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BMM3553 Mechanical Vibrations

Chapter 4: Two Degree of Freedom Vibration System (Part 2)

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Chapter Description

- Expected Outcomes

  Students will be able to:

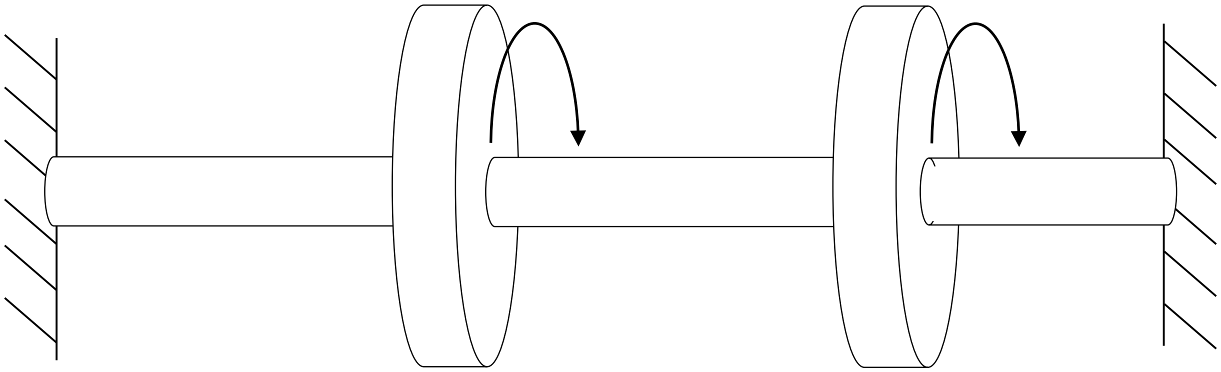
  – Develop Equation of Motion (EOM) for Undamped Two-DOF Rotational Free and Forced Vibration
  – Determine natural frequencies and mode shape of Undamped Two-DOF Rotational Free and Forced Vibration
  – Solving vibration problems 2 dof (combination 1 dof translational and 1 dof rotational).

- References

  – Singiresu S. Rao. Mechanical Vibrations. 5th Ed
  – Abdul Ghaffar Abdul Rahman. BMM3553 Mechanical Vibration Note. UMP.
  – Md Mustafizur Rahman. BMM3553 Mechanical Vibration Lecture Note. UMP

Torsional System 2 DOF

- Consider a following 2dof torsional system.



- Free Body Diagram?
- Equation of Motion?

Torsional System 2 DOF

$$J_1\ddot{\theta}_1 + (k_{t1} + k_{t2})\theta_1 - k_{t2}\theta_2 = M_{t1}$$

$$J_2\ddot{\theta}_2 - k_{t2}\theta_1 + (k_{t2} + k_{t3})\theta_2 = M_{t2}$$

For the free vibration analysis of the system, Eq. above reduces to

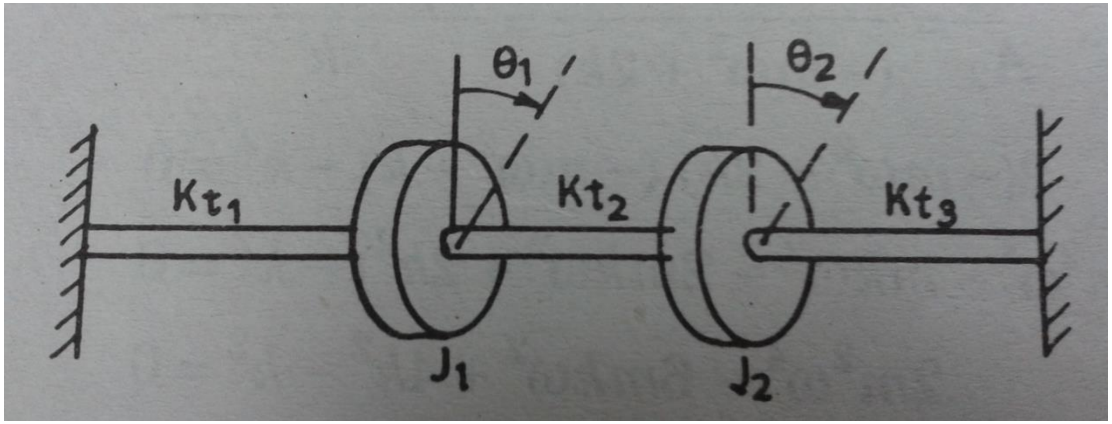
$$J_1\ddot{\theta}_1 + (k_{t1} + k_{t2})\theta_1 - k_{t2}\theta_2 = 0$$

