Automatic Control

Modelling of Dynamic Systems
(Transfer Function: Electro-Mechanical Systems)

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Chapter Description

- Aims
  - To expose students to construction of transfer function of Electro-mechanical system

- Expected Outcomes
  - Student will be able to create transfer function for a mechanical system actuated by a motor including gear trains

Reference: Norman S. Nise, 2008. Control Systems Engineering, sixth Edition, John Wiley & Sons, Inc

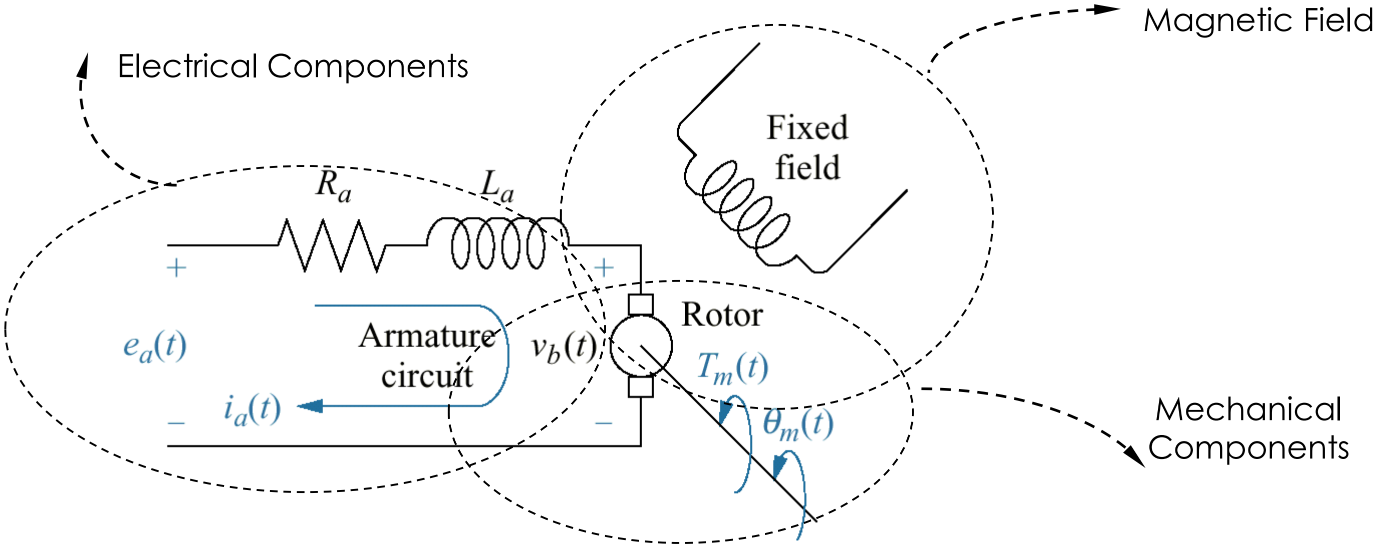
Content

- Create transfer function of a rotational or translational mechanical system, actuated by a motor and including gear train.

Introduction

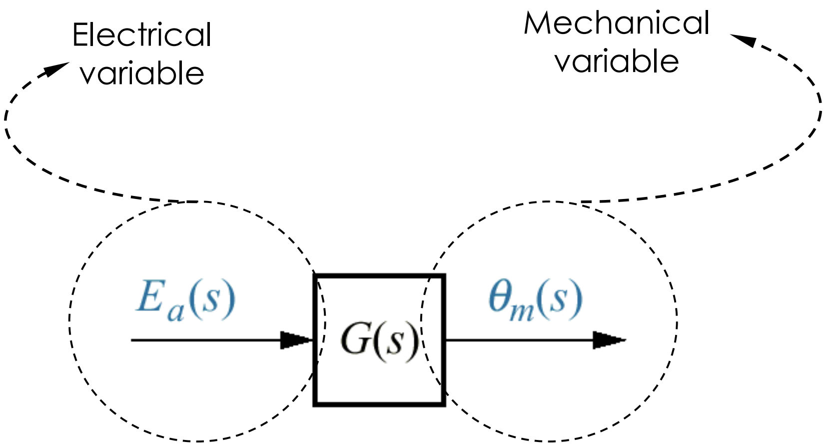
This chapter deals with systems involving electrical and mechanical components. System whereby power flows between electrical and mechanical components. Sometimes called mechatronic system.

