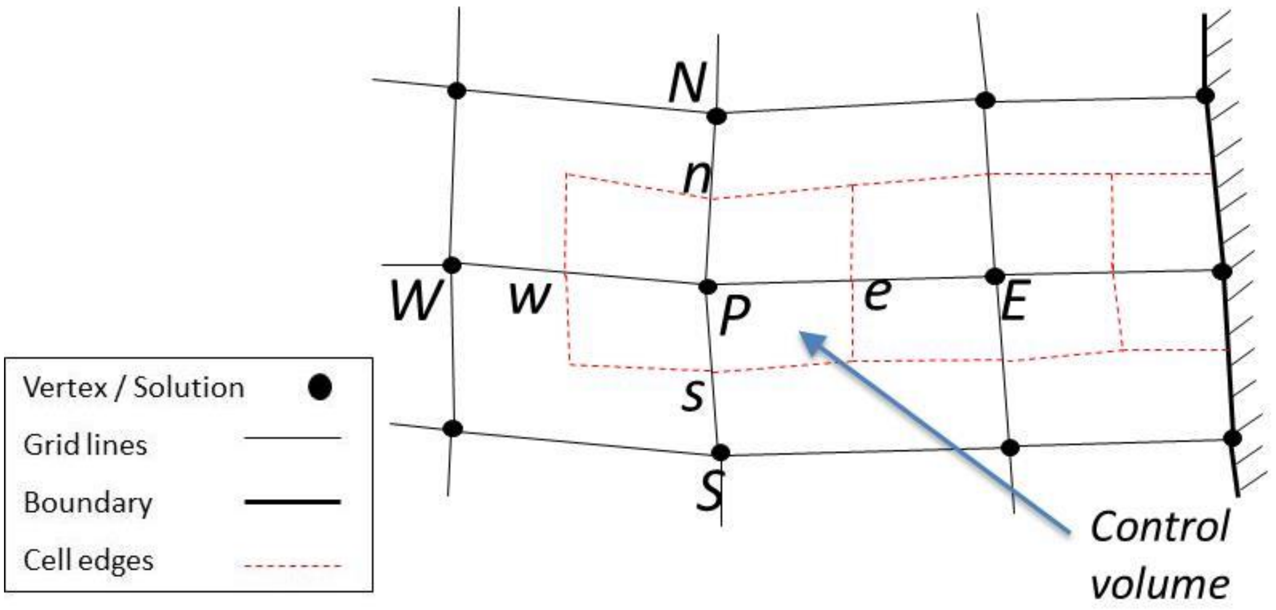
- For a structured grid in a 2D quadrilateral element, there are 4 enclosing surfaces (*N=4)* of the finite-volume cell. On the other hand, in 3D, *N* = 6.

- Similar to the *x*-direction , the first-order derivative for ψ in the y-direction will have the form:

