BMM3553 Mechanical Vibrations

Chapter 2: Undamped Vibration of Single Degree of Freedom System (Part 1)

by

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Chapter Description

- Expected Outcomes

  Students will be able to:

  - Develop Equation of Motion (EOM) for Undamped SDOF (Translational and Rotational System)
  - Analyze the total response of Undamped SDOF (Translational and Rotational System)

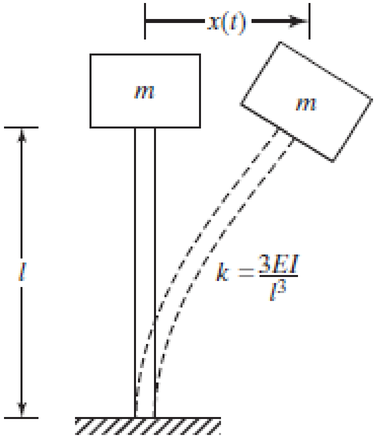
- References

  - Singiresu S. Rao. Mechanical Vibrations. 5th Ed
  - Abdul Ghaffar Abdul Rahman. BMM3553 Mechanical Vibration Note. UMP.
  - Md Mustafizur Rahman. BMM3553 Mechanical Vibration Lecture Note. UMP

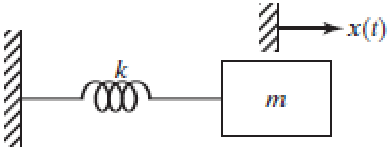
Single Degree of Freedom System

- Single degree of freedom system (SDOF) is the simplest form to represent a system or structure.
- Several structural systems can be modelled as single degree of freedom systems, which consists of the mass and stiffness of a system

Image source: S.S, Rao 5th Ed.



