BMM3553 Mechanical Vibrations

Chapter 5: Multi Degree of Freedom Vibration System

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Chapter Description

- Expected Outcomes

  Students will be able to:

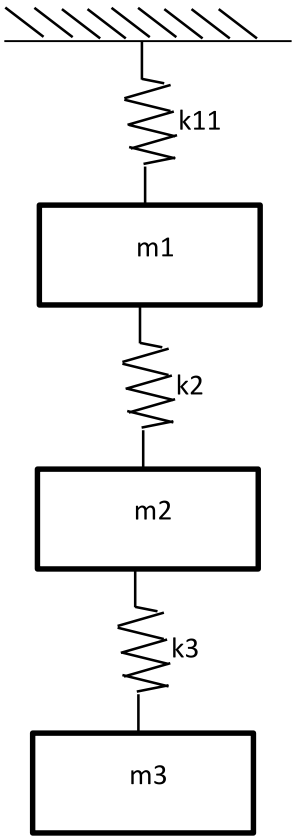
  - Develop Equation of Motion (EOM) for Undamped Multi-DOF Free Vibration
  - Determine natural frequencies and mode shape of Undamped Multi-DOF Free Vibration

- References

  - Singiresu S. Rao. Mechanical Vibrations. 5th Ed
  - Abdul Ghaffar Abdul Rahman. BMM3553 Mechanical Vibration Note. UMP.
  - Md Mustafizur Rahman. BMM3553 Mechanical Vibration Lecture Note. UMP

Multiple Degree of Freedom System

- The **natural frequencies**, **mode shapes** and **damping** formed the three dynamic characteristics of the system.
- Once these parameters are evaluated, by either measurement, analytical or numerical techniques, a mathematical model of the system can be developed.
- This model can then be used for further analysis and modification.
- If there are **n degrees of freedom** with which mass terms are associated, then **n differential equations** are required to describe the motions of the system.



Frequencies And Mode Shapes For Undamped Systems

- For a system with **n degrees of freedom**, the action equations for undamped free vibrations take the general form

$$\begin{bmatrix} M_{11} & M_{12} & \dots & M_{1n} \\ M_{21} & M_{22} & \dots & M_{2n} \\ \dots & \dots & \dots & \dots \\ M_{n1} & M_{n2} & \dots & M_{nn} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \dots \\ \ddot{x}_n \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \dots & \dots & \dots & \dots \\ K_{n1} & K_{n2} & \dots & K_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} \qquad (viii)$$

- Assume that in natural vibration all masses follow the harmonic function

$$X_r = A_r \sin(\omega_r t + \theta_r) \qquad (a)$$

- in which $\boldsymbol{\omega_r}$ and $\boldsymbol{\vartheta_r}$ are the angular frequency and phase angle of the **r**$^{\mathbf{th}}$ mode. The symbol $\boldsymbol{X_r}$ in equation (a) denotes the column matrix (or vector) of displacement of the **r**$^{\mathbf{th}}$ mode, and the $\boldsymbol{A_r}$ represents the corresponding vector of maximum values, or amplitudes.

Frequencies And Mode Shapes For Undamped Systems

$$X_r = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}_r \qquad A_r = \begin{bmatrix} A_1 \\ A_2 \\ \dots \\ A_n \end{bmatrix}_r$$

- Substitution of equation (a) into equation (1) produces a set of algebraic equations that may be stated as

$$\boldsymbol{H_r A_r = 0} \qquad \boldsymbol{(ix)}$$

- where $\boldsymbol{H_r}$ is the characteristic matrix

$$H_r = K - \omega_r^2 M \qquad (x)$$

Frequencies And Mode Shapes For Undamped Systems

- For non-trivial solutions of equation (2) the determinant of the characteristic matrix is set equal to zero giving

$$|H_r| = \begin{vmatrix} K_{11} - \omega_r^2 M_{11} & K_{12} - \omega_r^2 M_{12} & \dots & K_{1n} - \omega_r^2 M_{1n} \\ K_{21} - \omega_r^2 M_{21} & K_{22} - \omega_r^2 M_{22} & \dots & K_{2n} - \omega_r^2 M_{2n} \\ \dots & \dots & \dots & \dots \\ K_{n1} - \omega_r^2 M_{n1} & K_{n2} - \omega_r^2 M_{n2} & \dots & K_{nn} - \omega_r^2 M_{nn} \end{vmatrix} = 0 \qquad (xi)$$

- If the polynomial cannot be factored, its n roots ω_1^2, ω_2^2, ... ω_r^2, ... ω_n^2 may be found by numerical procedure. Such roots, which were referred to previously as characteristic values, are also called **eigenvalues**. Vectors of modal amplitudes, any one of which is represented by A_r are called characteristic vectors or **eigenvectors**.

