BMM3553 Mechanical Vibrations

Chapter 2: Undamped Vibration of Single Degree of Freedom System (Part 2)

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Chapter Description

- Expected Outcomes

  Students will be able to:

  – Develop Equation of Motion (EOM) for Undamped SDOF Forced Vibration

  – Analyze the total response of Undamped SDOF Forced Vibration

- References

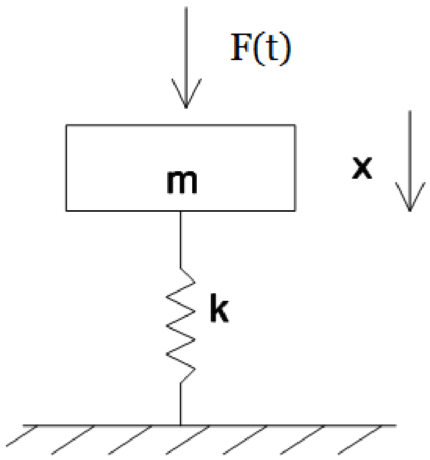
  – Singiresu S. Rao. Mechanical Vibrations. 5th Ed

  – Abdul Ghaffar Abdul Rahman. BMM3553 Mechanical Vibration Note. UMP.

  – Md Mustafizur Rahman. BMM3553 Mechanical Vibration Lecture Note. UMP

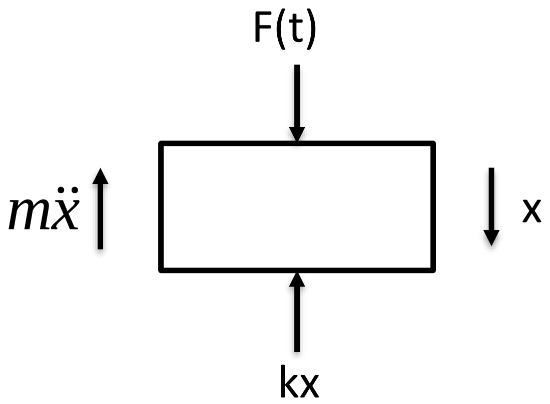
Equation of Motion : Undamped SDOF, Forced Vibration

- From the figure below, the equation of motion using Newton's Second Law of Motion states that



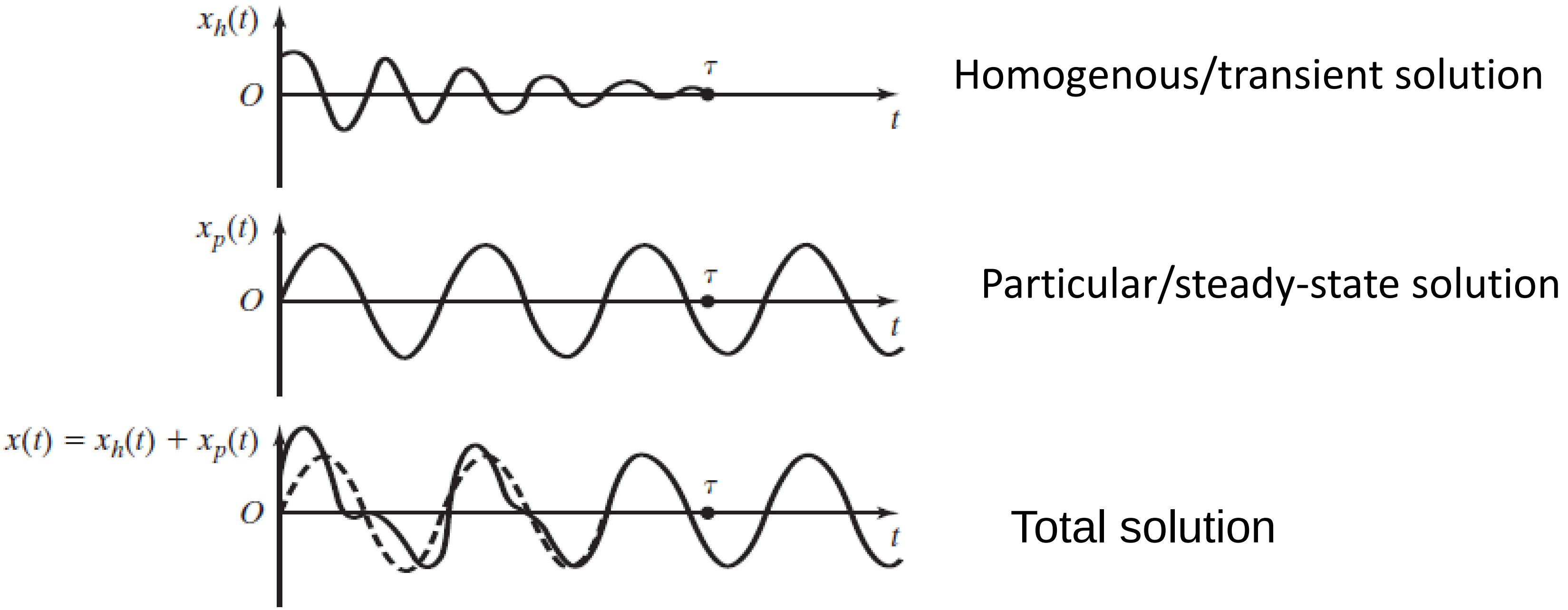
$$m\ddot{x} + kx = F(t)$$

- Free Body Diagram?