Permanent Magnet DC Motor – Schematic Diagram



Reference: Norman S. Nise, 2008. Control Systems Engineering, sixth Edition, John Wiley & Sons, Inc

Input & Output

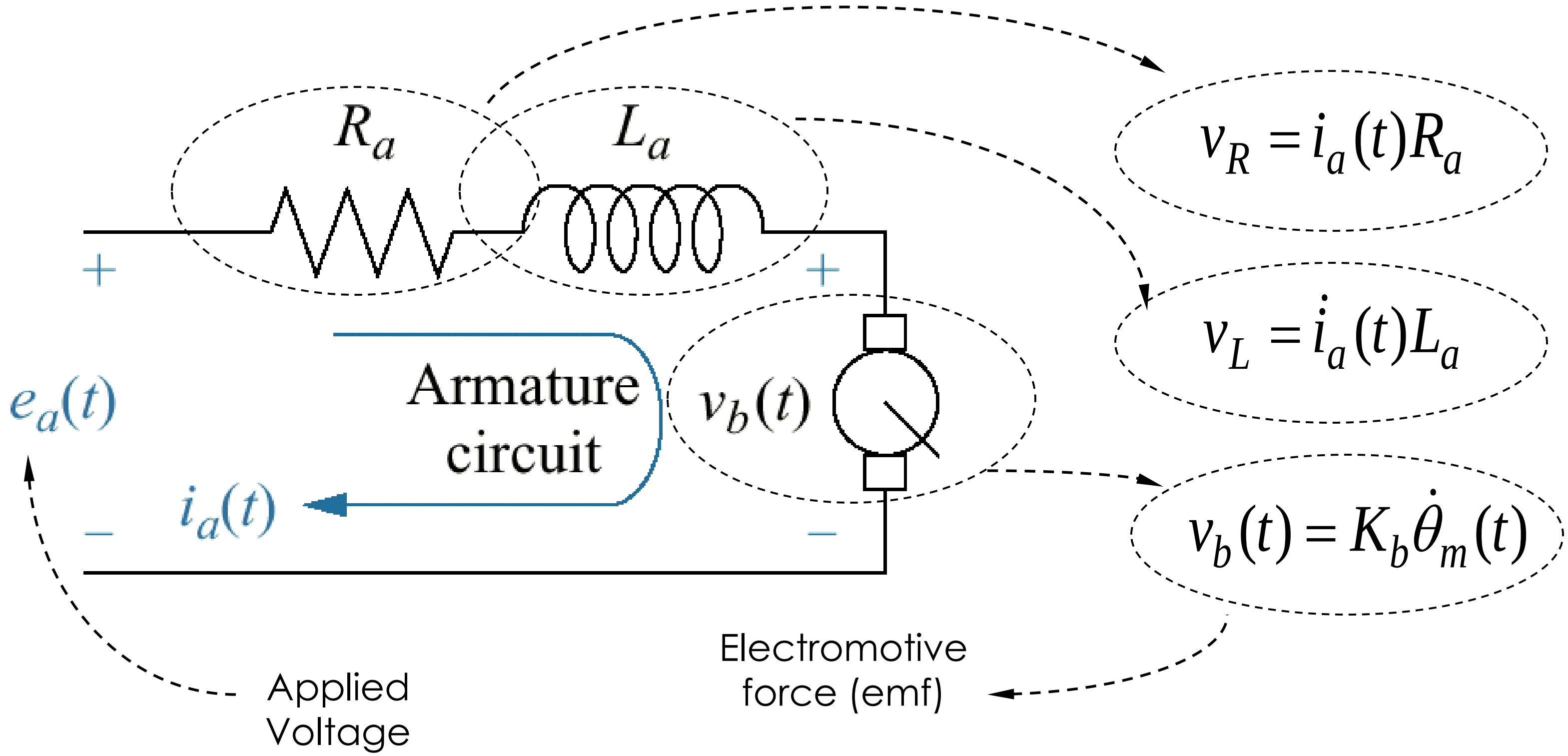


Electrical variable CONTROLS mechanical variable

Input: Electrical variable (applied voltage)
Output: mechanical variable (angular position)