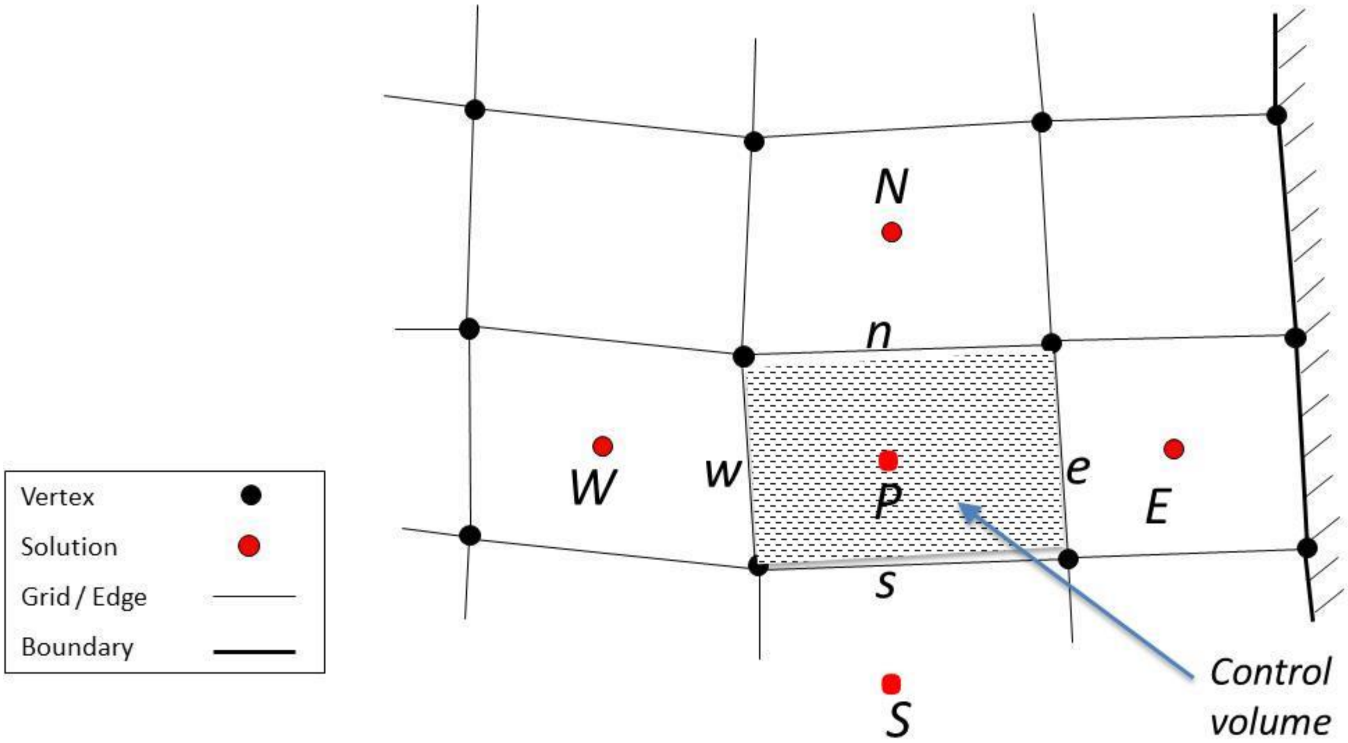
$$\left(\frac{\partial \psi}{\partial y}\right) = \frac{1}{\Delta V}\int_{\Delta V} \frac{\partial \psi}{\partial y} dV = \frac{1}{\Delta V}\int_{A} \psi dA^{y} \approx \frac{1}{\Delta V}\sum_{i=1}^{N} \psi_{i} A_{i}^{y}$$

Position of the flow and heat transfer solution in the finite-volume cell with respect to the mesh may have two forms:

i. Node-Centered: The flow and heat transfer solutions are positioned at the vertices of the mesh grid. The finite-volume cell is formed about the vertex.



ii. Cell-Centered: The flow and heat transfer solutions are positioned at the centroid of the cell

