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Principles of Communication Systems

Chapter 4 (Part 2): Pulse Code Modulation

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By the end of this class you should be able to

- Explain the basic concept of PCM
- Solve problems involving PCM

Pulse-Code Modulation (PCM)

- The most commonly used digital modulation scheme.
- In PCM the available range of signal voltages is divided into levels, and each is assigned a binary number.
- Each sample is then represented by the binary number representing the level closest to its amplitude, and this number is transmitted in serial form.
- In *linear* PCM, levels are separated by equal voltage gradations.

Pulse-Code Modulation (PCM)

Three steps involved in PCM:

Sampling :

- Periodically sampling the continually changing analog input voltage and convert it to a series of constant amplitude pulses (PAM)

Quantization:

- rounding off the amplitude of flat-top samples to a manageable number of levels

Encoding:

- Determine code of binary for each PAM signal based on their levels

STEP 1: Sampling

TWO types of sampling:

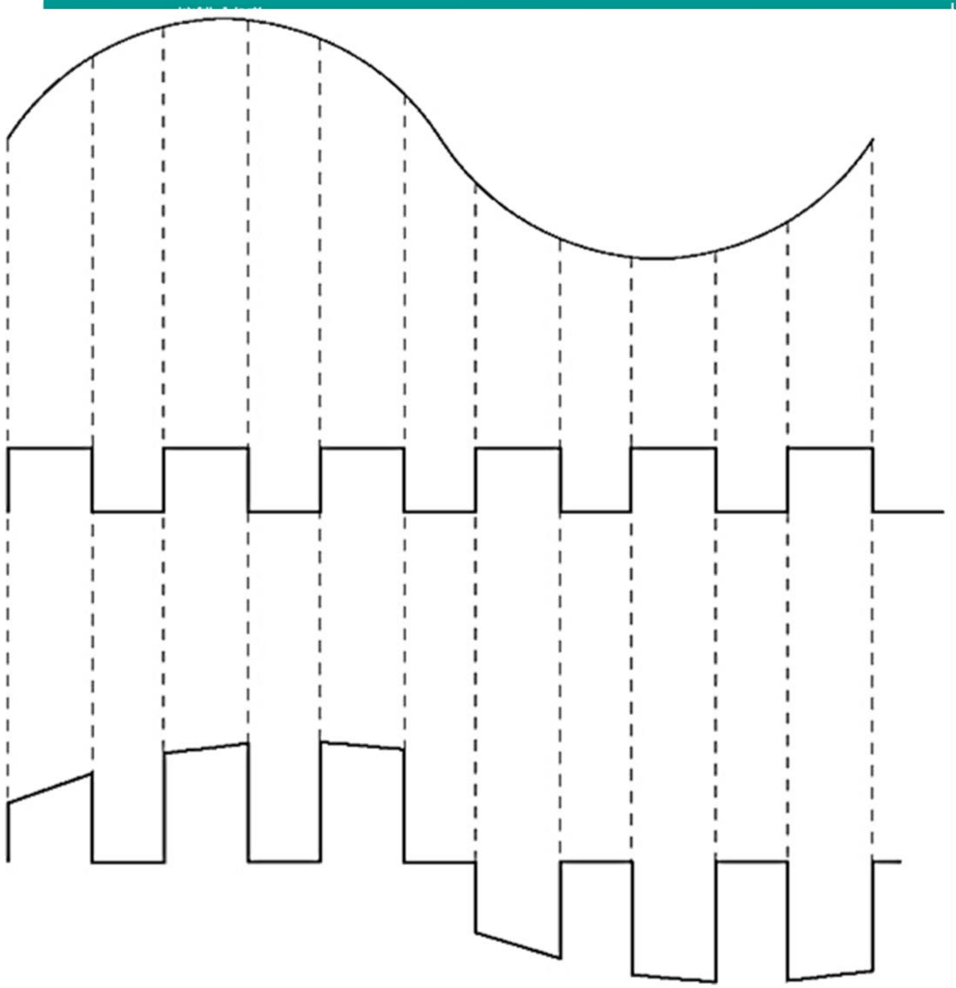
1. **Natural Sampling**

   tops of the sample pulses retain their natural shape, making it difficult for ADC to convert to PCM codes

2. **Flat-top Sampling**

   input voltage is sampled with narrow pulses and then held relatively constant until next sampling