$$J_2\ddot{\theta}_2 - k_{t2}\theta_1 + (k_{t2} + k_{t3})\theta_2 = 0$$

Example

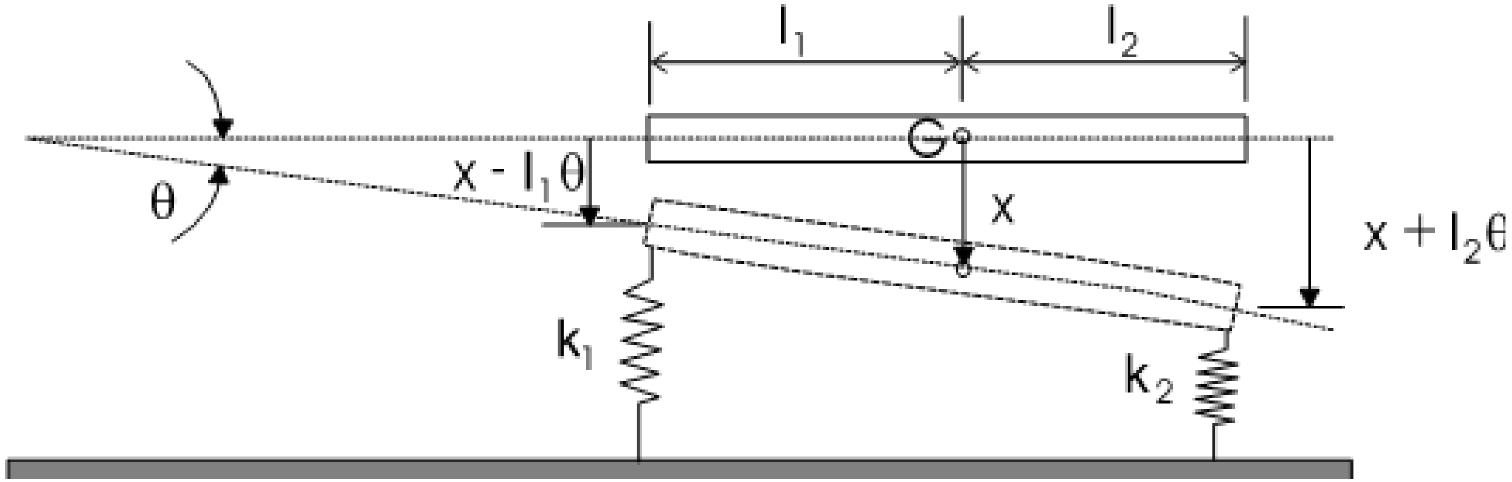
Determine the natural frequencies and amplitude ratio of mode shapes for 2 dof torsional system shown in figure below.

Given, $J_1 = J_o$, $J_2 = 2J_o$ $\quad$ $k_{t1} = k_{t2} = k_{t3} = k_t$



Two Degree Of Freedom - Automobile Simulation Undamped

- An automobile has many degrees of freedom but if we confine our attention to motion in a vertical plane and consider the unsprung mass (wheel, etc.) small as compared to the mass of the automobile then, neglecting damping, the system behaviour can be simulated by a two degree of freedom mathematical model in which the **translation(x)** of the centre of mass and **rotation(θ)** around the **centre of mass (G)** are the two possible modes of vibration.



Two Degree Of Freedom - Automobile Simulation Undamped

- The equations of motion are

For motion in the vertical direction

$$m\ddot{x}+k_1(x-\ell_1\theta)+k_2(x+\ell_2\theta)=0$$

$$m\ddot{x}+(k_1+k_2)x-(k_1\ell_1-k_2\ell_2)\theta=0$$

- For rotation about G

$$I_G\ddot{\theta}-k_1(x-\ell_1\theta)\ell_1+k_2(x+\ell_2\theta)\ell_2=0$$

$$I_G\ddot{\theta}-(k_1\ell_1-k_2\ell_2)x+(k_1\ell_1^2+k_2\ell_2^2)\theta=0$$

- Let the two motions be

$X(t) = A \sin(\omega t + \phi)$

$\theta(t) = B \sin(\omega t + \phi)$

Two Degree Of Freedom - Automobile Simulation Undamped

- Substitution for x(t) and θ(t) leads to the characteristic equation;-

$$\begin{vmatrix} k_1 + k_2 - m\omega^2 & k_2\ell_2 - k_1\ell_1 \\ k_2\ell_2 - k_1\ell_1 & k_1\ell_1^2 + k_2\ell_2^2 - I_G\omega^2 \end{vmatrix} = 0$$