Response: Undamped SDOF, Forced Vibration

- Vibration responses of homogeneous and particular solutions with time for a typical case:



Response of an Undamped System Under Harmonic Force

- If a harmonic force $F(t) = F_0 \cos \omega t$ acts on the mass *m* of an undamped system, the general equation of motion can be write as:

$$m\ddot{x} + kx = F_0 \cos \omega t$$

- The transient/homogeneous solution of the system is given by:

$$x_h(t) = C_1 \cos \omega_n t + C_2 \sin \omega_n t$$

where ω_n is the natural frequency

Response of an Undamped System Under Harmonic Force

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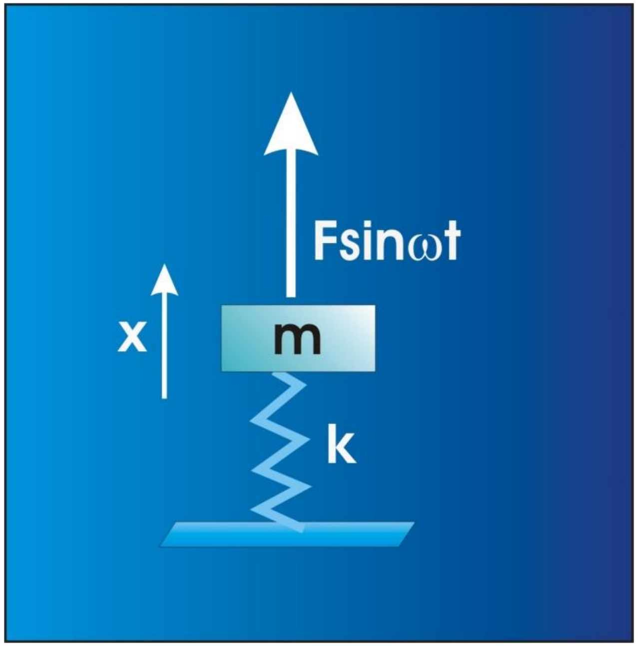


In Dynamics (Undamped)

$$m\ddot{x} + kx = F\sin\omega t$$

$$\Rightarrow -m\omega^2 x + kx = F\sin\omega t$$

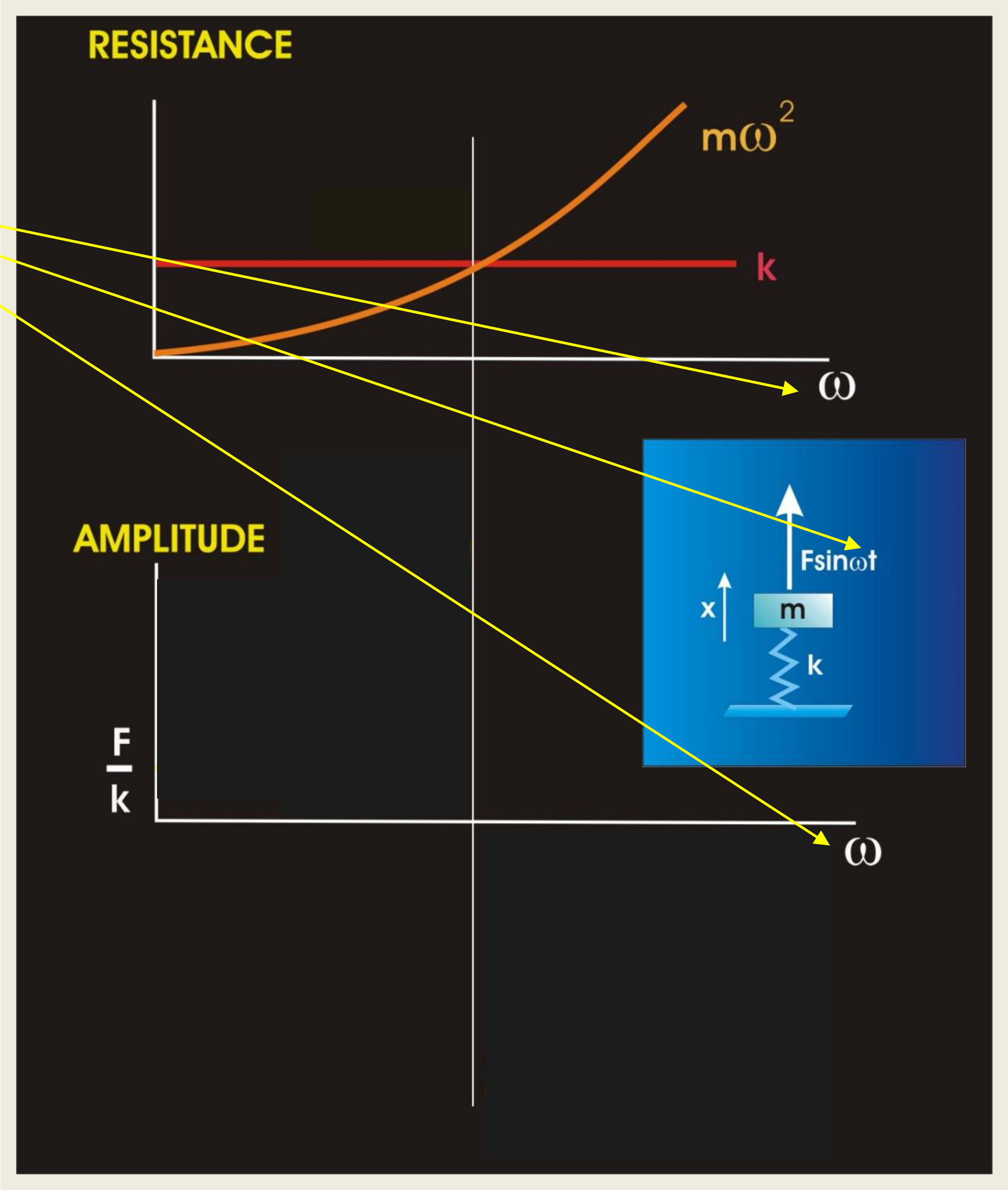
$$\Rightarrow x = \left(\frac{F}{k - m\omega^2}\right)\sin\omega t$$