Sampling: Natural sampling

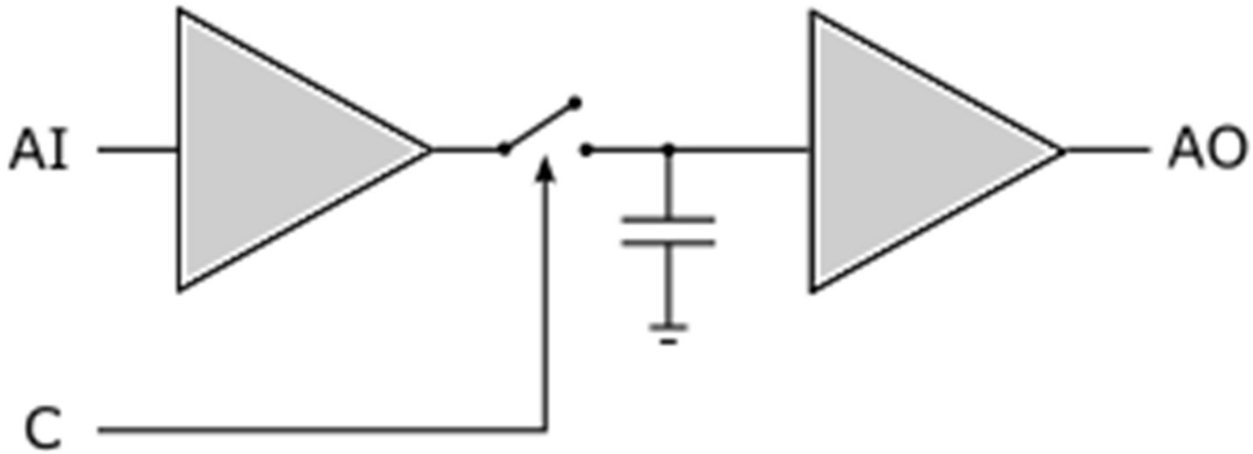


Input analog signal

Sampling pulse

Sampled output

Sampling: Flat-top sampling



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Sample-and-hold circuit

Sampling Frequency

- To retain the high-frequency information in the analog signal, a sufficient number of samples must be taken to adequately represent the waveform.
- The minimum sampling frequency is twice the highest analog frequency content of the signal.
- This minimum sampling frequency is known as the **Nyquist frequency**.
- In practice the sampling rate is much higher (typically 2.5 to 3 times more) than the Nyquist minimum.

Sampling Frequency

Nyquist sampling theorem states that an analog signal is completely described by its samples, taken at equal time intervals, the sampling frequency/sampling rate is greater than, or equal to, twice the maximum frequency component of the analogue signal

$$f_s \geq 2 f_m$$

f_s = 2 x (bandwidth of analog signal)
= 2B Hz

Sampling Frequency

- If the sampling frequency is not high enough, aliasing occurs.
- Aliasing causes a new signal near the original to be created.
- This signal has a frequency of $f_s - f_m$.
- When the sampled signal is converted back to analog by a D/A converter, the output will be the alias, not the original signal.

STEP 2: Quantization

- In PCM, we need to encode each sample value with *n* bits of binary value.
- But as number of bits, *n* is limited, it's impossible to represent each real values of sampled input.
- To solve this, we must define a finite set of discrete values called levels- which will then be used to represent the value of sampled input.
- Then we 'round-off' the the sampled input to the closest levels.
- This process is called quantization

Example Quantization:

Consider a PCM table below:

LEVEL	Quantized value (V)	VOLTAGE RANGE (V)
7	5.5	5 to 6
6	4.5	4 to 5
5	3.5	3 to 4
4	2.5	2 to 3
3	1.5	1 to 2
2	0.5	0 to1
1	-0.5	-1 to 0
0	-1.5	-2 to -1

- Let's say the output of sampling are as follow:
- at t_1 , V_1 = 4.4V
- at t_2 , V_2 = 4.45V
- at t_3 , V_3 = 4.6V
- at t_4 , V_4 = 3.5 V