- The solution of which is

$$\omega_{1,2}^2 = \frac{1}{2}\left[\frac{k_1 + k_2}{m} + \frac{k_1\ell_1^2 + k_2\ell_2^2}{I_G}\right] \pm \sqrt{\left(\frac{k_1 + k_2}{m} + \frac{k_1\ell_1^2 + k_2\ell_2^2}{I_G}\right)^2 - \frac{4k_1k_2(\ell_1 + \ell_2)^2}{mI_G}}$$

Two Degree Of Freedom: Coupling

- The equation of motion were found to be;-

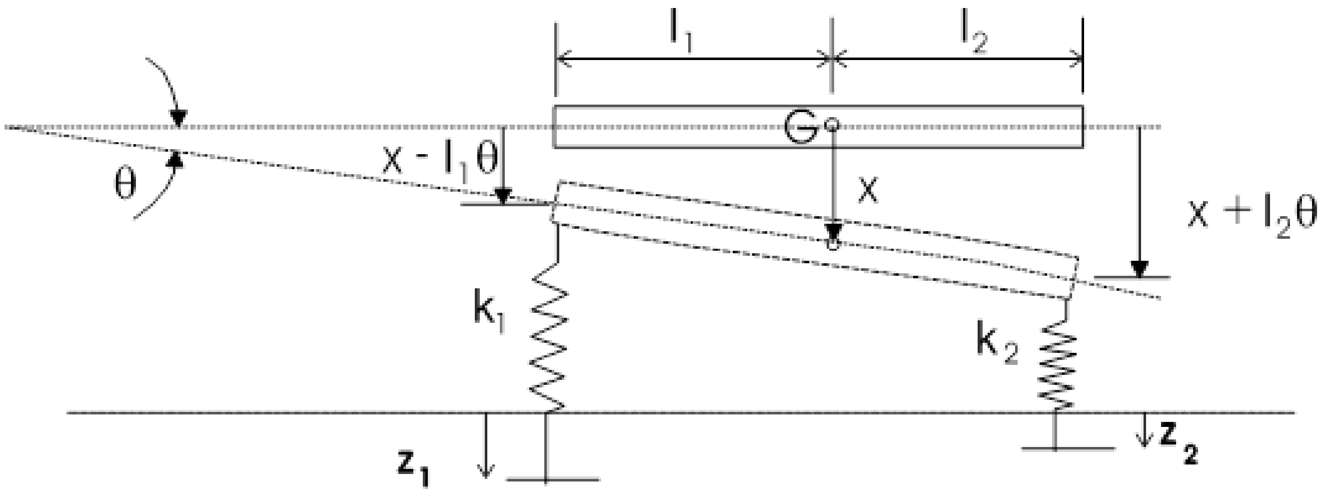
$$m\ddot{x} + (k_1 + k_2)x - (k_1\ell_1 - k_2\ell_2)\theta = 0.........(1)$$
$$-(k_1\ell_1 - k_2\ell_2)x + I_G\ddot{\theta} + (K_1\ell_1^2 + K_2\ell_2^2)\theta = 0......(2)$$

- Now, if $\mathbf{k_1 l_1 = k_2 l_2}$, equation (1) becomes **independent** of θ and equation (2) becomes independent of x. As they stand, the equations are said to be coupled and making $\mathbf{k_1 l_1 = k_2 l_2}$ uncouples them as below:

$$m\ddot{x} + (k_1 + k_2)x = 0.........(1a)$$
$$I_G\ddot{\theta} + (K_1\ell_1^2 + K_2\ell_2^2)\theta = 0......(2a)$$

- If the coupling term in equation (1) and (2) is dependent upon displacement, the coupling is said to be elastically or statically coupled. If it is dependent upon velocity the equations are said to be coupled dynamically. The type of coupling depends entirely on the choice of the coordinates for the equations of motion.

Two Degree Of Freedom: Coupling



Two Degree Of Freedom - Automobile Simulation Undamped – Ground Excitation

Two Degree Of Freedom: Coupling

- The equation of motion are;-
  - For motion in the vertical direction

$$m\ddot{x}+k_1(x-l_1\theta-z_1)+k_2(x+l_2\theta-z_2)=0$$

  - For rotation about G

$$I_G\ddot{\theta}-k_1(x-l_1\theta-z_1)l_1+k_2(x+l_2\theta-z_2)l_2=0$$

- Thus

$$\boldsymbol{-(k_1l_1-k_2l_2)x+I_G\ddot{\theta}+(k_1l_1^2+k_2l_2^2)\theta=l_2k_2z_2-l_1k_1z_1}$$

$$\boldsymbol{m\ddot{x}+(k_1+k_2)x-(k_1l_1-k_2l_2)\theta=k_1z_1+k_2z_2}$$

Two Degree Of Freedom: Coupling

- Decoupling by making

$$k_1 l_1 = k_2 l_2$$

we have heaving mode, $m\ddot{x} + (k_1 + k_2)x = k_1 z_1 + k_2 z_2$

and pitching mode, $I_G \ddot{\theta} + k_1 l_1 (l_1 + l_2)\theta = l_2 k_2 (z_2 - z_1)$

