Engine Design

ASSIGNMENT 2_SAMPLE ANSWER

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ENGINE SPECIFICATIONS

Engine Analysis of KTM DUKE 200

Bore, B	= 72mm
Stroke, S	= 49mm
Connecting Rod, r	= 98mm
Crank Radius, a	= 24.5mm
Idle Speed	= 3000rpm
Power Max	= 18.6425 kW @10000rpm
Torque Max	= 19.2 Nm @ 8000rpm

Derivation Formula

l = rod length
r = crank radius (half stroke)
A = crank angle (from TDC)
x = position of piston pin from crank center

$$l^2 = r^2 + x^2 - 2.r.x.\cos(A)$$

P : piston pin
N : crank pin
O : crank center

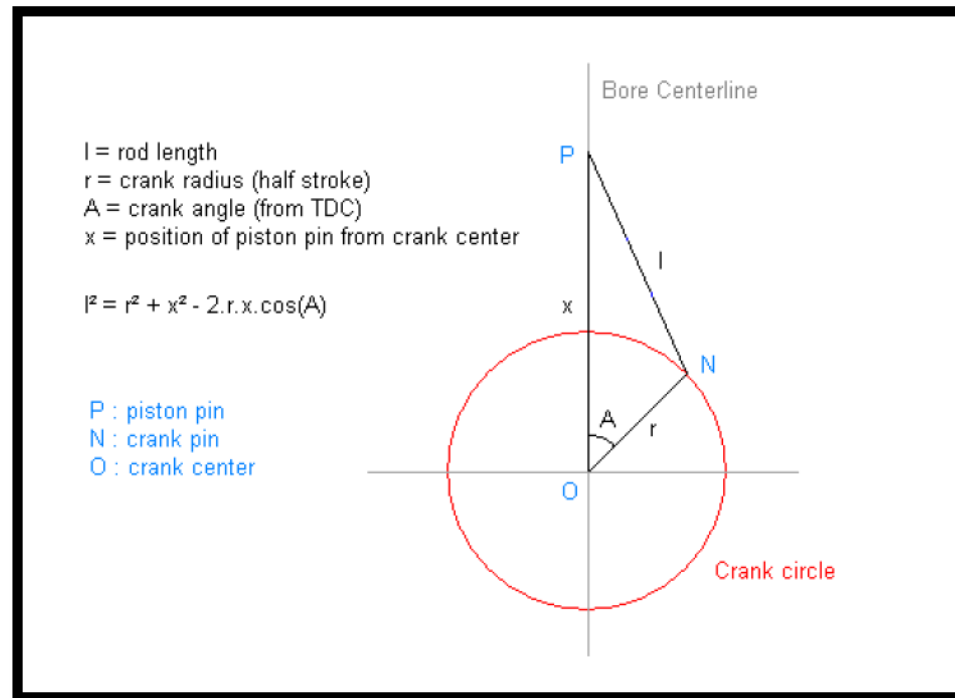Bore Centerline

Crank circle

Figure 1: Diagram showing geometric layout of piston pin, crank pin and crank centre

1) Piston Position/ Displacement

Position with respect to crank angle (by rearranging the triangle relation):

$$l^2 - r^2 = x^2 - 2 \cdot r \cdot x \cdot \cos A$$
$$l^2 - r^2 = x^2 - 2 \cdot r \cdot x \cdot \cos A + r^2 [(\cos^2 A + \sin^2 A) - 1]$$
$$l^2 - r^2 + r^2 - r^2 \sin^2 A = x^2 - 2 \cdot r \cdot x \cdot \cos A + r^2 \cos^2 A$$
$$l^2 - r^2 \sin^2 A = (x - r \cdot \cos A)^2$$
$$x - r \cdot \cos A = \sqrt{l^2 - r^2 \sin^2 A}$$
$$x = r \cos A + \sqrt{l^2 - (r \sin A)^2}$$