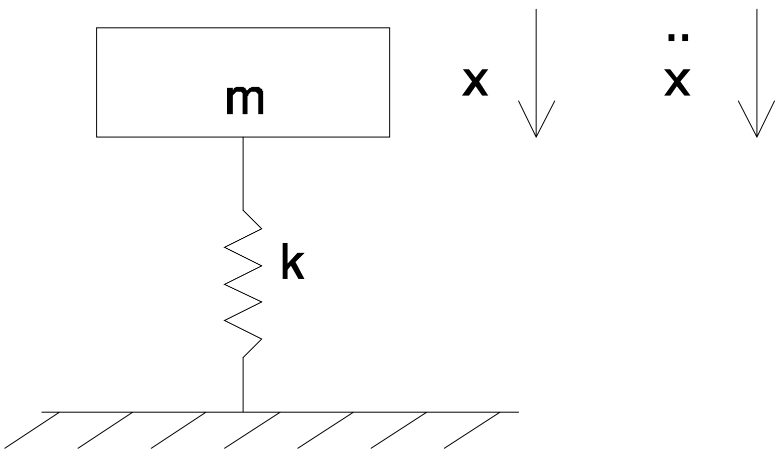
(a) Idealization of the tall structure

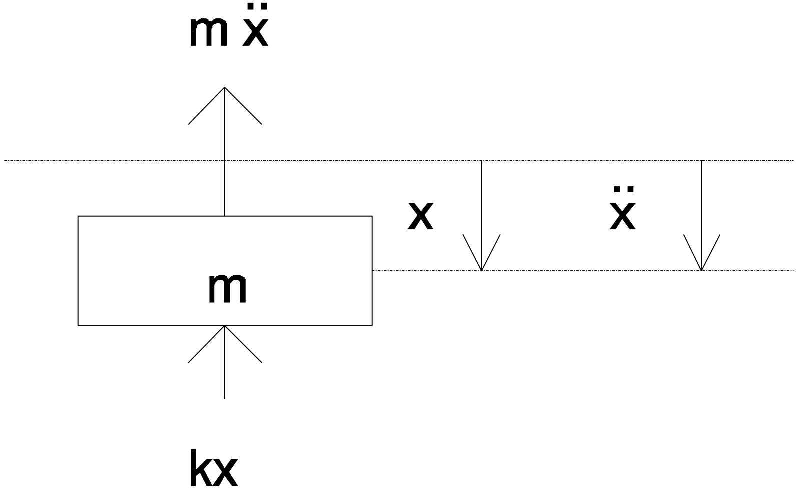


(b) Equivalent spring-mass system

SINGLE DEGREE OF FREEDOM SYSTEM: Free Vibration

- Free Vibration: No external force induced to the system



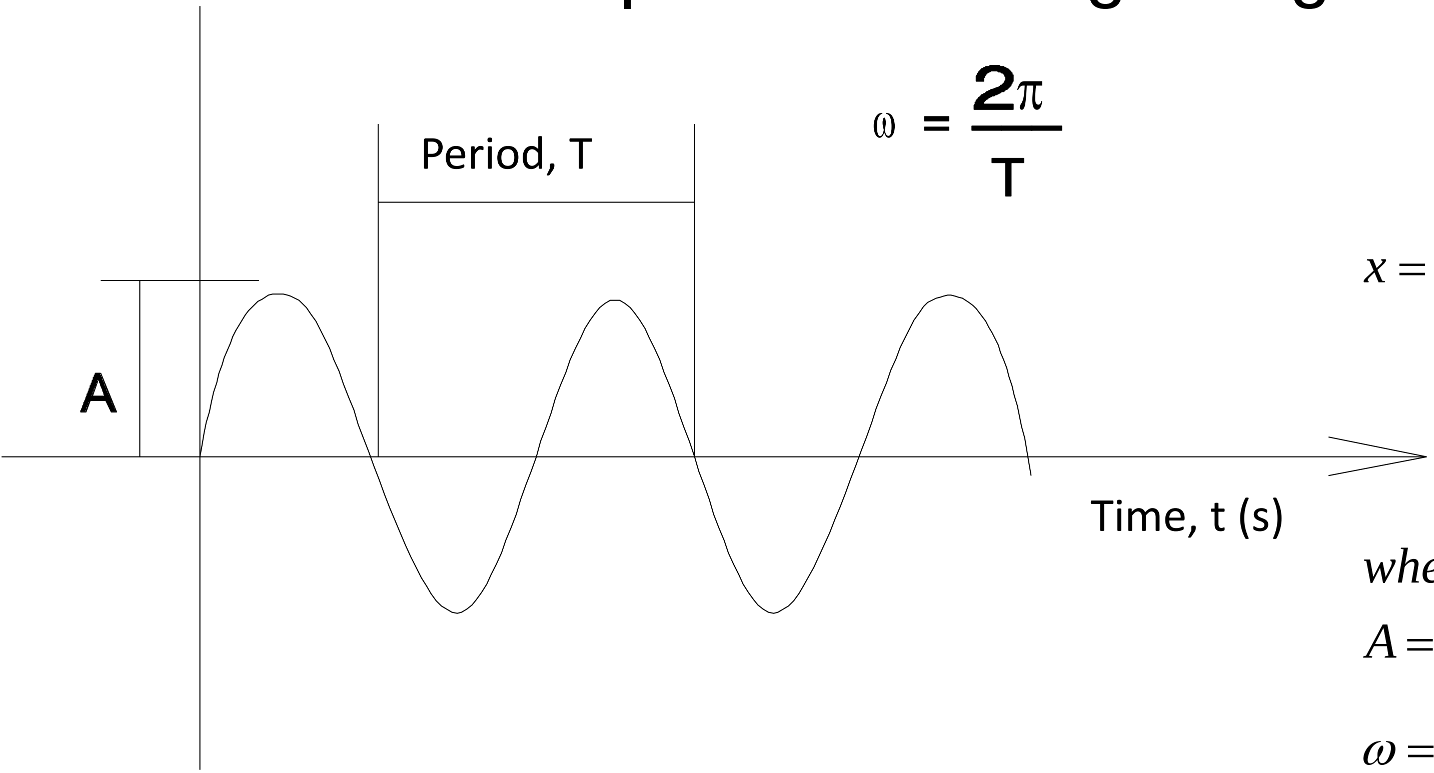


$$\sum F = 0$$

$$kx + m\frac{d^2x}{dt^2} = 0 \qquad (1.1)$$

SINGLE DEGREE OF FREEDOM SYSTEM: Free Vibration

- Vibration response of a single degree of freedom



$$\omega = \frac{2\pi}{T}$$

$$x = A\sin(\omega t + \beta)$$

where

$A = amplitude$

$\omega = Frequency \quad (\frac{rad}{\sec})$

$\beta = Phase \quad Angle$

SINGLE DEGREE OF FREEDOM SYSTEM: Free Vibration

Displacement $$x = A\sin(\omega t + \beta)$$

Velocity $$\frac{dx}{dt} = A\omega\cos(\omega t + \beta)$$

Acceleration $$\frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \beta) = -\omega^2 x$$

SINGLE DEGREE OF FREEDOM SYSTEM: Free Vibration

$$\sum F = m\ddot{x}$$

$$-kx = m\ddot{x} \quad , \ddot{x} = -\omega^2 x$$

$$kx - \omega^2 mx = 0$$

$$kx = \omega^2 mx$$

Here, there are two forces always acting on mass m, namely,