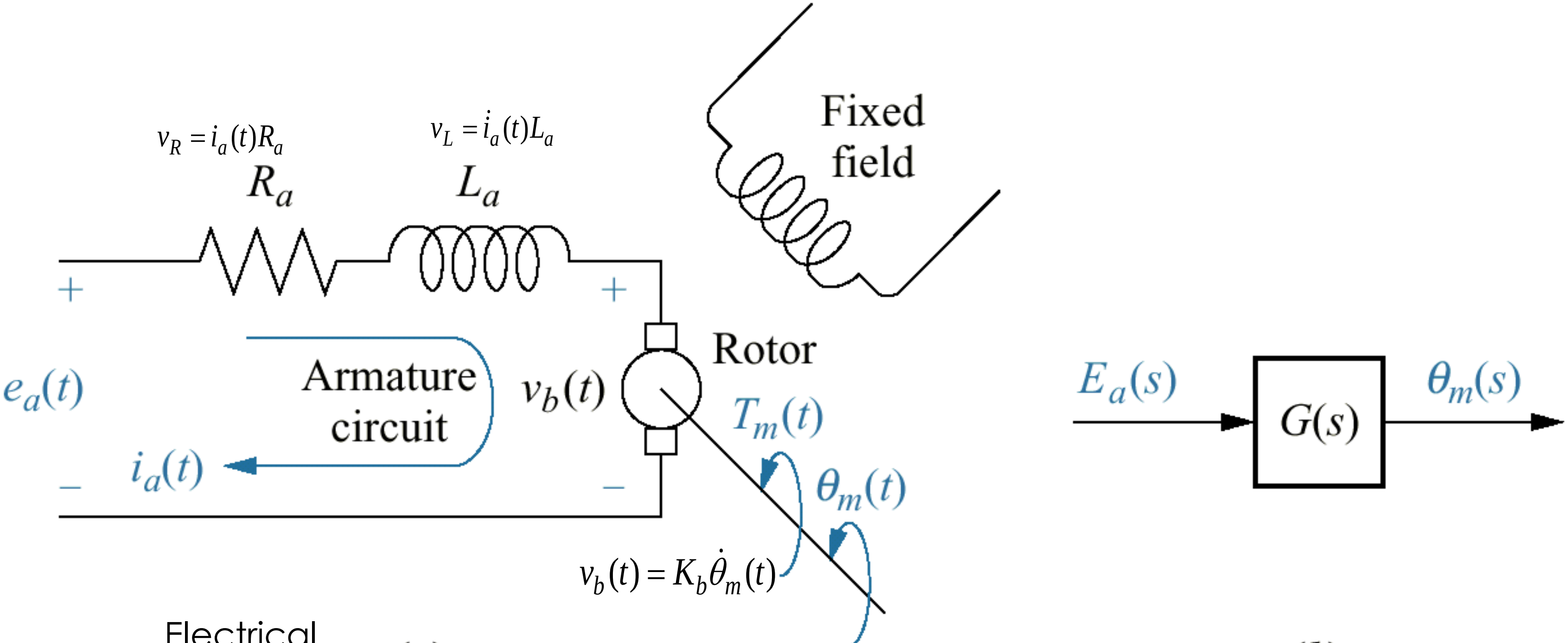
Equation: Electrical Components

Using Kirchoff's Voltage Law (KVL) around the loop



Equation: Electrical Components

Using Kirchoff's Voltage Law (KVL) around the loop



Electrical Equation

(a) **(b)**

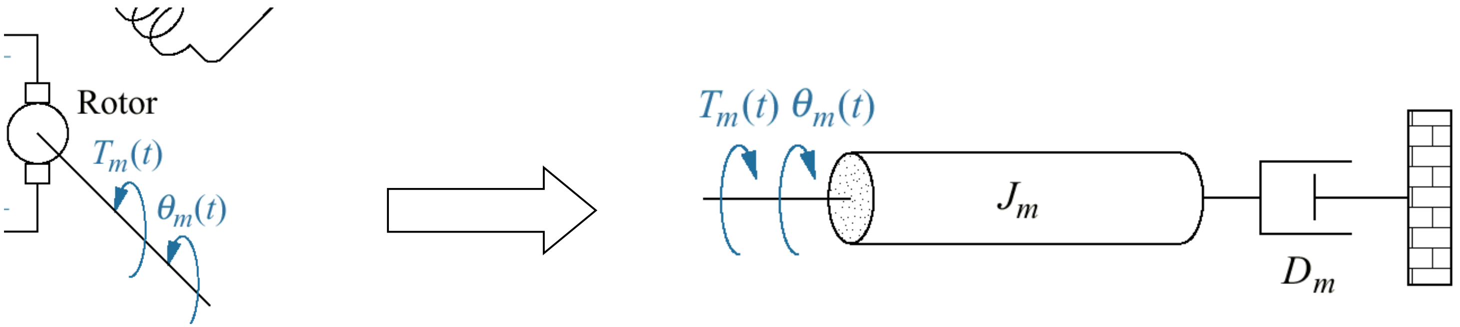
$$V_R + V_L + V_b = e_a(t)$$

$$i_a(t)R_a + \dot{i}_a(t)L_a + K_b\dot{\theta}_m(t) = e_a(t)$$

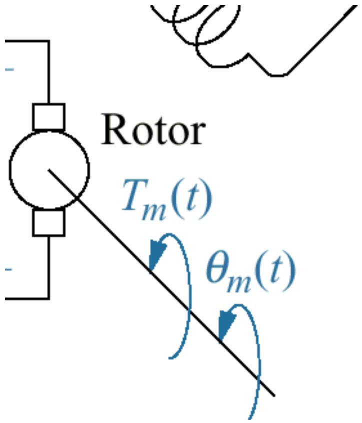
Equation: Mechanical Component

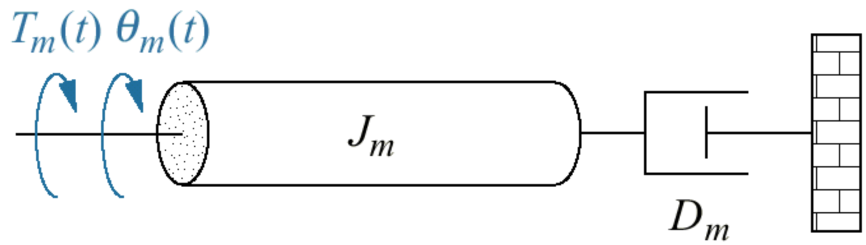
Rotor Schematic Model

Mechanical Components

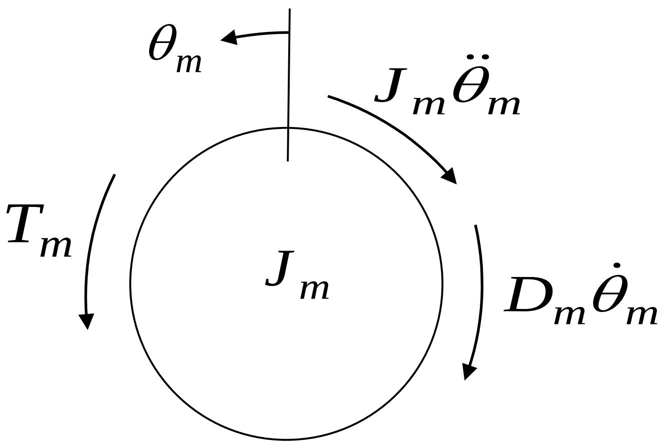


Equation: Mechanical Component





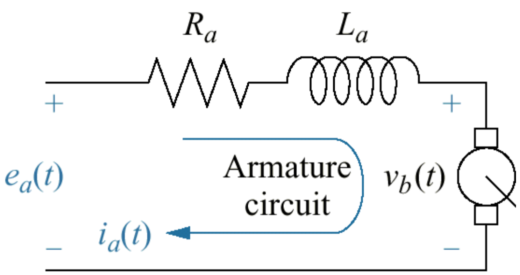
Use Newton's Second Law and Free Body Diagram



$$J_m\ddot{\theta}_m + D_m\dot{\theta}_m = T_m$$

