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Computational Fluid Dynamics

Lecture Two

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Equations Describing Fluids in Motion

- Aims
  - The aim of this lecture is to recall the basic equations related to fluid flow and heat transfer.
- Expected Outcomes: At the end of this lecture, students should be able to
  - understand the governing continuity, momentum, and energy equations
  - derive 2D continuity and momentum equations
  - solve 1D flow problems analytically
- References

1) J. Tu, G.H. Yeoh, C. Liu, Computational Fluid Dynamics : A Practical Approach, Elsevier, 1st Edition, 2013.
2) C.T. Shaw, Using Computational Fluid Dynamics, Prentice Hall, 1992

Contents

Feature of some Common Flows

- Lagrangian description
- Eulerian descriptions

Governing Equations of Flow

- Conservation of mass

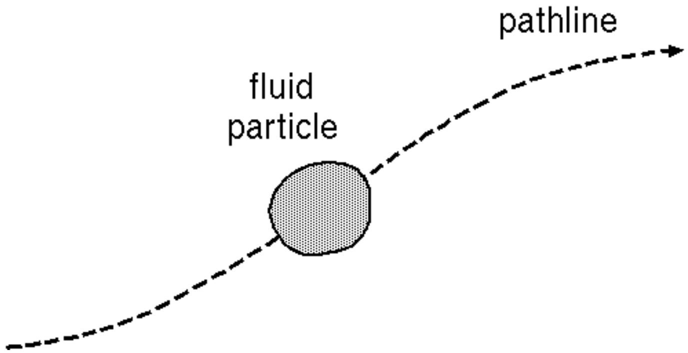
Feature of some Common Flows

- For people who use CFD in an industrial environment engrossing in the computational aspect and excluding everything else can be a disastrous mistake.
- The purpose of computer hardware and software is ONLY to help us understand how fluids flow and their interaction with other objects.
- Thus, in order to use CFD it is important to have good understanding of the physical phenomena in fluid flow.

Lagrangian and Eulerian Descriptions

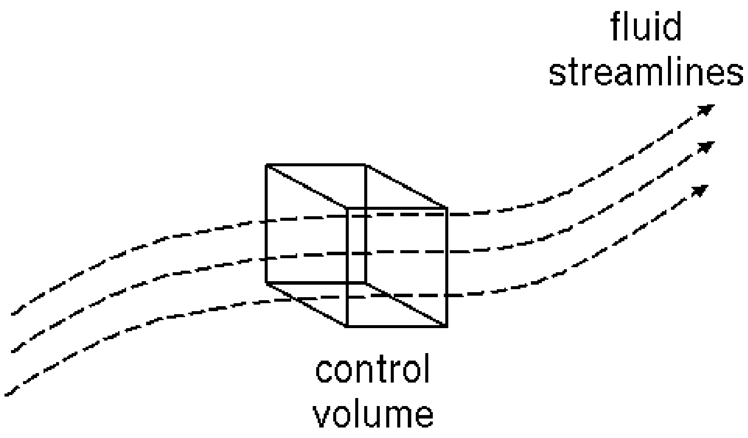
Lagrangian Description of fluid motion

- This method focuses on the motion (movement) of fluid particles by tracking each particle's vector position and determines the fluid property (such as velocity, mass flow, change in energy etc) as function of time.
- In this case, a mathematical equation is given for each fluid particle.



Eulerian Description of fluid motion

- This method focuses on the motion (movement) of fluid particles inward and outward to the control volume.
- Instead of following each fluid particle, the Eulerian description states the flow variables values of location only, i.e., it doesn't care which fluid particle occupies which location in the control volume



