Translational Transfer Function with 2 DoF
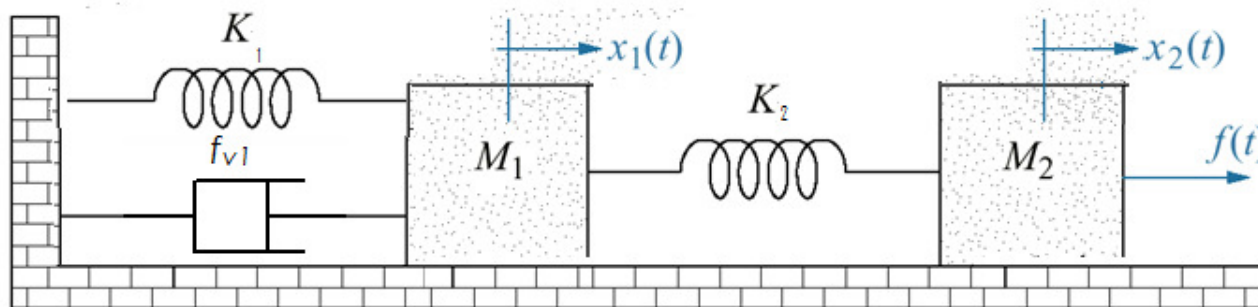

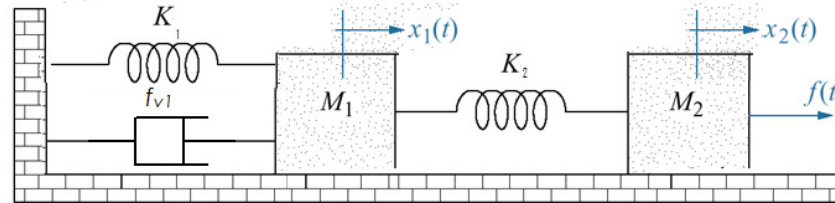Find the transfer function G(s) = X2(s)/F(s)

Reference : Norman S. Nise, 2008. Control Systems Engineering, sixth Edition, John Wiley & Sons, Inc. (**Chapter 2 – Problem 55**)

Quiz: Chapter 3

Answer :

Input variable: Applied force $f(t)$

Output variable: Mass position $x_2(t)$

FPD on M1 $\Big|$ $M_1 s^2 X_1(s) + f_{v1} s X_1(s) + k X_1(s) + k2 X_1(s) - k_2 X_2(s) = 0$

FPD on M2 $\Big|$ $-k_2 X_1(s) + M_2 s^2 X_2(s) + k_2 X_2(s) = F(s)$

$$\leftrightarrow \begin{bmatrix} M_1 s^2 + f_{v1} s + k_1 + k_2 & -k_2 \\ -k_2 & M_2 s^2 + k_1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ F \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{\begin{bmatrix} 0 \\ F \end{bmatrix}}{\begin{bmatrix} M_1 s^2 + f_{v1} s + k_1 + k_2 & -k_2 \\ -k_2 & M_2 s^2 + k_1 \end{bmatrix}}$$

$$X_1 = \frac{\begin{bmatrix} 0 & -k_2 \\ F & M_2 s^2 + k_1 \end{bmatrix}}{\begin{bmatrix} M_1 s^2 + f_{v1} s + k_1 + k_2 & -k_2 \\ -k_2 & M_2 s^2 + k_1 \end{bmatrix}} \quad X_2 = \frac{\begin{bmatrix} M_1 s^2 + f_{v1} s + k_1 + k_2 & 0 \\ -k_2 & F \end{bmatrix}}{\begin{bmatrix} M_1 s^2 + f_{v1} s + k_1 + k_2 & -k_2 \\ -k_2 & M_2 s^2 + k_1 \end{bmatrix}}$$

$$G(s) = \frac{X_2(s)}{F(s)} = ...$$