$$Displacement = \frac{Force}{Stiffness - Inertia}$$

*If one needs to reduce movement, one has to look at **stiffness** as well as **inertia***

Stiffness is not a function of ω. However, Inertia is a function of ω

Let us increase the excitation frequency, ω, from small to large values



At low frequency ω ~ 0

Inertia resistance, $m\omega^2 \sim 0$

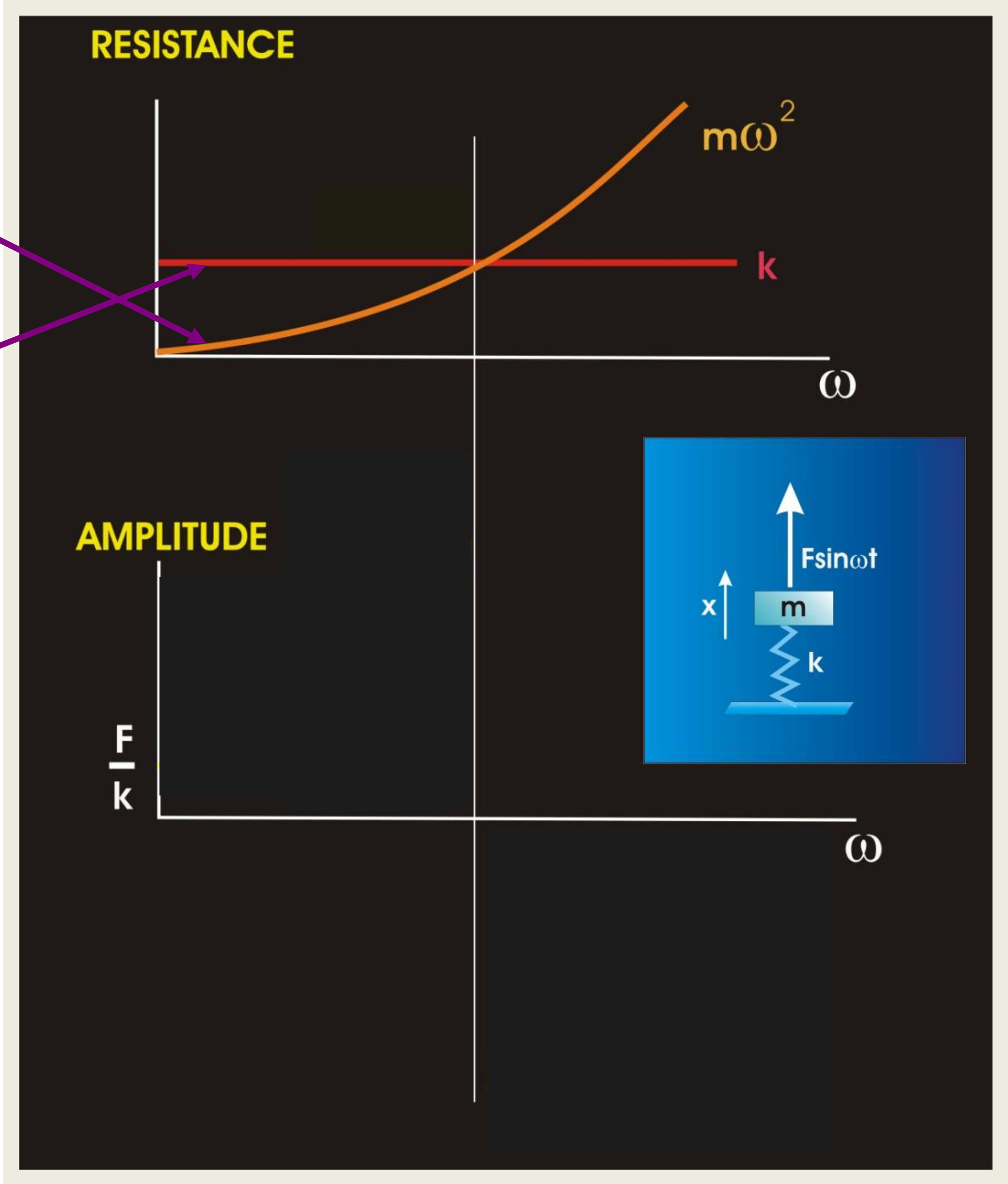
At this instant, motion is controlled by the stiffness resistance, k