Flow Governing Equations

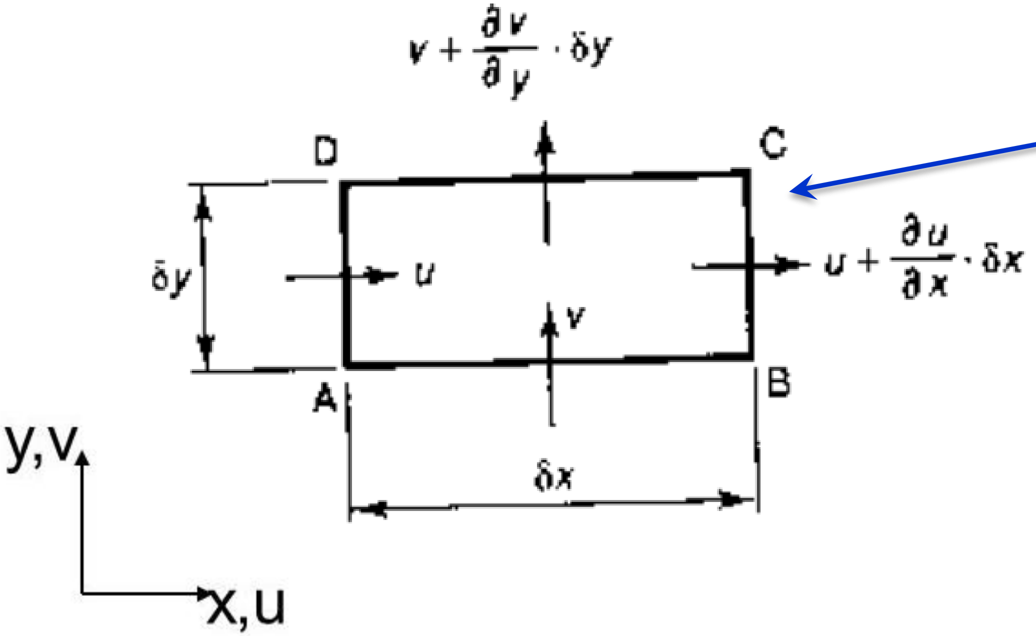
- The main governing equations in any fluid flow are representation of the following laws:
  - Conservation of mass:- for any closed system, the mass of the system must remain conserved
  - Conservation of momentum:- if no external forces are applied to the system, the momentum of the system should not change (remain constant) (Newton's second law)
  - Conservation of energy:- The rate of change of energy is equal to the sum of the rate of heat addition to and the rate of work done on a fluid particle (first law of thermodynamics)

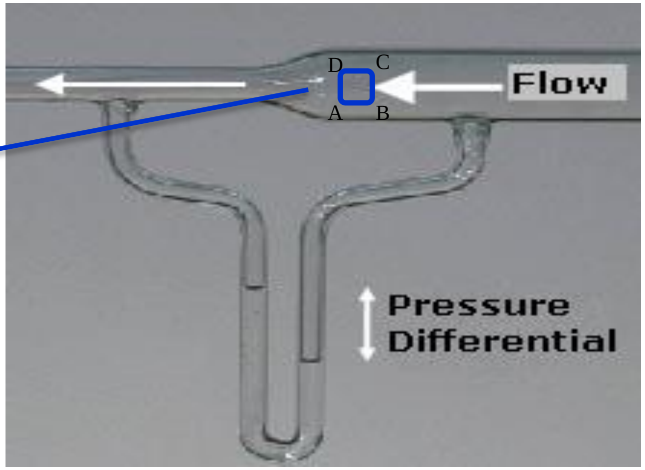
Developing the Governing Equations

- Every Computational Fluid Dynamics software package need predict the nature of the fluid flow as well as the heat and mass transfer for a given situation.

- To perform this, the numerical solutions for the flow governing equations must be calculated.

a) Conservation of mass





https://commons.wikimedia.org/wiki/File:VenturiFlow.png

Rate of increase of mass in fluid element	=	Net rate of flow of mass into fluid element

- Assuming the flow to be incompressible, no fluid will accumulate in the control volume. This is due to the fact that the fluid can not be compressed as its density is assumed to be a constant.

- Thus, the total mass flow rate of fluid flowing into the control volume must be zero.