Stiffness force - which work to bring the mass back to the position of equilibrium. This force is constant with respect to frequency.

Inertia force - which work to eliminate the acceleration of the mass. This force is a function of frequency.

SDOF Undamped: Free Vibration with initial conditions

Equation of Motion of a Spring-Mass System in Vertical Position

$$x(t) = C_1 e^{i\omega_n t} + C_2 e^{-i\omega_n t}$$

where C_1 and C_2 are constants

By using the identities

$$x(t) = A_1 \cos \omega_n t + A_2 \sin \omega_n t$$

where A_1 and A_2 are new constants

Apply initial condition

$$x(t=0) = A_1 = x_0$$

$$\dot{x}(t=0) = \omega_n A_2 = \dot{x}_0$$

$$A_1 = x_0 \text{ and } A_2 = \dot{x}_0 / \omega_n$$

SDOF Undamped: Free Vibration with initial conditions

Substitute to identities equation,

$$x(t) = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

Amplitudes

$$A = \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega_n} \right)^2 \right]^{1/2}$$

Phase Angle

$$\beta = \tan^{-1} \left(\frac{x_0 \omega_n}{\dot{x}_0} \right)$$

$$x = A \sin(\omega_n t + \beta)$$

SDOF Undamped: Free Vibration with initial conditions

Harmonic motion in graphical representation.