$$x = \frac{F \sin \omega t}{k - m\omega^2}$$

$$\Rightarrow x \approx \frac{F \sin \omega t}{k}$$

This is termed as Stiffness Controlled

Situation is more towards static condition



Displacement x is in phase with force F

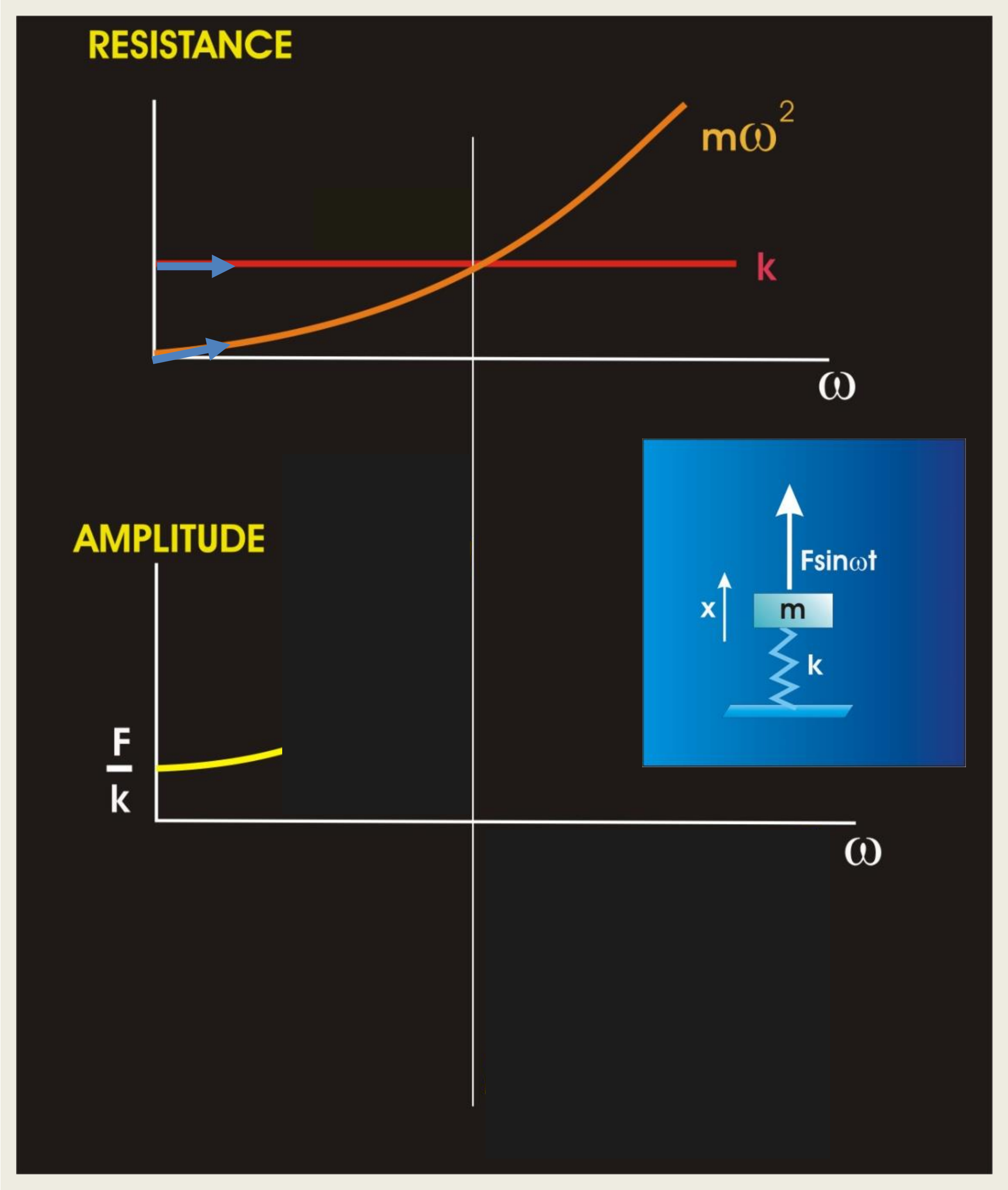
From low frequency to resonance

Stiffness resistance is constant with ω.

However, as ω is increased, the inertia resistance will increase

It will come to an instant where the inertia will cancel the stiffness, i.e.