2) Piston Velocity

Velocity with respect to crank angle (take first derivative, using the chain rule):

$$x' = \frac{dx}{dA}$$

$$= -r\sin A + \frac{(\frac{1}{2}).(-2).r^2 \sin A \cos A}{\sqrt{l^2 - r^2 \sin^2 A}}$$

$$= -r\sin A - \frac{r^2 \sin A \cos A}{\sqrt{l^2 - r^2 \sin^2 A}}$$

3) Piston Acceleration

Acceleration with respect to crank angle (take second derivative, using the chain rule and the quotient rule):

$$x'' = \frac{d^2 x}{dA^2}$$

$$= -r\cos A - \frac{r^2 \cos^2 A}{\sqrt{l^2 - r^2 \sin^2 A}} - \frac{-r^2 \sin^2 A}{\sqrt{l^2 - r^2 \sin^2 A}} - \frac{r^2 \sin A \cos A.(-\frac{1}{2}).(-2).r^2 \sin A \cos A}{\left(\sqrt{l^2 - r^2 \sin^2 A}\right)^3}$$

$$= -r\cos A - \frac{r^2 (\cos^2 A - \sin^2 A)}{\sqrt{l^2 - r^2 \sin^2 A}} - \frac{r^4 \sin^2 A \cos^2 A}{\left(\sqrt{l^2 - r^2 \sin^2 A}\right)^3}$$

MATLAB CODING

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Editor - C:\Users\SARINDRAN RAMAYES\Downloads\Sem 6\Engine Design\Assignments\engineanalysis.m

engineanalysis.m

1    %Assignment 2: Engine Analysis of KTM Duke 200
2    %Single Cylinder Oversquare
3    %Engine Specifications:
4    %Bore Length,B : 72mm
5    %Stroke Length,S : 49mm
6    %Con-Rod Length,r : 98mm
7    %Crank Radius,a : 24.5mm
8    %Idling Speed : 3000rpm
9    %Power Max : 18.6425kw@10000rpm
10   %Torque Max : 19.2N.m@8000rpm
11
12   clc
13   clear
14   r=98;
15   a=24.5;
16   crankangle=linspace(0,2*pi,60);
17   pistonheight= (a*cos(crankangle)) + (sqrt(r.^2 - a.^2*sin(crankangle).^2))-(r-a);
18   pistonvelocity=diff(pistonheight);
19   pistonacceleration=diff(diff(pistonheight));
```

```matlab
21      %Piston Position vs Crank Angle
22 -    figure(1);
23 -    plot(crankangle,pistonheight,'b', 'LineWidth',2);
24 -    indexmin = find(min(pistonheight) == pistonheight);
25 -    crankanglemin = crankangle(indexmin);
26 -    pistonaccelerationmin = pistonheight(indexmin);
27 -    indexmax = find(max(pistonheight) == pistonheight);
28 -    crankanglemax = crankangle(indexmax);
29 -    pistonaccelerationmax = pistonheight(indexmax);
30 -    strmin = ['Minimum = ',num2str(pistonaccelerationmin)];
31 -    text(crankanglemin,pistonaccelerationmin,strmin,'HorizontalAlignment','left');
32 -    strmax = ['Maximum = ',num2str(pistonaccelerationmax)];
33 -    text(crankanglemax,pistonaccelerationmax,strmax,'HorizontalAlignment','right');
34 -    ylabel('Piston Position (mm)')
35 -    xlabel('Crank Angle (rad)')
36 -    title('Plot of Piston Position vs Crank Angle','FontSize',12)
37 -    hold on;
38
```

```matlab
39    %Piston Velocity vs Crank Angle
40 -  figure(2);
41 -  plot(crankangle(2:end),pistonvelocity,'r','LineWidth',2);
42 -  indexmin = find(min(pistonvelocity) == pistonvelocity);
43 -  crankanglemin = crankangle(indexmin);
44 -  pistonaccelerationmin = pistonvelocity(indexmin);
45 -  indexmax = find(max(pistonvelocity) == pistonvelocity);
46 -  crankanglemax = crankangle(indexmax);
47 -  pistonaccelerationmax = pistonvelocity(indexmax);
48 -  strmin = ['Minimum = ',num2str(pistonaccelerationmin)];
49 -  text(crankanglemin,pistonaccelerationmin,strmin,'HorizontalAlignment','left');
50 -  strmax = ['Maximum = ',num2str(pistonaccelerationmax)];
51 -  text(crankanglemax,pistonaccelerationmax,strmax,'HorizontalAlignment','right');
52 -  ylabel('Piston Velocity (m/s)')
53 -  xlabel('Crank Angle (rad)')
54 -  title('Plot of Piston Velocity vs Crank Angle','FontSize',12)
55 -  hold on;
```