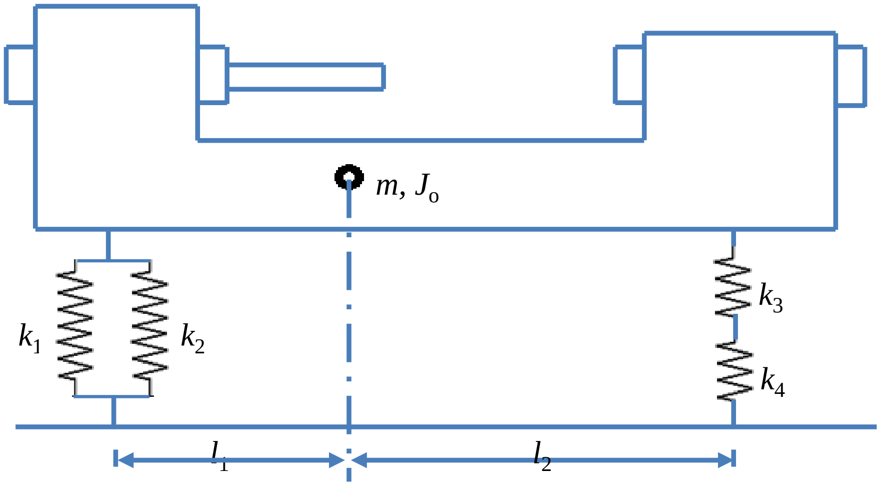
- Natural frequencies
  For heaving (x) mode, $\omega_1 = \sqrt{\dfrac{k_1 + k_2}{m}}$

  For pitching (θ) mode, $\omega_2 = \sqrt{\dfrac{k_1 \ell_1 (l_1 + \ell_2)}{I_G}}$

Exercise

A lathe machine tool, having a mass of m = 1000 kg and mass moment of inertia of Jo = 300 kg.m2, is supported on elastic supports as shown in Figure Q3. Given the stiffness of the supports are k1 = k2 = 1500 N/m and k3 = k4 = 4000 N/m. The supports are located at l1 = 0.5 m and l2 = 0.8 m.

a) Develop equation of motion for lathe machine tool. (7 Marks)
b) Determine natural frequencies (10 Marks)
c) Amplitude ratio for both mode shapes and locate nodal points for each mode. (8 Marks)
d)Develop new equation of motions and what is your observation about these equations when this lathe machine tool is uncoupled system. (5 Marks)



Exercise

A car model shown in Figure Q2 simplified by considering its rigid body supported on rear and front springs, is considered to study vertical linear vibration and angular oscillations. Car weighs 2000 N, wheel base, *L* of 3 m, centre of gravity (CG) is located 1.4 m (l_1) behind the front wheel axis and has a radius gyration about its CG as 1.1 m. The front springs have a combined stiffness, k_1 of 6000 N/m and rear spring, k_2 of 6500 N/m.

a) Develop equation of motion for the car model. (6 Marks)
b) Determine natural frequencies (11 Marks)
c) Amplitude ratio for both mode shapes and locate nodal points for each mode. (10 Marks)
d) Develop new equation of motions and what is your observation about these equations when this car model is uncoupled system. Then shows the new natural frequencies of this system as below: (6 Marks)

$$\omega_1 = \sqrt{\frac{k_1 + k_2}{m}} \qquad \omega_2 = \sqrt{\frac{k_1 l_1^2 + k_2 l_2^2}{I}}$$

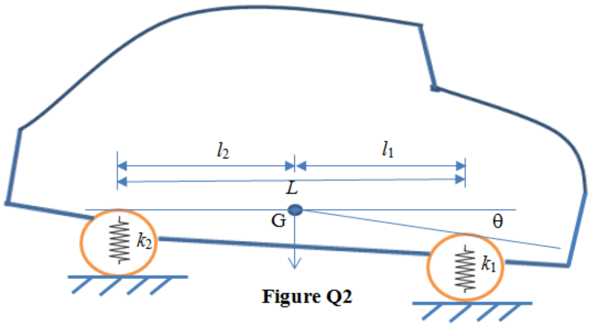


Figure Q2

Thank You

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