$$m\omega^2 = k$$

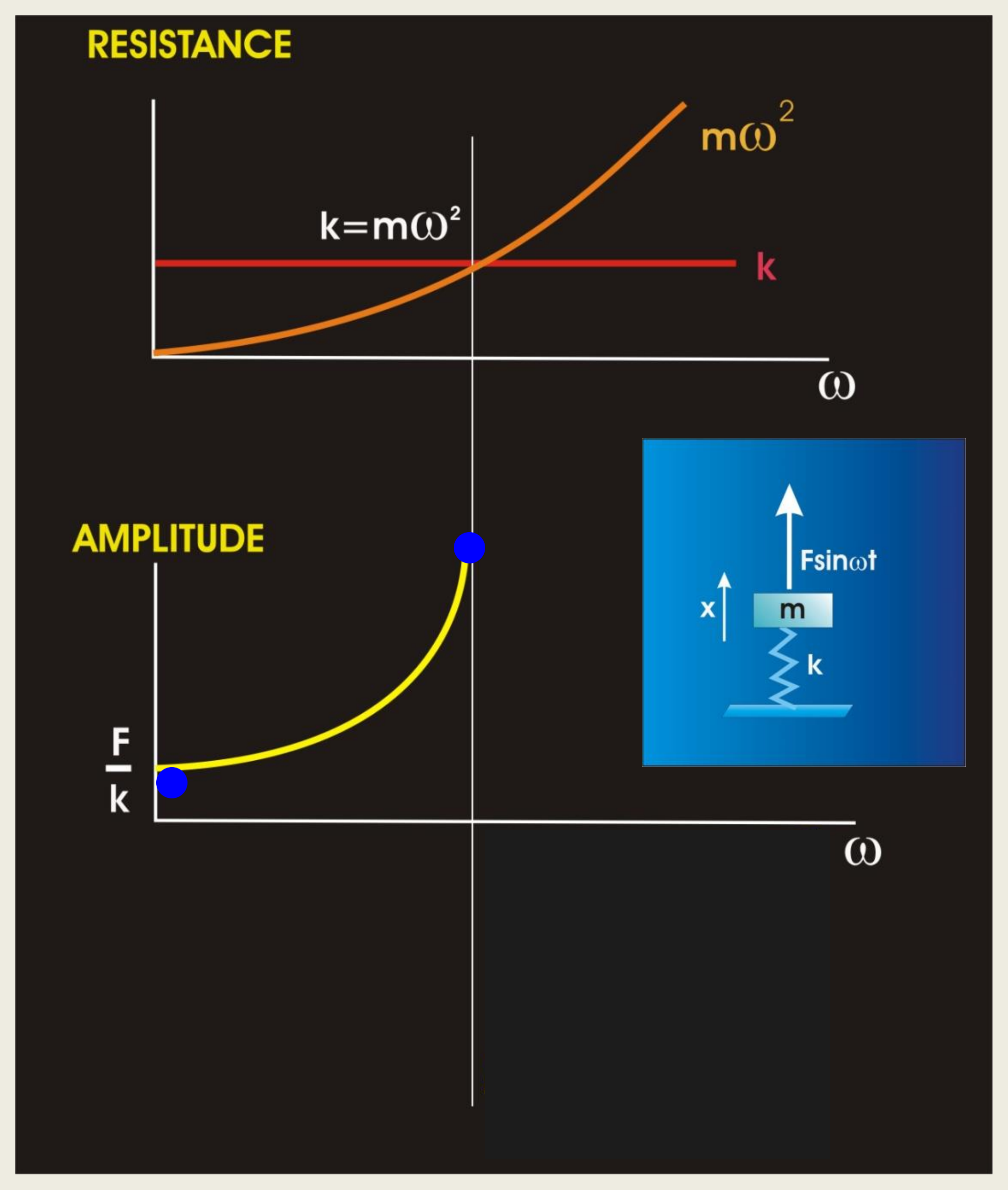


As a result, the excitation force F will now act on the mass without any resistance. This will cause the mass to oscillate with large amplitude.

If this oscillation is allowed to continue, the amplitude will get larger until it is restricted by damping, non-linearity or part of the system will break.