- The Quantized value will be?
- V_1 =4.5
- V_2 = 4.5
- V_3 =4.5
- V_4 =3.5

Quantization

- The number of levels available depends on the number of bits used to express the sample value.
- The number of levels is given by **$L = 2^n$**

where L = Number of levels

n = number of bits per sample

EXAMPLE 1: the number of levels with 2 bits per sample is: $L = 2^2 = 4$

Exercise 1: Find the number of levels for the following PCM system:

a) 3-bit PCM

b) 8-bit PCM

Quantization Interval (ΔV)/ resolution

- Represent the voltage value for each quantized level
- Smallest analog voltage change that can be distinguished

$$\Delta V = \frac{m_{\max} - m_{\min}}{L}$$

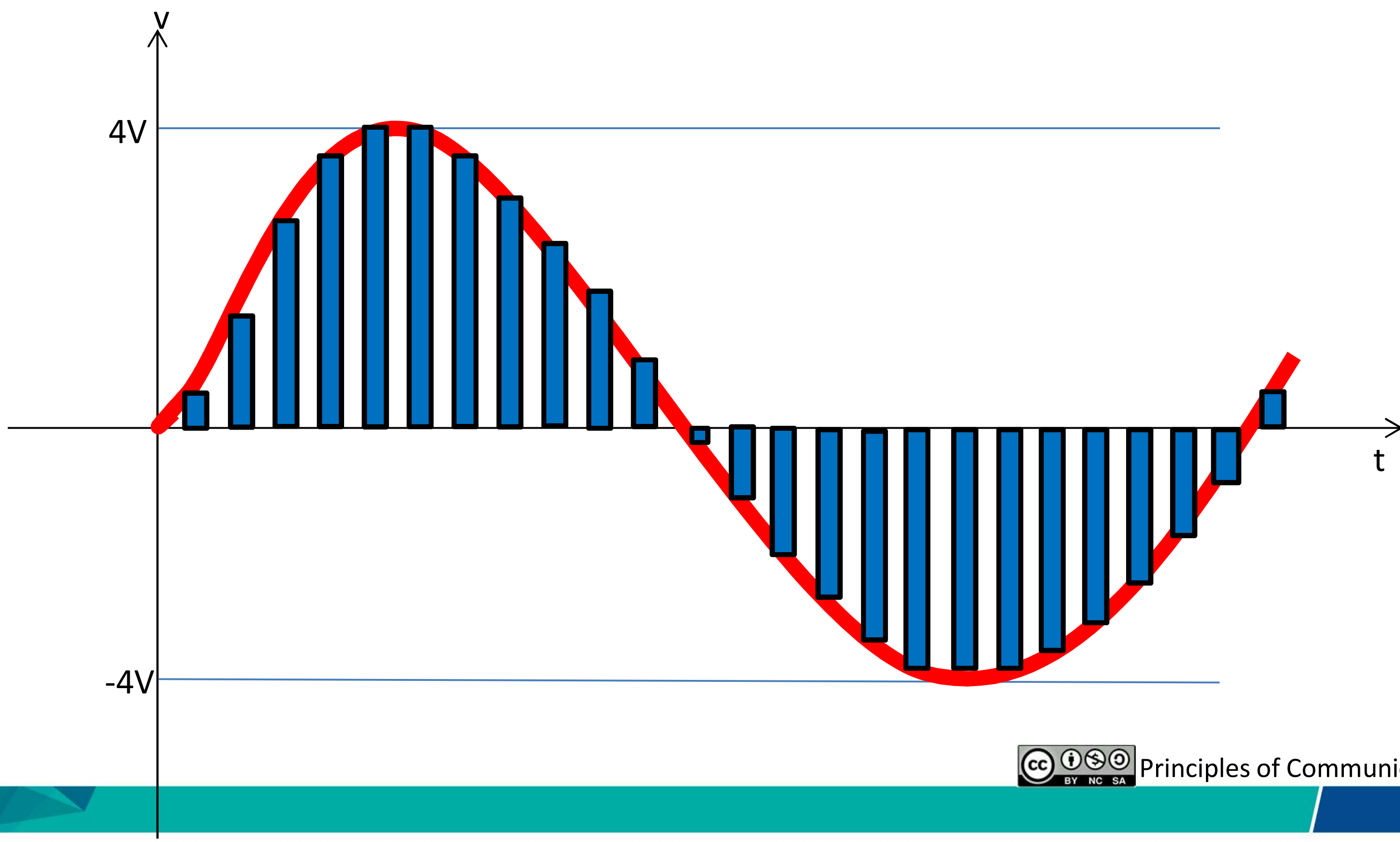
– Where m_{max} = Highest voltage value for the analog signal

m_{min} = lowest voltage value for the analog signal

L = number of levels

Example 1

Given an analog signal as shown in the figure below, with the highest amplitude of 4V, lowest amplitude -4V. The signal is then sampled by sampling pulses. The analog signal and the output are shown below:



Example 1

- Let's say we use 2 bits per sample, thus L = 4
- Therefore, quantized interval ,

$$\Delta V = \frac{m_{\max} - m_{\min}}{L} = \frac{8\text{ V}}{4} = 2\text{ V}$$

- Thus we can say the minimum value in a level is 2V.
- Next step is to build up the PCM table
- Find the voltage value for each levels: The middle voltage for each quantized level

Quantization Value V_k

- The middle voltage for each quantized level
- For example 1: The middle quantized value for level 0 should be:

$$V_0 = m_{\min} + \frac{\Delta V}{2} = -4 + \frac{2}{2} = -3 \text{ V}$$

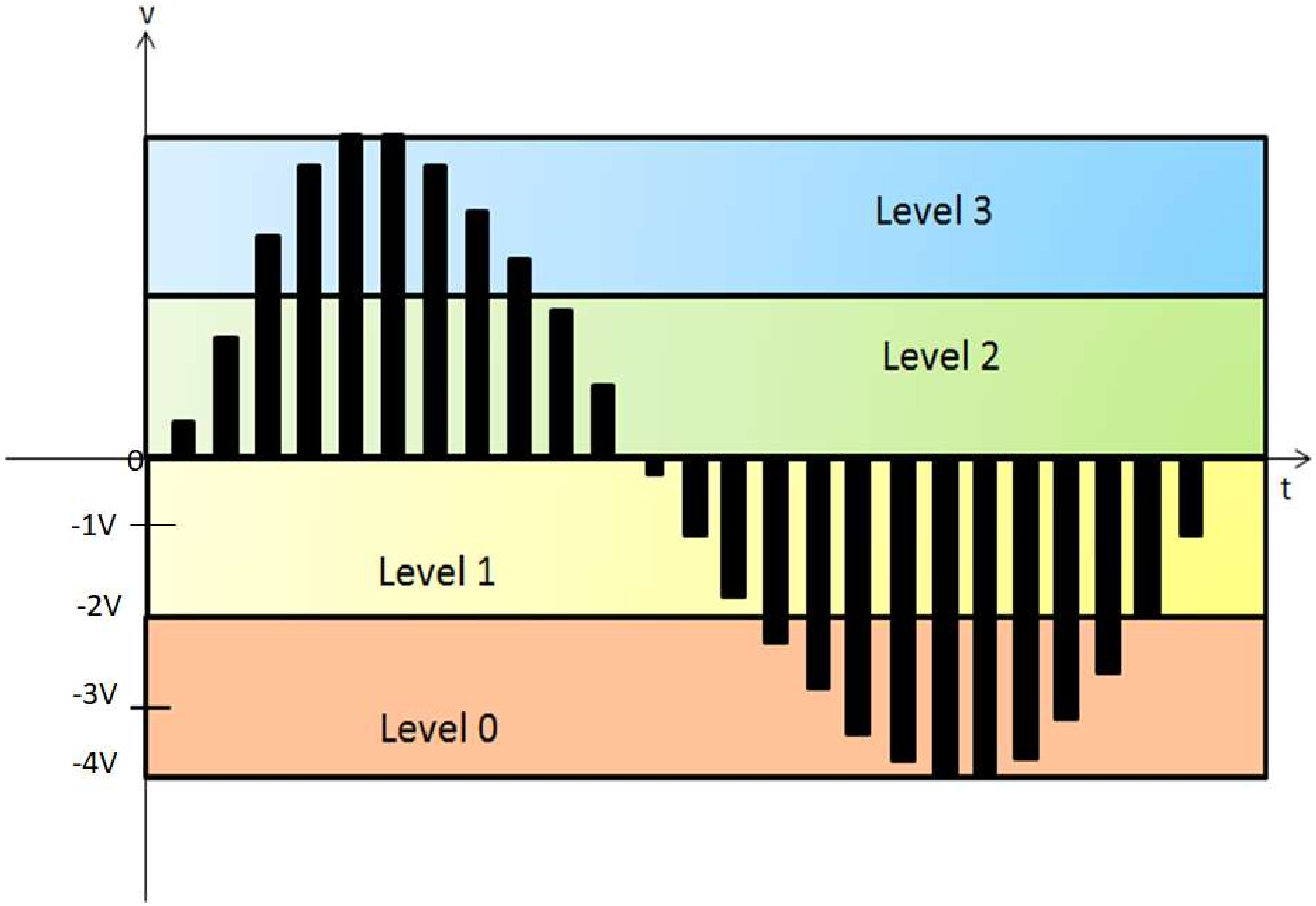
- Quantized value for level 1 is $V_1 = -3 + 2 = \text{-}1\text{V}$

...calculate quantized value for level 2 and 3.