- The net mass flow given by the sum of the masses flowing across each face must be zero.

$$\sum_{in} \dot{m} = \rho \left(u \, \delta y + v \, \delta x \right)$$

$$\sum_{out} \dot{m} = \rho \left(u + \frac{\partial u}{\partial x} \delta x \right) \delta y + \rho \left(v + \frac{\partial v}{\partial y} \delta y \right) \delta x$$

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

$$\Rightarrow \rho \left(u \, \delta y + v \, \delta x \right) = \rho \left(u + \frac{\partial u}{\partial x} \delta x \right) \delta y + \rho \left(v + \frac{\partial v}{\partial y} \delta y \right) \delta x$$

Which can be simplified as :

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

This is called two-dimensional continuity equation

The above equation can be extended to three-dimensional case as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- For compressible flow, the density term should be considered. Thus the continuity equation will have the form:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

- In vector notation:

$$\boxed{\frac{\partial \rho}{\partial t}+div(\rho \mathbf{u})=0}$$

Change in density

Net flow of mass across boundaries
Convective term

- For incompressible fluids $\frac{\partial \rho}{\partial t}=0$, and the equation becomes: div **V** = 0.
- Alternative ways to write this: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ and $\frac{\partial V_i}{\partial x_i}=0$

Exercise 1

Consider unsteady, 2D velocity field shown below:

$$\vec{V} = (u, v) = \left(2 + 3.2x\right)\vec{i} + \left(6 + 10\boldsymbol{sin}(\omega t) - 3.2y\right)\vec{j}$$

Where, the angular velocity is expressed as $\omega = 2\pi \text{ rad/s}$

Show that the flow field can be approximated as incompressible.

Exercise 2

Consider a three dimensional, steady and incompressible flow field. Two of its velocity components are given as

$$u = \alpha x^2 + \beta y^2 + \gamma z^2 \text{ and}$$
$$w = \alpha xz + \beta yz^2$$

Where, $\alpha, \beta, \text{and } \gamma$ are constants. The y component is not known.

Develop an expression for the y velocity component, *v*, as a function of *x*, *y*, and *z*.

Exercise 3

An air-fuel mixture is compressed by a piston in a cylinder of an internal combustion engine. The piston is assumed to move up at constant speed V_p. The distance L between the top of the cylinder and the piston decreases with time according to the linear approximation

$$L = L_{bottom} - V_p t,$$

where L_{bottom} is the location of the piston when it is at the bottom of the cylinder at time $t=0$. At $t=0$, the density of the air-fuel mixture in the cylinder is equal to ρ_o.

Determine the density of the air-fuel mixture as a function of time and the given parameters during the piston's up stroke.

Solution

- ✓ First develop linear approximation of the vertical velocity *v* as function of *L* and V_p.

$v=0$ *at* $y=0$, and

$v=V_p$ *at* $y=L$.

$$\Rightarrow v=-V_p\frac{y}{L}$$

- ✓ The compressible continuity equation is appropriate for this problem,

$$\frac{\partial\rho}{\partial t}+\cancel{\frac{\partial(\rho u)}{\partial x}}+\frac{\partial(\rho v)}{\partial y}+\cancel{\frac{\partial(\rho w)}{\partial z}}=0 \qquad \Rightarrow\frac{\partial\rho}{\partial t}+\rho\frac{\partial v}{\partial y}=0$$

- ✓ Substituting the velocity value we get

$$\frac{\partial\rho}{\partial t}+\rho\frac{\partial}{\partial y}\left(-V_p\frac{y}{L}\right)=0 \qquad \Rightarrow\frac{\partial\rho}{\partial t}=\frac{\rho V_p}{L}=\frac{\rho V_p}{L_{bottom}-V_p t}$$

✓ Rearranging similar variables integrating, we get

$$\int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = \int_{0}^{t} \frac{V_p}{L_{bottom} - V_p t} \qquad \Rightarrow \boldsymbol{ln} \frac{\rho}{\rho_o} = \boldsymbol{ln} \frac{1}{L_{bottom} - V_p t}$$

✓ Then, the desired expression for density as function of time is:

$$\Rightarrow \rho = \rho_o \frac{1}{L_{bottom} - V_p t}$$

Conclusion of The Chapter

- Lagrangian description of flow
  - requires tracing each fluid particle,
  - Is computationally expensive.
- Eulerian description of flow
  - how flow properties vary at a fluid element that is fixed in space and time.
  - Easy for computational programming
- Two and three dimensional conservation of mass

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

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Research interest:

- Computational Fluid Dynamics,
- Thermo-fluids,
- Multidisciplinary Numerical Modelling and Simulation

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