$$x = \left(\frac{F}{k - m\omega^2} \right) \sin \omega t$$

$$\Rightarrow x \approx \left(\frac{F}{0} \right) \sin \omega t$$

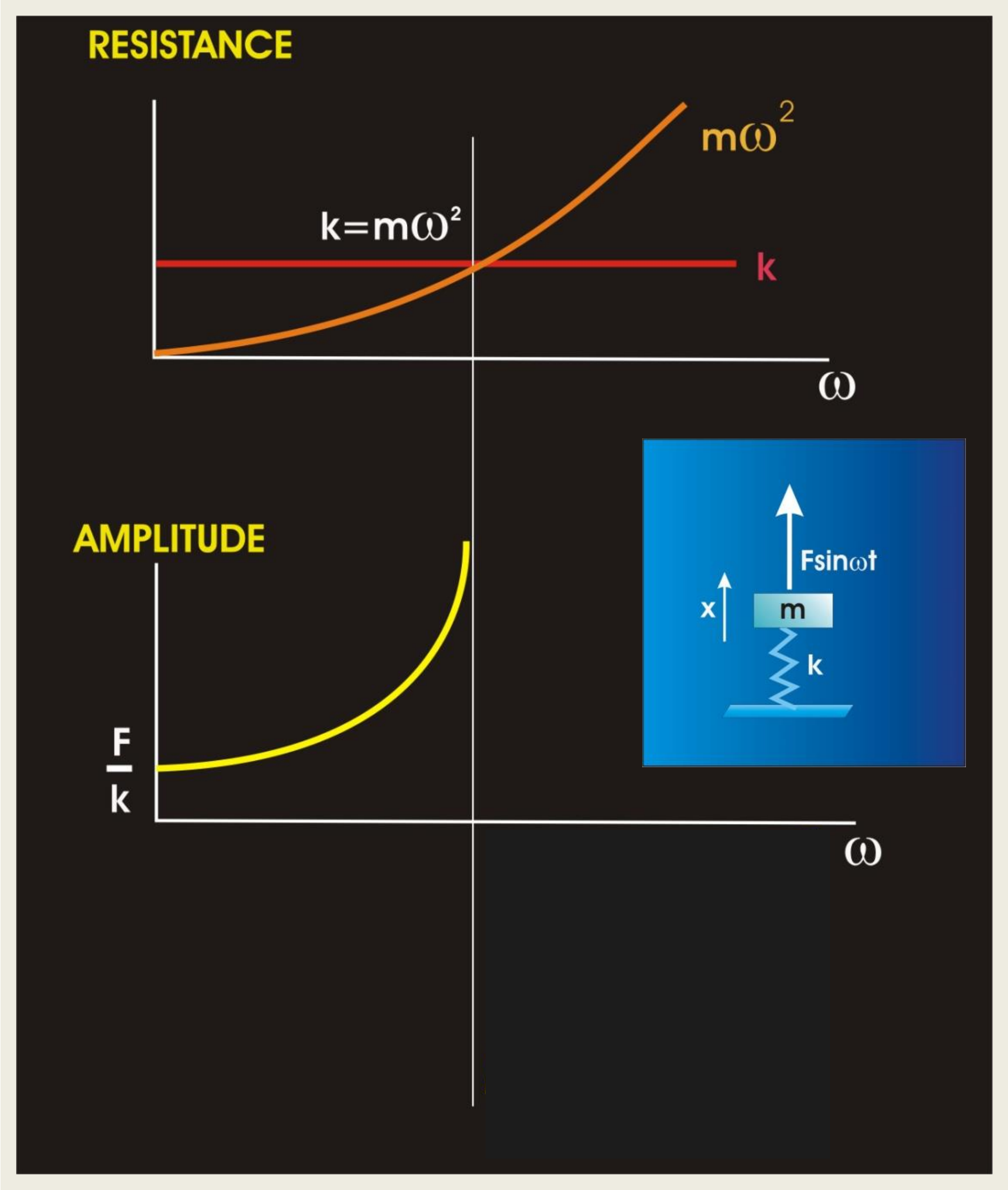


- This condition is termed as **Resonance.**
- The system has totally lost the ability to resist.
- The frequency at this instant is called **natural frequency,**
- Mathematically, ω_0

$$k - m\omega^2 = 0$$

$$\Rightarrow \quad m\omega^2 = k$$

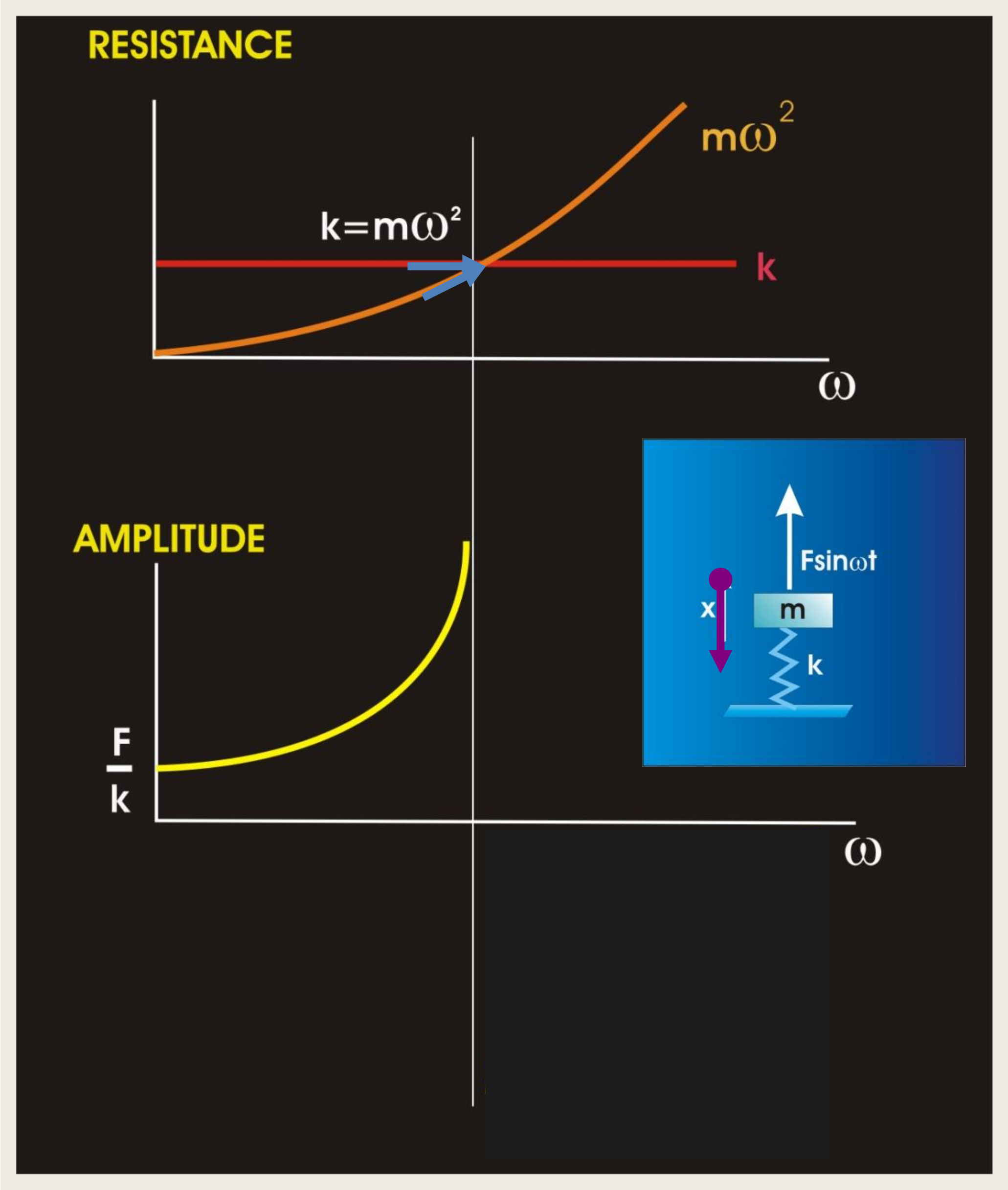
$$\Rightarrow \quad \omega = \omega_0 = \sqrt{\frac{k}{m}}$$



