Computational Fluid Dynamics

Lecture 4

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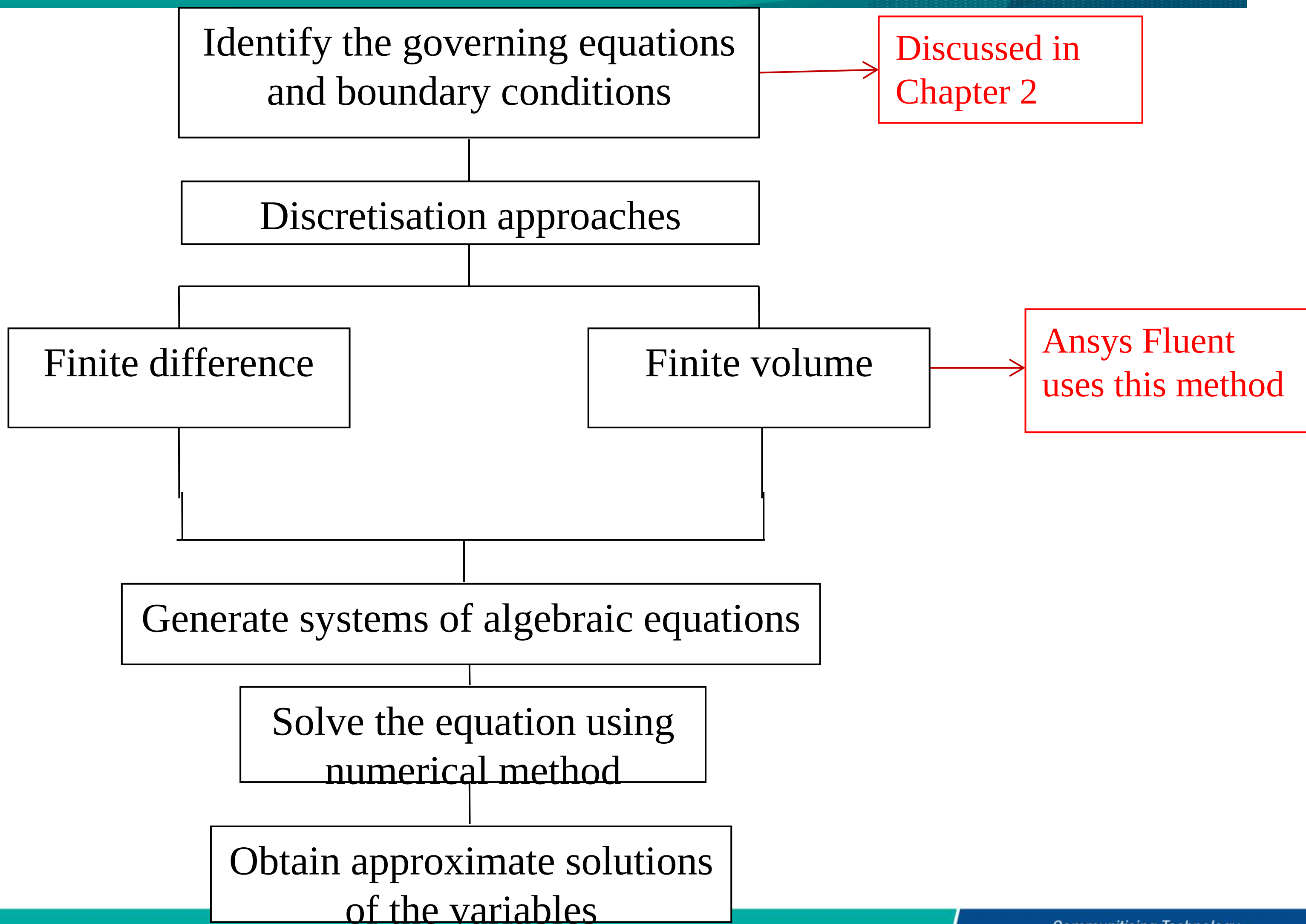
Numerical Discretization

- Aims
  - The aim of this chapter is to identify and understand the common methods used in process of discretization

- Expected Outcomes: At the end of this lecture, students should be able to understand
  - types of numerical discretization techniques
  - numerical solutions to algebraic equations
  - How to apply common discretization techniques to typical flow equations
- References

1) J. Tu, G.H. Yeoh, C. Liu, Computational Fluid Dynamics : A Practical Approach, Elsevier, 1st Edition, 2013.
2) C.T. Shaw, Using Computational Fluid Dynamics, Prentice Hall, 1992

Overall Computational Solution Procedure



Numerical Discretization Techniques

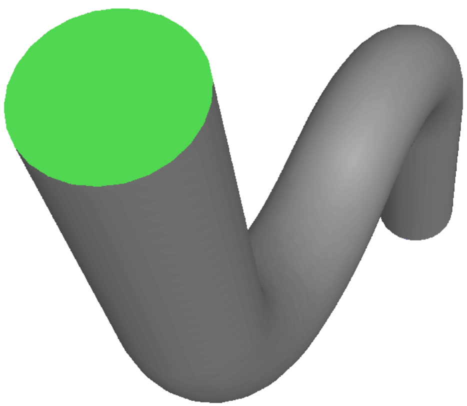
What is Discretization?