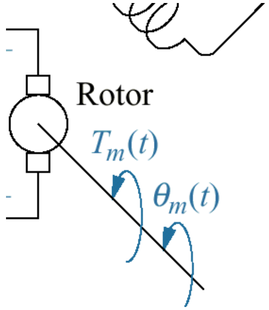
Mechanical Equation

All the Equations



$$i_a R_a + \dot{i}_a L_a + K_b \dot{\theta}_m = e_a$$

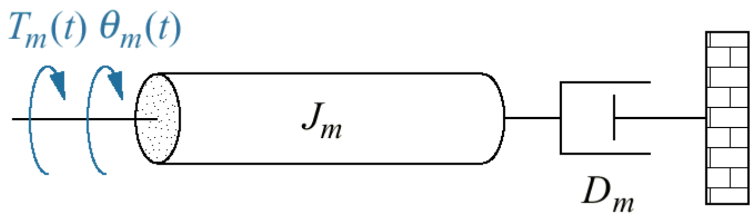
Electrical Equation



$$T_m = K_t i_a$$

Torque-Current Equation

Electro-mechanical Equation



$$J_m \ddot{\theta}_m + D_m \dot{\theta}_m = T_m$$

Mechanical Equation

All the Equations

Electrical Equation

$$i_a R_a + \dot{i}_a L_a + K_b \dot{\theta}_m = e_a$$

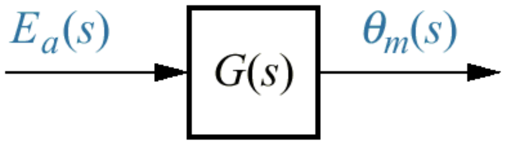
Mechanical Equation

$$J_m \ddot{\theta}_m + D_m \dot{\theta}_m = T_m$$

Electro-mechanical Equation

$$T_m = K_t i_a$$

Combined Equation in order to obtain the transfer function



Finding the Transfer Function