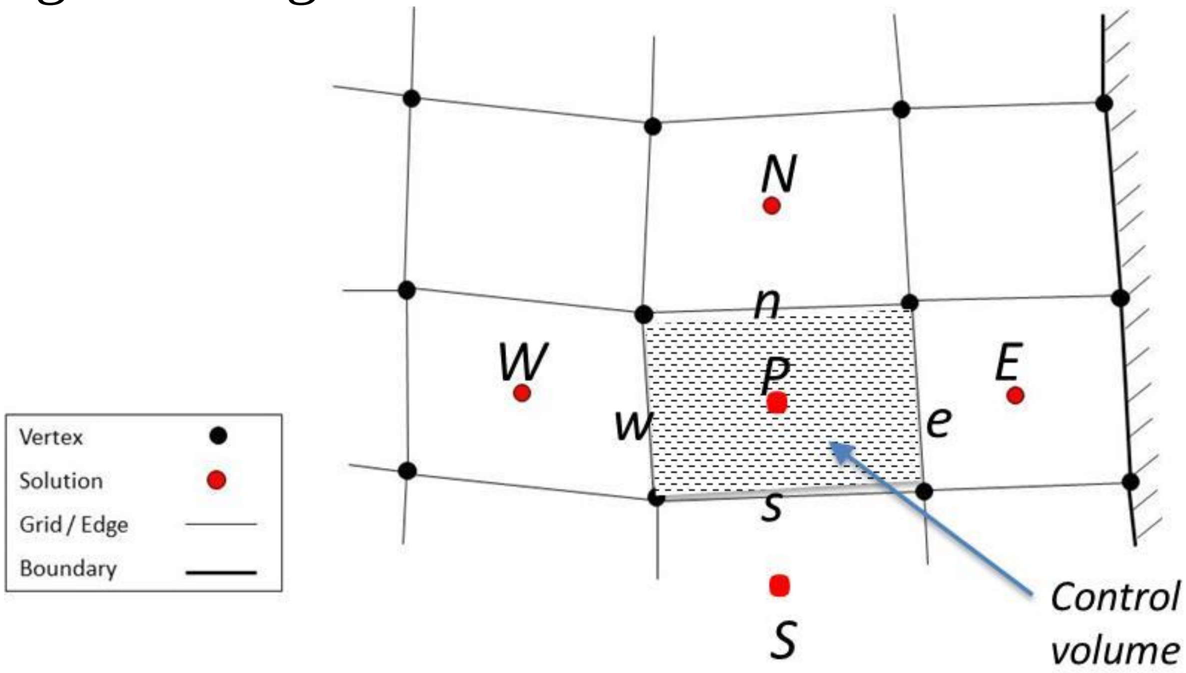


Example

Determine the discreditised form of the following two dimensional continuity equation using the FVM.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Use structured uniform grid arrangement.



Solution

- Applying the finite volume equations and either of the solution locations (node-centered or cell-centered) provided in the previous slides, the following expressions can be obtained

$$\frac{\partial u}{\partial x} \approx \frac{1}{\Delta V}\sum_{i=1}^{4} u_i A_i^x$$

$$= \frac{1}{\Delta V}\left(u_e A_e^x - u_w A_w^x + u_n A_n^{x=0} - u_s A_s^{x=0} \right)$$

$$\frac{\partial v}{\partial y} \approx \frac{1}{\Delta V}\sum_{i=1}^{4} v_i A_i^y$$

$$= \frac{1}{\Delta V}\left(v_e A_e^{y=0} - v_w A_w^{y=0} + v_n A_n^y + v_s A_s^y \right)$$

- The face velocities u_e , u_w ,v_n and v_s are positioned halfway between centroid of each finite-volume cells,
- This makes it easy for us to calculate the face velocities from the values located at the centroid of each finite-volume cells.

- Thus:

$$u_e = \frac{u_P + u_E}{2};\ u_w = \frac{u_P + u_W}{2};\ v_n = \frac{v_P + v_N}{2};\ v_s = \frac{v_P + v_S}{2}$$

- By substituting the above expressions to the discretized form of the velocity first-order derivatives, the final form of the discretized continuity equation becomes:

$$\left(\frac{u_P + u_E}{2}\right) A_e^x - \left(\frac{u_P + u_W}{2}\right) A_w^x + \left(\frac{v_P + v_N}{2}\right) A_n^y - \left(\frac{v_P + v_S}{2}\right) A_s^y = 0$$

- $A_e^x = A_w^x = \Delta y$ and $A_n^y = A_s^y = \Delta x$, the above equation can then be expressed by:

$$\left(\frac{u_P + u_E}{2}\right)A_e^x - \left(\frac{u_P + u_W}{2}\right)A_w^x + \left(\frac{v_P + v_N}{2}\right)A_n^y - \left(\frac{v_P + v_S}{2}\right)A_s^y = 0$$

- and reduced to:

$$\left(\frac{u_P + u_E}{2}\right)\Delta y - \left(\frac{u_P + u_W}{2}\right)\Delta y + \left(\frac{v_P + v_N}{2}\right)\Delta x - \left(\frac{v_P + v_s}{2}\right)\Delta x = 0$$

- or in another form: $\left(\frac{u_E - u_W}{2}\right)\Delta y + \left(\frac{v_N - v_S}{2}\right)\Delta x = 0$

$$\frac{u_E - u_W}{2\Delta x} + \frac{v_N - v_S}{2\Delta y} = 0$$

Summary

- FVM can be applied to greater than the second order representation of the flow.
- First order derivative of any flow variable ψ

$$\left(\frac{\partial \psi}{\partial x}\right) = \frac{1}{\Delta V}\int_V \frac{\partial \psi}{\partial x} dV = \frac{1}{\Delta V}\int_A \psi dA^x \approx \frac{1}{\Delta V}\sum_{i=1}^{N} \psi_i A_i^x$$

$$\left(\frac{\partial \psi}{\partial y}\right) = \frac{1}{\Delta V}\int_{\Delta V} \frac{\partial \psi}{\partial y} dV = \frac{1}{\Delta V}\int_A \psi dA^y \approx \frac{1}{\Delta V}\sum_{i=1}^{N} \psi_i A_i^y$$

- Location of cells
  - Node-Centered
  - Cell-Centered

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Research interest:

- Computational Fluid Dynamics,
- Thermo-fluids,
- Multidisciplinary Numerical Modelling and Simulation

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