- Discretization can be defined as conversion of partial differential equations (that represent the thermal-fluid process) to a numerical analogue of systems of algebraic equation.
- In this case, each components (terms) of the differential equation should be converted to an algebraic equation that can easily be calculated by computer through programing.

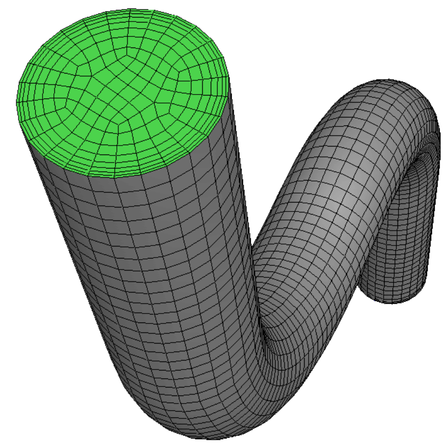
Numerical Discretization Techniques

Discretization of the physical domain

- When the continuous domain is transformed into a domain in which the flow governing equations can be solved it is called discretizing the domain



Continuous domain



Discretized domain

Source:- Fluent, Inc.

http://www.engr.uconn.edu/~barbertj/CFD%20Training/Fluent/4%20Solver%20Settings.pdf

Numerical Discretization Techniques

Discretization Techniques

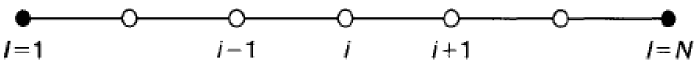
- There are many discretization methods. However, only two of the most common techniques are discussed in this chapter;
  - the finite difference method (FDM), and
  - the finite volume method (FVM).

3.1. The Finite-Difference Method

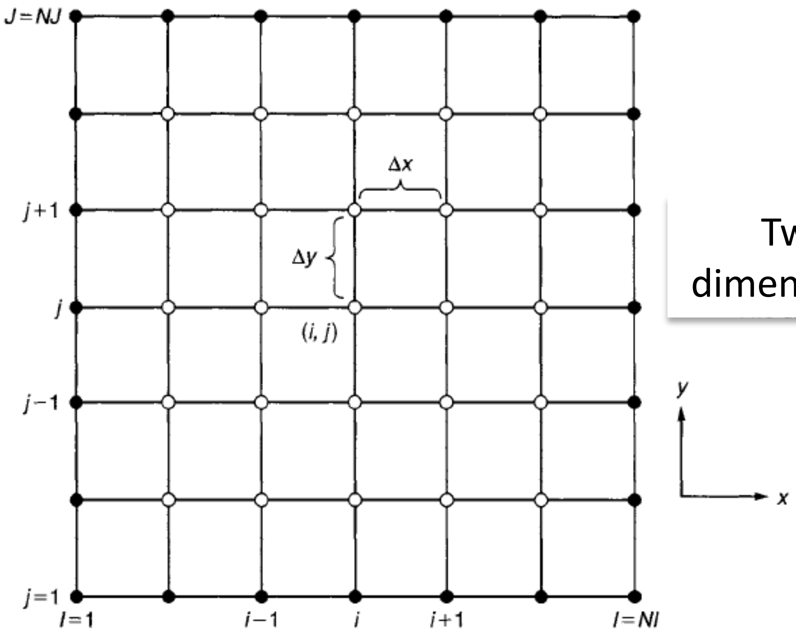
- The finite difference method is the oldest type of discretization technique developed by Euler in 1768,
- Before converting the differential equation to algebraic equations, the geometric domain should first be discretized to define a numerical grid
- One unknown variable will be assigned for each node and need one algebraic equation.
- The approach is to represent each term of the PDE at the particular node by a finite-difference approximation.
- The numbers of equations and unknown variables must be equal

3.1.The Finite-Difference Method

- The figures shown below illustrate typical examples of 1D and 2Dl grids commonly used in the FDM [1].
- The grids are uniformly distributed in Cartesian coordinates.



One dimensional



Two dimensional

3.1.The Finite-Difference Method

- Referring the above figures, if there exist a flow field ψ at the vortex (*i,j*) then the Taylor series expansion about point (*i,j*) along the *x*-direction produces the following equations for the variable at points (*i*+*1, j*) and (*i*-*1,j*) [1].

$$\psi_{i+1,j} = \psi_{i,j} + \left(\frac{\partial \psi}{\partial x}\right)_{i,j} \Delta x + \left(\frac{\partial^2 \psi}{\partial x^2}\right)_{i,j} \frac{\Delta x^2}{2} + \left(\frac{\partial^3 \psi}{\partial x^3}\right)_{i,j} \frac{\Delta x^3}{6} + \ldots \qquad (1)$$

$$\psi_{i-1,j} = \psi_{i,j} - \left(\frac{\partial \psi}{\partial x}\right)_{i,j} \Delta x + \left(\frac{\partial^2 \psi}{\partial x^2}\right)_{i,j} \frac{\Delta x^2}{2} - \left(\frac{\partial^3 \psi}{\partial x^3}\right)_{i,j} \frac{\Delta x^3}{6} + \ldots \qquad (2)$$