Example

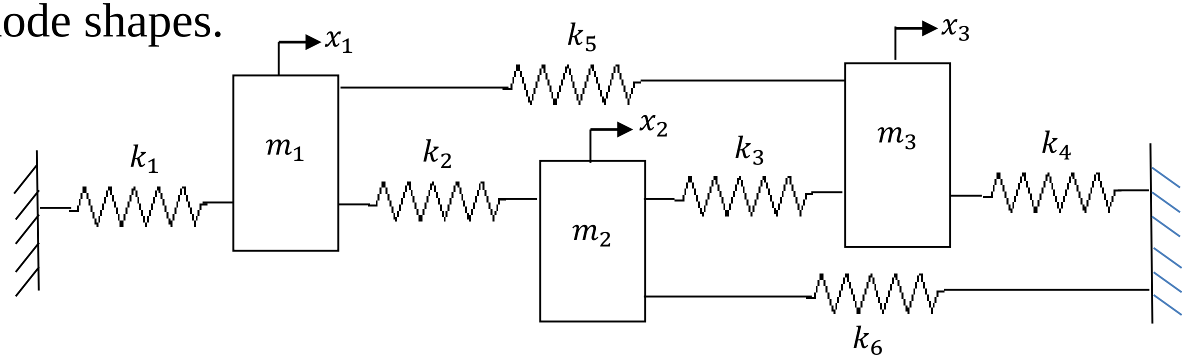
A simplified multi-degree of freedom of an undamped-structural-system is shown in **figure below**. The equations of motion derived using the displacements of the masses,x_1,x_2 andx_3 as degree of freedom system. By assuming that, $m_1 = m_3 = m$, $m_2 = 2m$, $k_1 = k_2 = k_3 = k_4 = k$, $k_5 = 3k$ and $k_6 = 2k$,

a) Develop the equation of motion for the system; (7 Marks)

b) Determine the natural frequencies in term of m and k; (16 Marks)

c) Determine the principle mode shapes of the system which normalized to the first mass. (9 Marks)

d) Sketch the mode shapes. (3 Marks)



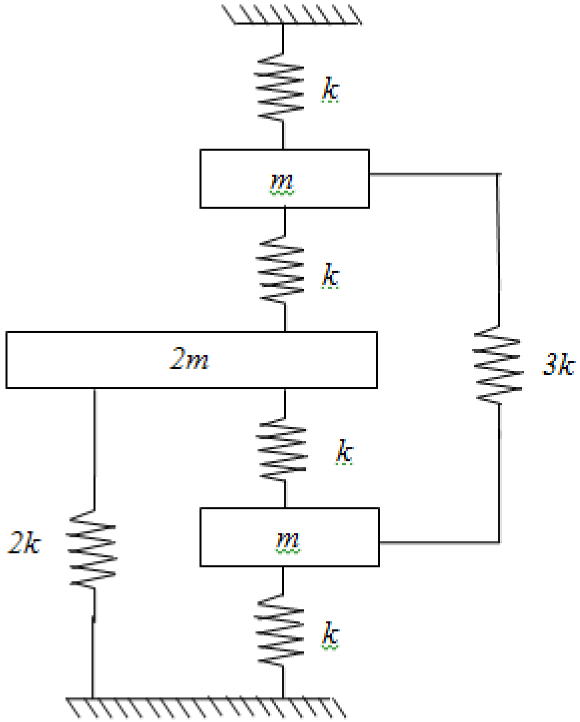
Example

Figure shows a complex multi-degree of freedom spring-mass system.

a) Develop the equation of motion of the system.

b) Determine the natural frequencies and mode shape of the system.

c) Estimate the largest strain that can occur to any of the spring in the system. State which spring in your answer



Thank You

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