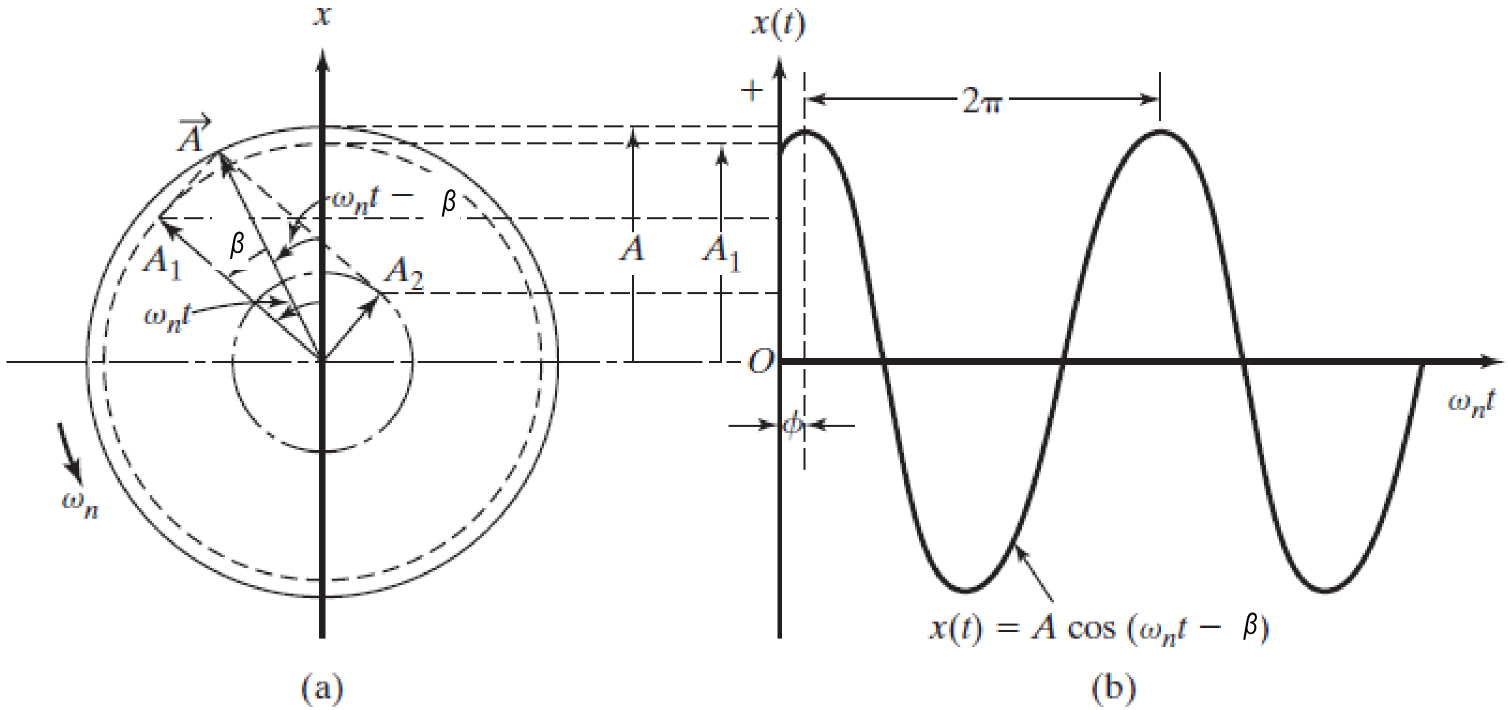


Image source: S.S. Rao 5th Ed

Problem 2.4, S.S. Rao 5th Ed.

A helical spring, when fixed at one end and loaded at the other, requires a force of 100N to produce an elongation of 10 mm. The ends of the spring are now rigidly fixed, one end vertically above the other , and a mass of 10 kg is attached at the middle point of its length. Determine the time taken to complete one vibration cycle when the mass is set vibrating in the vertical direction.

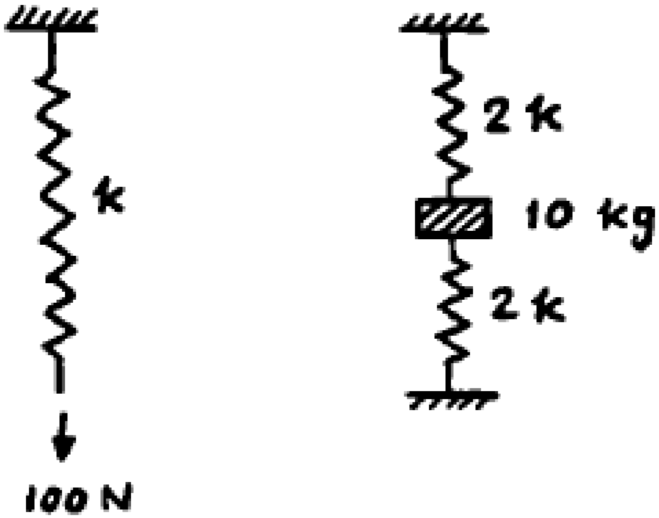
Solution:

$$k = \frac{F}{\delta} = \frac{100\ N}{10\ \text{mm}} = 10\ \text{N/mm} = 10\times10^{3}\ \text{N/m}$$

$$k_{eq} = 2k + 2k = 4k$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4\times10000}{10}} = 63.2456\ \text{rad/sec}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{63.2456} = 0.0993\ \text{sec}$$