After resonance

If the frequency is increased further:

- The inertia will overcome the stiffness resistance.
- The magnitude of oscillation will become small until a point where the motion is controlled by the mass m.
- This is termed as **Mass Controlled**
- The system is said to be in **isolation**.



$$x = \left(\frac{F}{k - m\omega^2} \right) \sin \omega t$$

$$\Rightarrow x \approx \left(\frac{F}{-m\omega^2} \right) \sin \omega t$$

Displacement x is out of phase with force F

Response of an Undamped System Under Harmonic Force

- Because the exciting force and particular solution (steady state) is harmonic and has the same frequency, we can assume a solution in the form:

$$x_p(t) = X\cos\omega t$$

where X is the max amplitude of $x_p(t)$

$$X = \frac{F_0}{k - m\omega^2} = \frac{\delta_{st}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

where $\delta_{st} = F_0 / k$ denotes the static deflection

Thus final solution or response undamped force vibration,

$$x(t) = C_1\cos\omega_n t + C_2\sin\omega_n t + \frac{F_0}{k - m\omega^2}\cos\omega t$$

Response of an Undamped System Under Harmonic Force

- Using initial conditions

$$C_1 = x_0 - \frac{F_0}{k - m\omega^2}, \qquad C_2 = \frac{\dot{x}_0}{\omega_n}$$

Hence

$$x(t) = \left(x_0 - \frac{F_0}{k - m\omega^2} \right) \cos \omega_n t + \left(\frac{\dot{x}_0}{\omega_n} \right) \sin \omega_n t$$

$$+ \left(\frac{F_0}{k - m\omega^2} \right) \cos \omega t$$

The max amplitude can be expressed as

$$\frac{X}{\delta_{st}} = \frac{1}{1 - \left(\frac{\omega}{\omega_n} \right)^2}$$

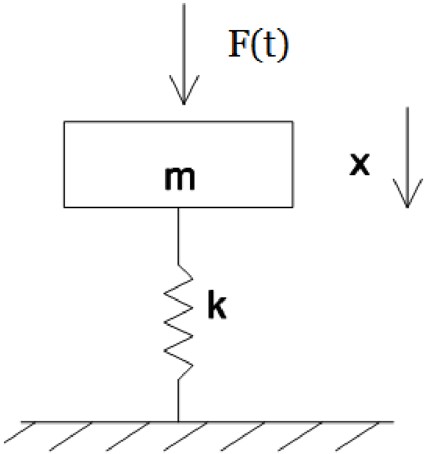
Example

A system as shown in figure below consist of a 10kg mass and 4kN/m spring. If a harmonic force, *F*(*t*) = 400 cos (10 t) N is acted to the system, determine the total response of the system under the following initial conditions:

(a), $x_0 = 0.1\,\text{m}, \quad \dot{x}_0 = 0$

(b) $x_0 = 0, \quad \dot{x}_0 = 10\,\text{m/s}$

(c) $x_0 = 0.1\,\text{m}, \quad \dot{x}_0 = 10\,\text{m/s}$



$k = 4000$ N/m, $m = 10$ kg, $F(t) = 400\cos 10t$, $F_0 = 400$ N, $\omega = 10$ rad/s

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4000}{10}} = 20 \text{ rad/s}, \quad \text{r} = \frac{\omega}{\omega_n} = \frac{10}{20} = 0.5 < 1$$

The response of the system is

$$x(t) = \left(x_0 - \frac{F_0}{k - m\omega^2} \right)\cos\omega_n t + \left(\frac{\dot{x}_0}{\omega_n} \right)\sin\omega_n t + \left(\frac{F_0}{k - m\omega^2} \right)\cos\omega t$$

$$\text{(a)} \quad x_0 = 0.1, \; \dot{x}_0 = 0$$

$$x(t) = \left(0.1 - \frac{400}{4000 - 10\times(10)^2} \right)\cos 20t + \left(\frac{0}{20} \right)\sin 20t + \left(\frac{400}{4000 - 10\times(10)^2} \right)\cos 10t$$

$$\Rightarrow x(t) = -0.033333\cos 20t + 0.133333\cos 10t$$

(b) $x_0 = 0,\ \dot{x}_0 = 10$

$$x(t) = \left(0 - \frac{400}{4000 - 10 \times (10)^2}\right)\cos 20t + \left(\frac{10}{20}\right)\sin 20t + \left(\frac{400}{4000 - 10 \times (10)^2}\right)\cos 10t$$