Transform differential equations into frequency domain

$$i_a R_a + \dot{i}_a L_a + K_b \dot{\theta}_m = e_a \qquad I_a(s)R_a + sI_a(s)L_a + s\theta_m(s)K_b = E_a(s)$$

$$J_m \ddot{\theta}_m + D_m \dot{\theta}_m = T_m \qquad s^2 J_m \theta_m(s) + sD_m \theta_m(s) = T_m(s)$$

$$T_m = K_t i_a \qquad T_m(s) = K_t I_a(s)$$

Actual condition is that Resistor is much greater than Inductor

$$I_a(s)R_a + \cancel{sI_a(s)L_a} + s\theta_m(s)K_b = E_a(s)$$

Finding the Transfer Function

Combine Equations

$$I_a(s)R_a + sI_a(s)L_a + s\theta_m(s)K_b = E_a(s)$$

$$T_m(s) = K_t I_a(s)$$

$$\frac{T_m(s)}{K_t}R_a + s\theta_m(s)K_b = E_a(s)$$

$$s^2 J_m\theta_m(s) + sD_m\theta_m(s) = T_m(s)$$

$$\frac{s^2 J_m\theta_m(s) + sD_m\theta_m(s)}{K_t}R_a + s\theta_m(s)K_b = E_a(s)$$