SDOF: Undamped Torsional Vibration

- Torsional vibration consider the oscillation of the system based on the angle displacement (θ).
- The equation of motion is derived from torque applied to the system

torsion of circular shafts

$$M_t = \frac{GI}{l}$$

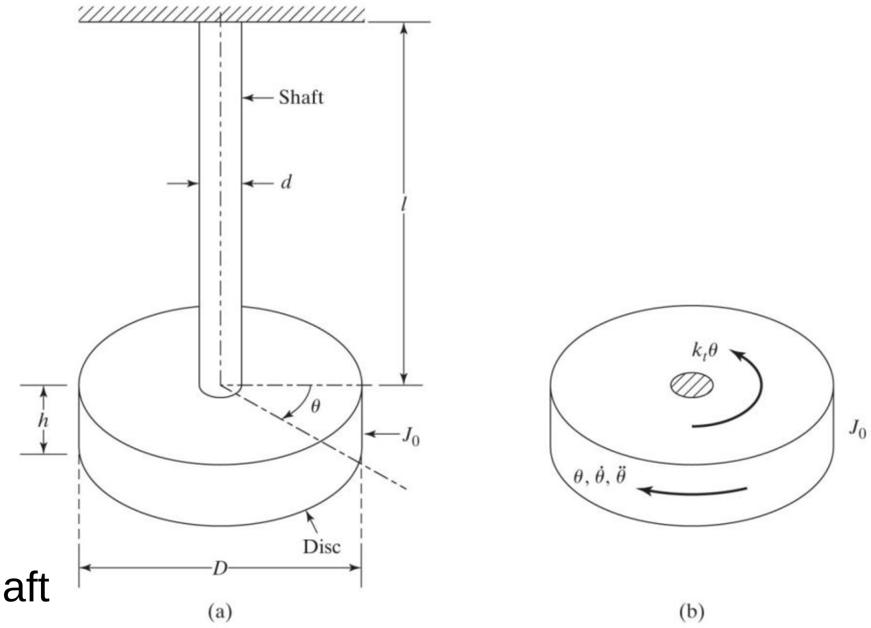
where

M_t = torque,

G = shear modulus,

l = length of shaft,

I = polar moment of inertia of cross section of shaft



SDOF: Undamped Torsional Vibration

$Equation\ of\ Motion,$

$$\sum M = 0$$

$$J_o\ddot{\theta} + k_T\theta = 0$$

$$\omega_n = \sqrt{\frac{k_T}{J_o}}$$

polar mass moment of inertia of a disc

$$J_o = \frac{\rho h\pi D^4}{32} = \frac{WD^2}{8g}$$

ρ = mass density

h = thickness of disc

D = diameter of disc

W = weight of disc

$From\ Strength\ of\ Material, torsional\ of\ shaft, k_T$ can be determined:

$$\frac{T}{I} = \frac{G\theta}{l}$$

$$k_T = \frac{T}{\theta} = \frac{GI}{l} = \frac{\pi G d^4}{32l}$$

where,

$$I = \frac{\pi d^4}{32}$$

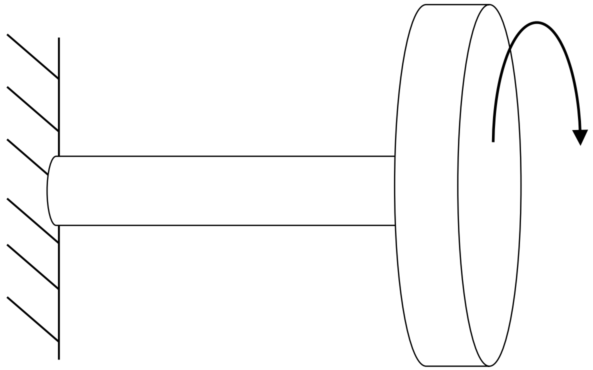
I = polar moment of inertia of the cross section of the shaft

G = shear modulus

l = length of shaft   d=diameter of shaft

Example: Torsional Vibration

- A torsional vibration system as shown in figure below oscillated at 2.3s for one complete cycle. If the radius and length of the shaft are 0.8 cm and 1.5m respectively, determine the mass moment of inertia of the wheel. (Gshaft=83 GPa)



Thank You

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