3.1. The Finite-Difference Method

- The finite difference expression for the first order derivative of ψ can be expressed by subtracting Eqn. (2) from (1).

$$\frac{\partial \psi}{\partial x} = \frac{\psi_{i+1,j} - \psi_{i-1,j} + \left(\frac{\Delta x^3}{3} \frac{\partial^3 \psi}{\partial x^3} \right)}{2\Delta x} \qquad (3)$$

- Neglecting the higher order derivatives in the Eq. (3), we get

$$\frac{\partial \psi}{\partial x} = \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta x} \qquad (4)$$

- Equation (4) is called Central Difference

3.1.The Finite-Difference Method

- The term "Central Difference" refers to show that the value of the variable depends on the values on both sides of the point (*i*,*j*).
- It is also possible to generate other expressions for the first derivative as:

$$\frac{\partial \psi}{\partial x} = \frac{\psi_{i+1,j} - \psi_{i,j}}{\Delta x} \qquad \textbf{Forward Difference} \qquad (5)$$

$$\frac{\partial \psi}{\partial x} = \frac{\psi_{i,j} - \psi_{i-1,j}}{\Delta x} \qquad \textbf{Backward Difference} \qquad (6)$$

3.1. The Finite-Difference Method

- Similarly, the *y* derivatives can be obtained in same manner.

$$\frac{\partial \psi}{\partial y} = \frac{\psi_{i,j+1} - \psi_{i,j-1}}{\Delta y} \qquad (7)$$

Central Difference

$$\frac{\partial \psi}{\partial y} = \frac{\psi_{i,j+1} - \psi_{i,j}}{\Delta y} \qquad (8)$$

Forward Difference

$$\frac{\partial \psi}{\partial y} = \frac{\psi_{i,j} - \psi_{i,j-1}}{\Delta y} \qquad (9)$$

Backward Difference

3.1. The Finite-Difference Method

- Moreover, the second derivative can be approximated using the Taylor series expansion. Thus, by adding Eqns. (1) and (2), we get

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x)^2} \qquad (10)$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y)^2} \qquad (11)$$

3.1. The Finite-Difference Method

- Sometimes, the derivative may be with respect to time. In this regards, the Taylor series expansions can be done similar to that of the derivatives for space. For instance, the expression for the forward difference approximation in time:

$$\frac{\partial \psi}{\partial t} = \frac{\psi_{i,j}^{k+1} + \psi_{i,j}^{k}}{\Delta t} \qquad (12)$$

3.1. The Finite-Difference Method

Exercise 3.1.

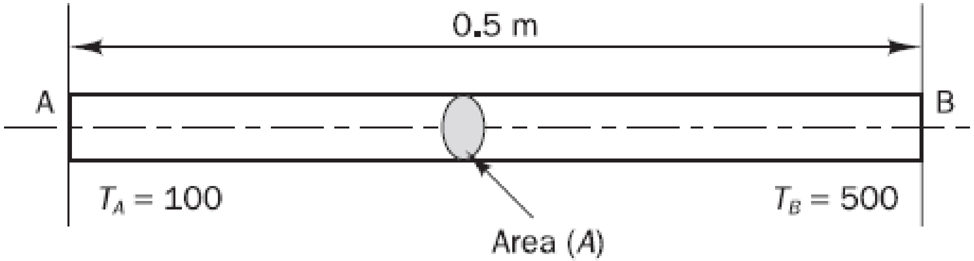
For a source-free heat conduction for the insulated rod shown below, ends A and B are kept at constant temperatures. The one-dimensional problem is governed by $k\dfrac{d^2T}{dx^2} = 0$,

i) obtain finite difference expression for the differential equation,

ii) the steady state temperature distribution in the rod.

Take k =1000 W/m.K,

$Area = 10 \times 10^{-3}$ m^2.



Exercise 3.2.

A circular rod of length 1 m and uniform cross sectional area is free from one end and connected to thin rectangular fin at the other end. The rod is cooled by means of convective heat transfer. Moreover, the free end has temperature of 30 ^oC and the one attached to the fin is kept at 200 ^oC. One-dimensional heat transfer in this situation is governed by

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(20 - T) = 0 .$$

Where $hP/(kA) = 25\ \text{m}^2$,

Calculate the temperature distribution along the fin.

Summary

- The partial differential equations representing the flow process should be converted to systems of algebraic equations using discretization techniques.

- The oldest discretization method is the finite difference method.
- The finite difference discretization technique in space can be written in one of the following forms: central difference, forwards difference, and backward difference.

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Research interest:

- Computational Fluid Dynamics,
- Thermo-fluids,
- Multidisciplinary Numerical Modelling and Simulation

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