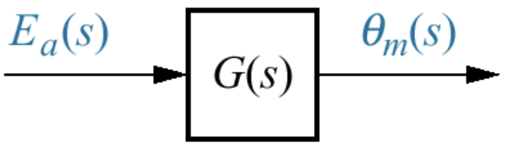
Finding the Transfer Function

Combine Equations

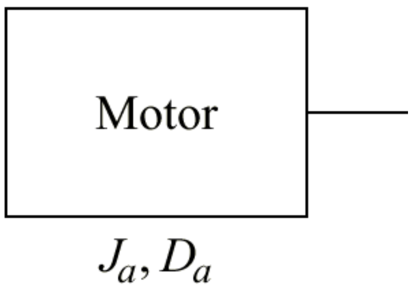
$$\frac{s^2 J_m \theta_m(s) + s D_m \theta_m(s)}{K_t} R_a + s\theta_m(s) K_b = E_a(s)$$

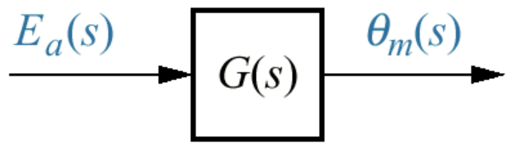
$$[s^2 J_m \frac{R_a}{K_t} + s D_m \frac{R_a}{K_t} + s K_b]\theta_m(s) = E_a(s)$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s^2 J_m \frac{R_a}{K_t} + s D_m \frac{R_a}{K_t} + s K_b}$$



Finding the Transfer Function

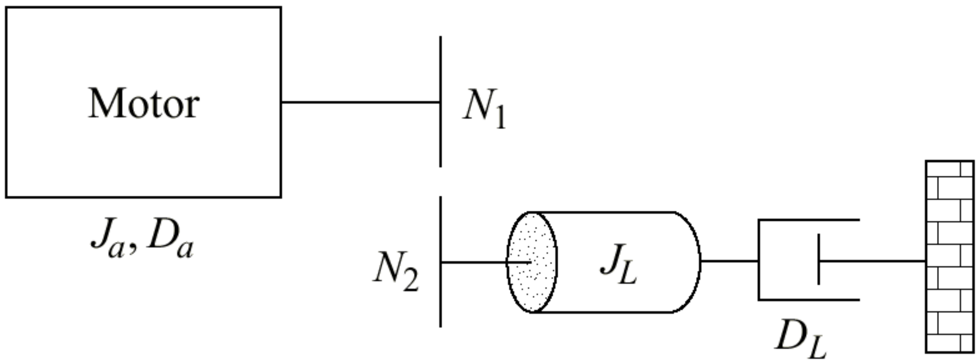


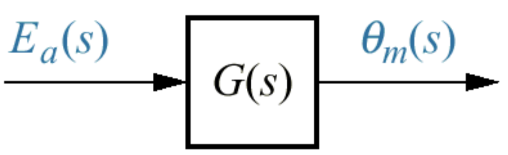


$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s^2 J_m \frac{R_a}{K_t} + sD_m \frac{R_a}{K_t} + sK_b}$$

DC Motor Connected to Load

What is the transfer function with load?





$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s^2 J_m \frac{R_a}{K_t} + sD_m \frac{R_a}{K_t} + sK_b}$$

DC Motor Connected to Load

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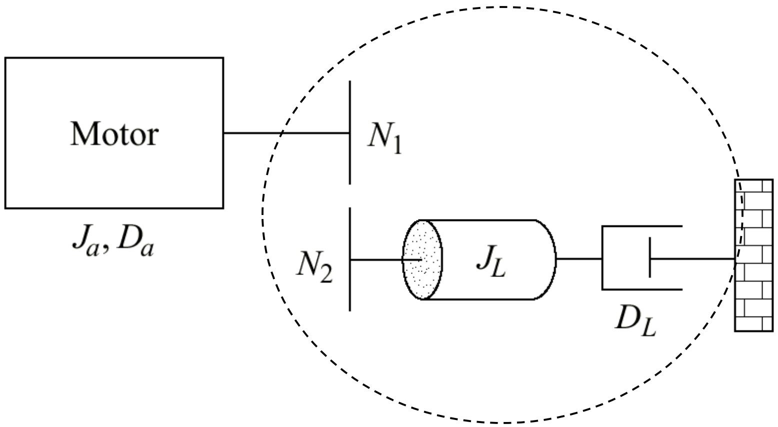