```matlab
57        %Piston Acceleration vs Crank Angle
58 -      figure(3);
59 -      plot(crankangle(3:end),pistonacceleration,'g','LineWidth',2);
60 -      indexmin = find(min(pistonacceleration) == pistonacceleration);
61 -      crankanglemin = crankangle(indexmin);
62 -      pistonaccelerationmin = pistonacceleration(indexmin);
63 -      indexmax = find(max(pistonacceleration) == pistonacceleration);
64 -      crankanglemax = crankangle(indexmax);
65 -      pistonaccelerationmax = pistonacceleration(indexmax);
66 -      strmin = ['Minimum = ',num2str(pistonaccelerationmin)];
67 -      text(crankanglemin,pistonaccelerationmin,strmin,'HorizontalAlignment','left');
68 -      strmax = ['Maximum = ',num2str(pistonaccelerationmax)];
69 -      text(crankanglemax,pistonaccelerationmax,strmax,'HorizontalAlignment','right');
70 -      ylabel('Piston Acceleration (m/s^2)')
71 -      xlabel('Crank Angle (rad)')
72 -      title('Plot of Piston Acceleration vs Crank Angle','FontSize',12)
73 -      hold on;
74
```
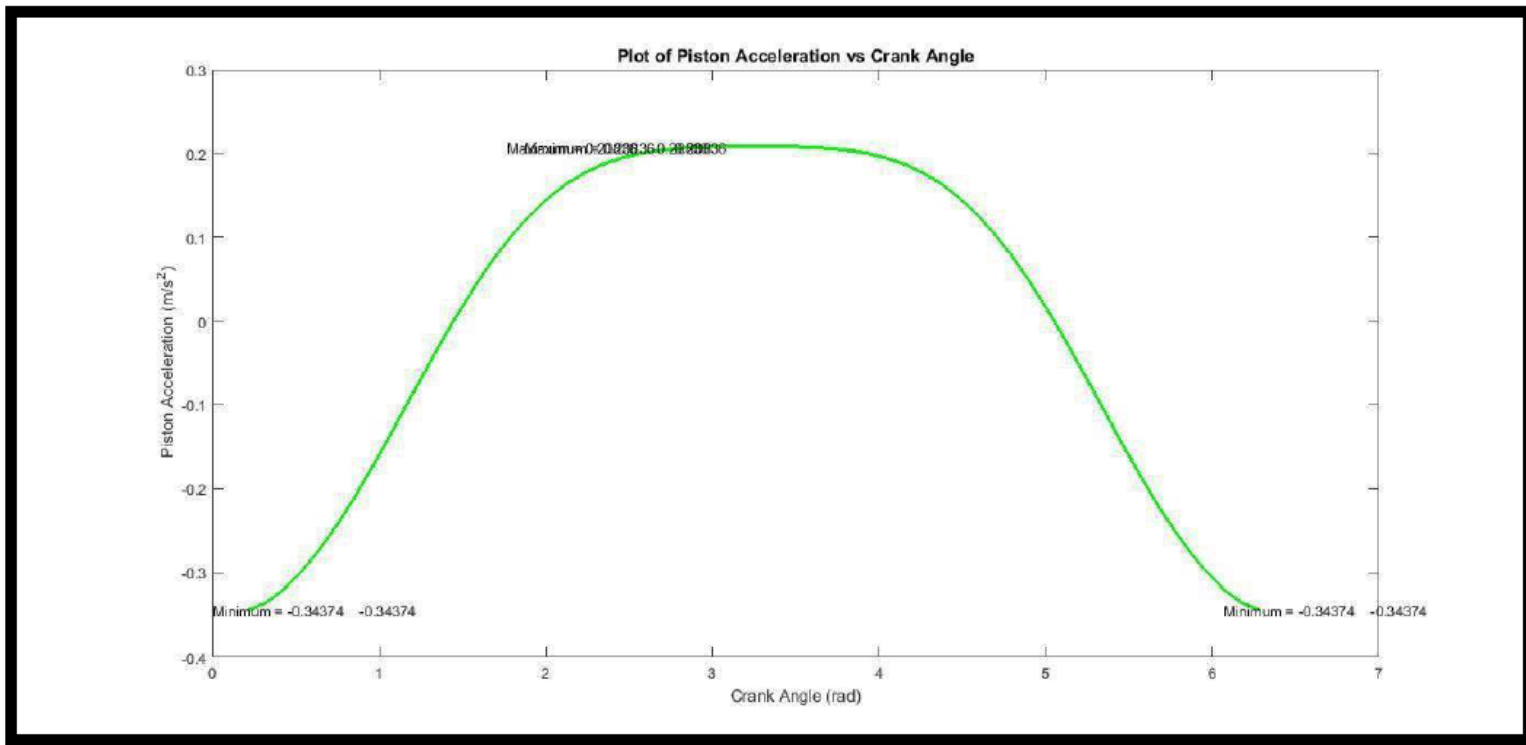
a) Piston Position vs. Crank Angle
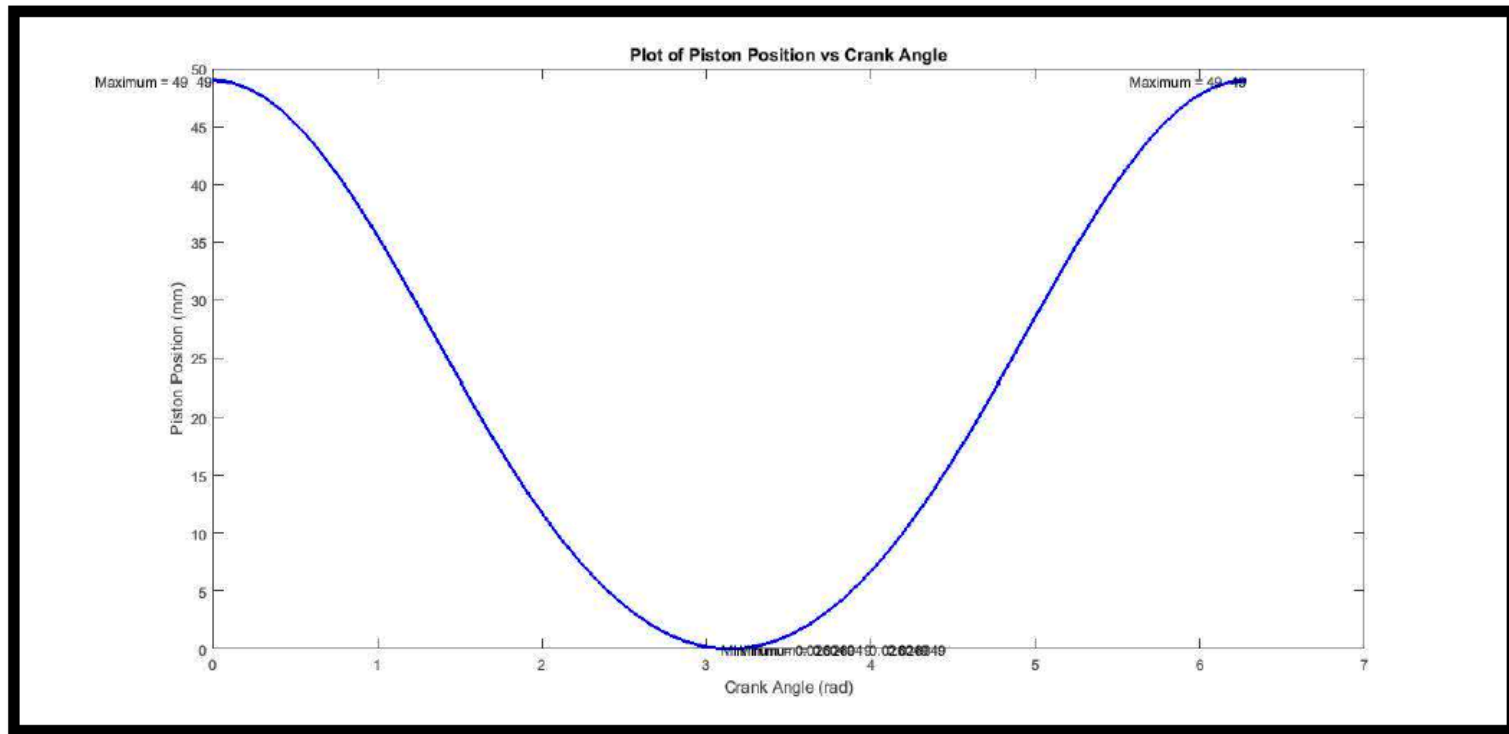
b) Piston Velocity vs. Crank Angle
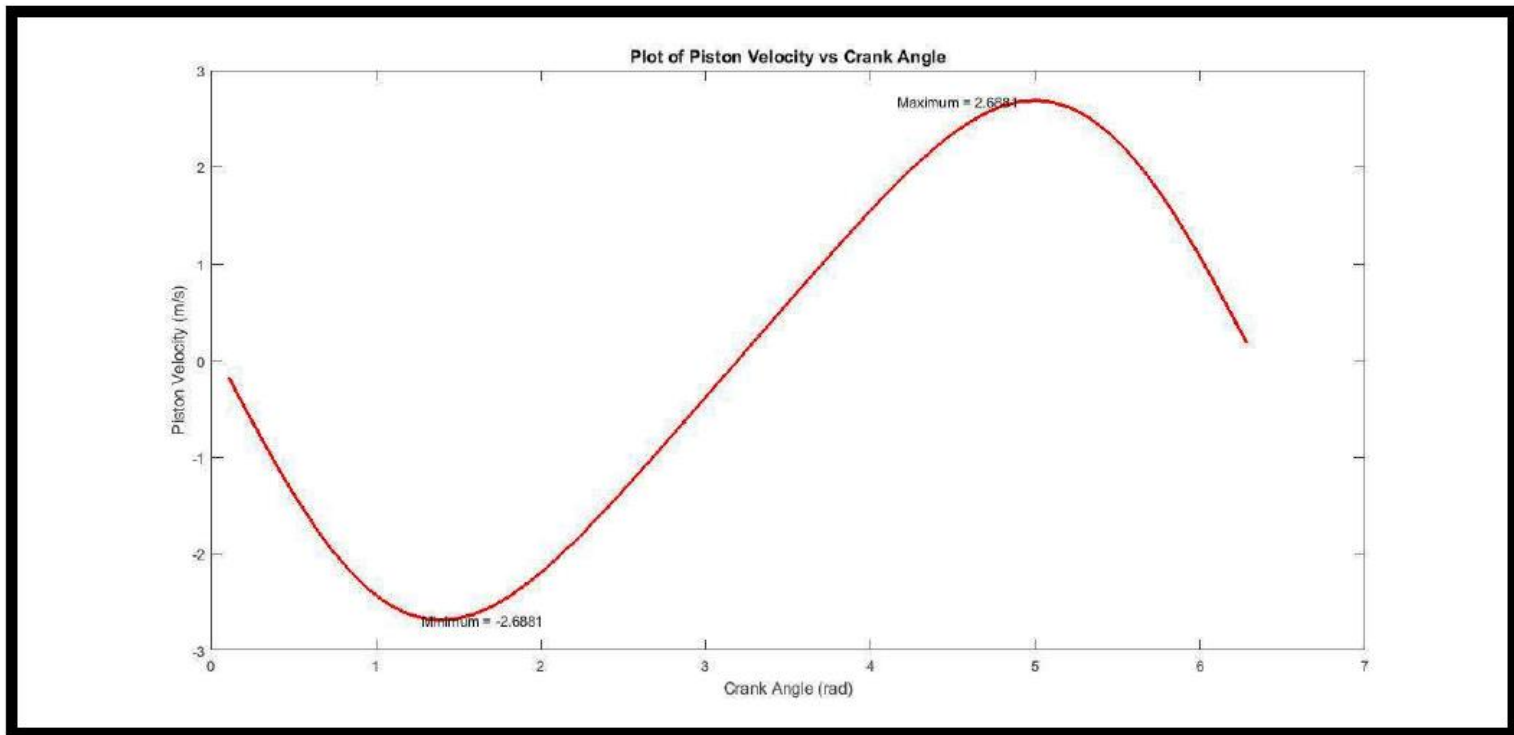
c) Piston Acceleration vs. Crank Angle

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