$$\Rightarrow x(t) = -0.133333\cos 20t + 0.5\sin 20t + 0.133333\cos 10t$$

(c) $x_0 = 0.1,\ \dot{x}_0 = 10$

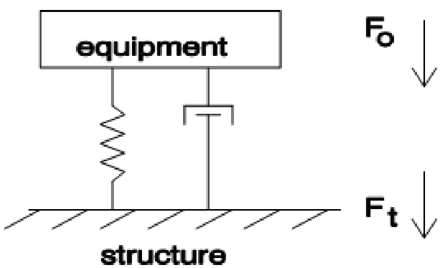
$$x(t) = \left(0.1 - \frac{400}{4000 - 10 \times (10)^2}\right)\cos 20t + \left(\frac{10}{20}\right)\sin 20t + \left(\frac{400}{4000 - 10 \times (10)^2}\right)\cos 10t$$

$$\Rightarrow x(t) = -0.033333\cos 20t + 0.5\sin 20t + 0.133333\cos 10t$$

Isolation Study

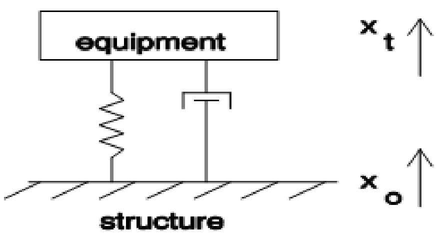
Apart from forming the foundation for the study of more complex systems, one important study of single degree of freedom system is in the area of vibration isolation

- ❑ Transmission to structure from the effect of equipment **vibration-force excitation**



eg. engine with the car structure
Washing machine with the floor

- ❑ Transmission to equipment from the structure vibration-**ground excitation**



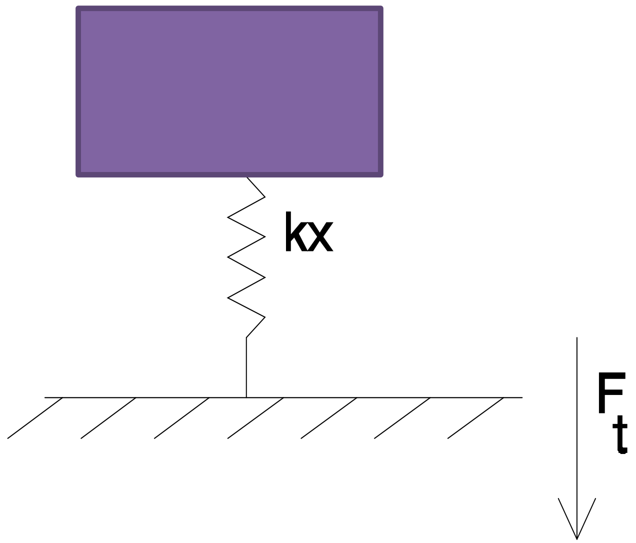
eg. sensitive electronic equipment

Vibration Transmission To The Structure

- The dynamic load experienced by the structure is the result of the spring tension and compression. This load is proportional to the amplitude A of the oscillation.

  Maximum force transmitted,

  $F_t = kA$

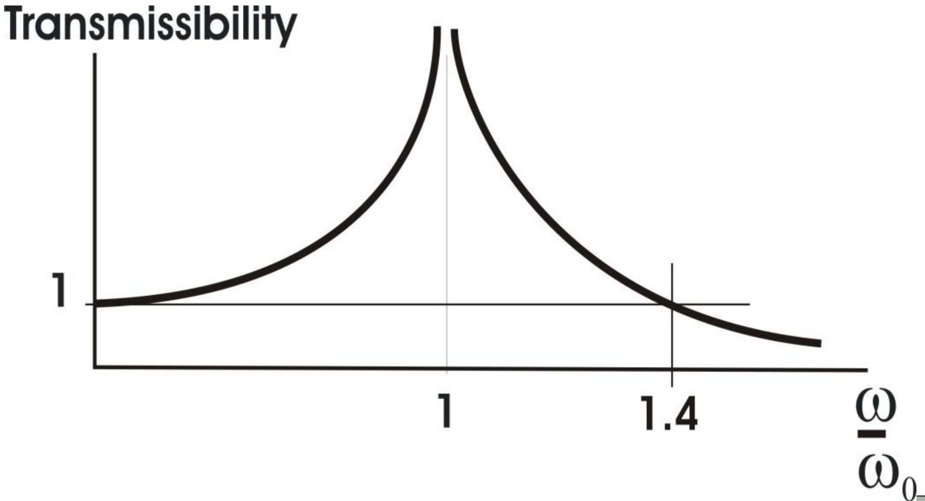


Vibration Transmission To The Structure

- **Transmissibility**

$$T = \frac{F_t}{F} = \frac{kA}{A(k-\omega^2 m)}$$

$$= \frac{1}{(1-\frac{\omega^2}{\omega_n^2})} = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2} \quad (1.10)$$



When $T = -1$,

$$-1 = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$\frac{\omega}{\omega_n} = \sqrt{2} \qquad \frac{\omega}{\omega_n} = 1.41$$

Example

A 400 kg exhaust fan with 0.15 kg-m rotating unbalance is supported on four parallel springs. The weight of exhaust fan resulting 50mm deflection to the spring. If the fan is running at 1500 rpm, determine:

a) Amplitude of vibration

b) Force transmitted to the ground at each supported spring

Thank You

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