Not equal because the load on motor shaft is now reflected by the load shaft

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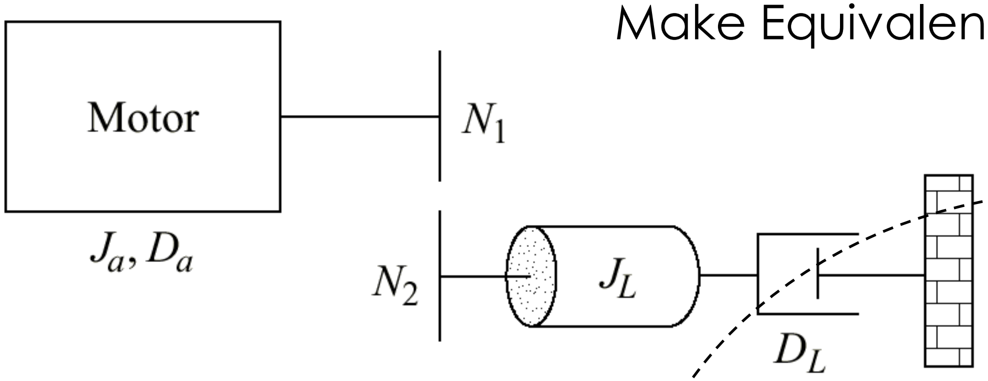
DC Motor Connected to Load

Make Equivalent Shaft



$$J_e = J_a + J_L \left(\frac{N_1}{N_2} \right)^2$$

$$D_e = D_a + D_L \left(\frac{N_1}{N_2} \right)^2$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s^2 J_e \frac{R_a}{K_t} + s D_e \frac{R_a}{K_t} + s K_b}$$

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DC Motor Connected to Load

Mechanical constants

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s^2 J_e \frac{R_a}{K_t} + s D_e \frac{R_a}{K_t} + s K_b}$$

J_e Equivalent Moment of Inertia

D_e Equivalent Damper

R_a Armature resistance

K_b EMF constant

K_t Torque constant

What parameters are required?

Electrical constants

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DC Motor Connected to Load

Mechanical constants

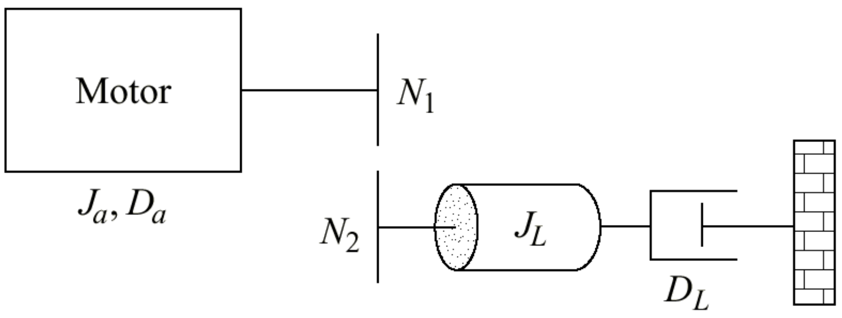
$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s^2 J_e \frac{R_a}{K_t} + sD_e \frac{R_a}{K_t} + sK_b}$$

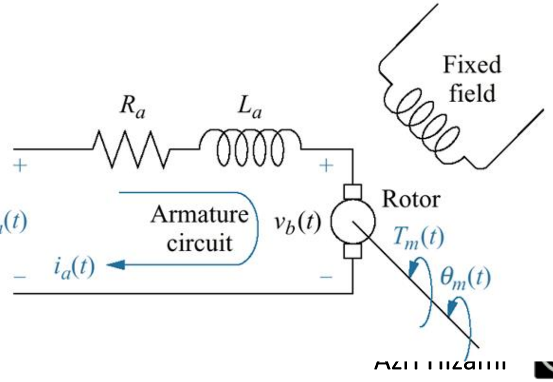
Equivalent Moment of Inertia

$$J_e = J_a + J_L \left(\frac{N_a}{N_L} \right)^2$$

Equivalent Damper

$$D_e = D_a + D_L \left(\frac{N_a}{N_L} \right)^2$$





Evaluation of Motor Parameters

Electrical constants

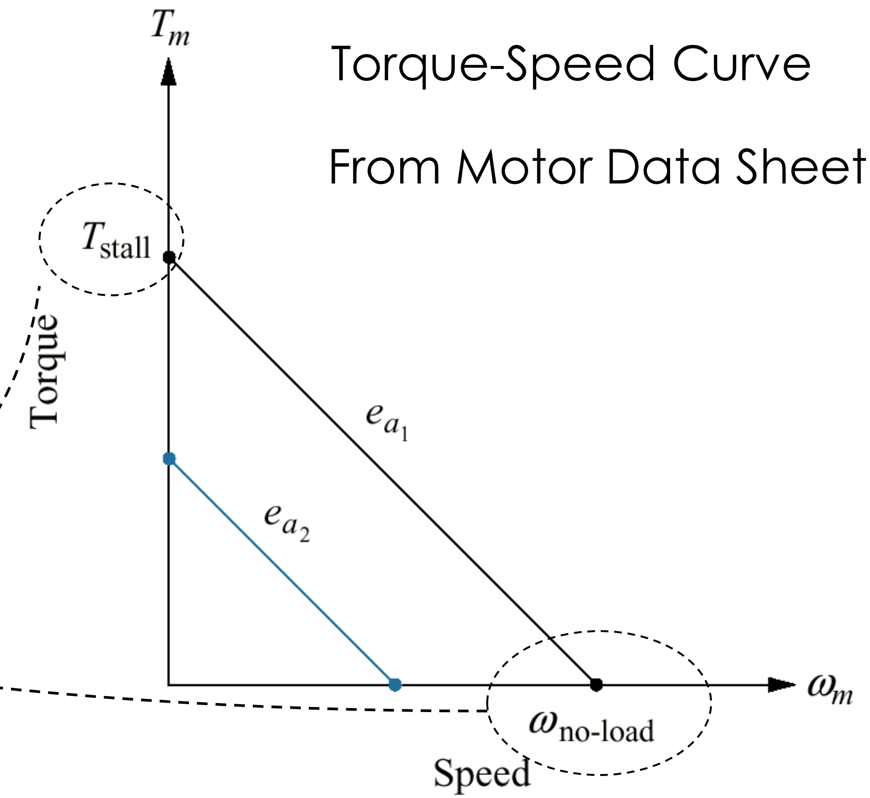
$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s^2 J_e \frac{R_a}{K_t} + s D_e \frac{R_a}{K_t} + s K_b}$$

$$K_b = \frac{e_a}{\omega_{no-load}}$$

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a}$$

Torque-Speed Curve

From Motor Data Sheet



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Evaluation of Motor Parameters

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s^2 J_e \frac{R_a}{K_t} + sD_e \frac{R_a}{K_t} + sK_b}$$

Mechanical constants

$$J_e = J_a + J_L \left(\frac{N_a}{N_L} \right)^2$$

$$D_e = D_a + D_L \left(\frac{N_a}{N_L} \right)^2$$

Electrical constants

$$K_b = \frac{e_a}{\omega_{no-load}}$$

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a}$$

Thank you

Should there be any question, please contact the author at
mahizami@ump.edu.my

Credit: The slides were developed together with Dr. Gigih Priyandoko of the Faculty of Mechanical Engineering, UMP