- Next, can build up our PCM system and determine the voltage range value for each level.
- For example, at level 0, the voltage range would be from -4V to -2V

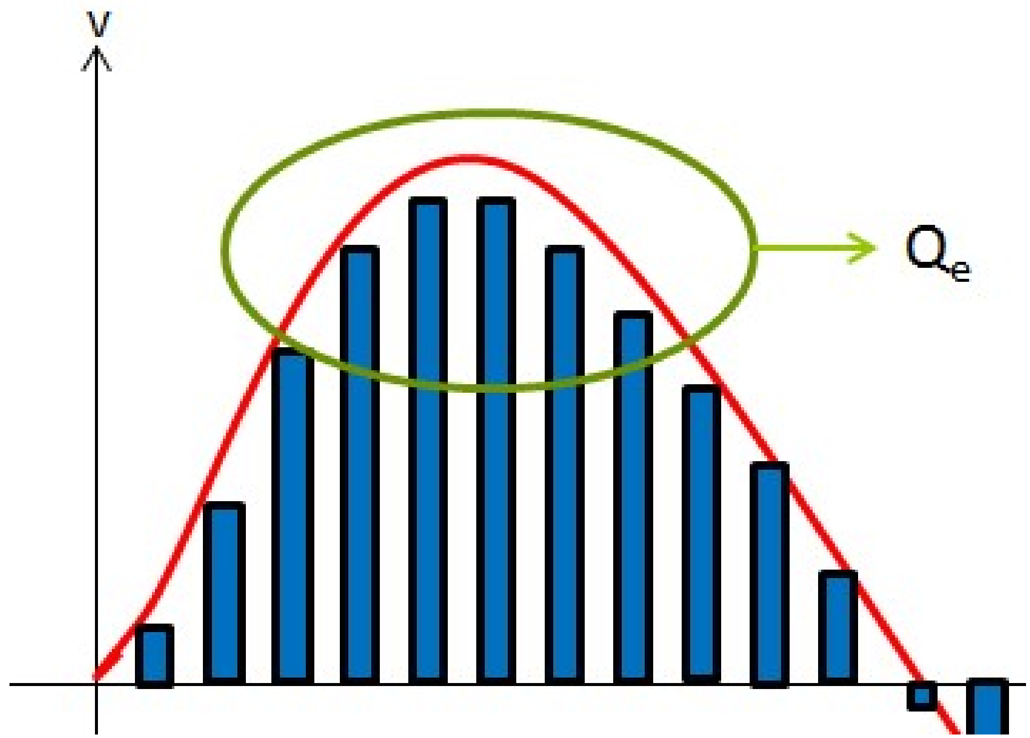
Exercise 1

Based on Example 1, Fill up and complete the middle value and range for each levels below



Quantization Error Q_e

- The quantization error Q_e is the difference between an input value and its **quantized** value
- $Q_e = quantized\ value - original\ analog\ signal$
- Maximum magnitude Q_e is equal to one-half a quantum



$$Q_e = \frac{\text{resolution}}{2}$$

- Lower resolution will produce a more accurate quantized signal that will better resemble the original analog sample

Dynamic Range

Ratio of the largest possible magnitude to the smallest (other than 0) magnitude that can be decoded by the digital-to-analog converter (DAC) in the receiver

$$DR = \frac{V_{max}}{V_{min}} = \frac{m_{max} - m_{min}}{\Delta V} = 2^n$$

$$DR(dB) = 20\log\frac{V_{max}}{V_{min}} = 20\log 2^n$$

DR = dynamic range (unitless)
V_{min} = the quantum value
V_{max} = the maximum voltage magnitude of the DACs
n = number of bits in a PCM code (excl. sign bit)

Types of Quantization

1. Linear/uniform Quantization
   1. Equally spaced quantization levels, ΔV

2. Non-Linear Quantization
   1. Quantization levels not evenly spaced
      - There is a greater number of quantizing steps for low amplitude.
   2. Reduces overall signal distortion
      - Improves the S/N for small amplitude signals.

Non-Linear/non-Uniform Quantization

Non-uniform: to improve SNR (SQR)

- ⇒ More levels is available for low level amplitudes compared to high amplitude
- ⇒ Increase SNR for low level amplitude and decrease SNR for higher amplitudes

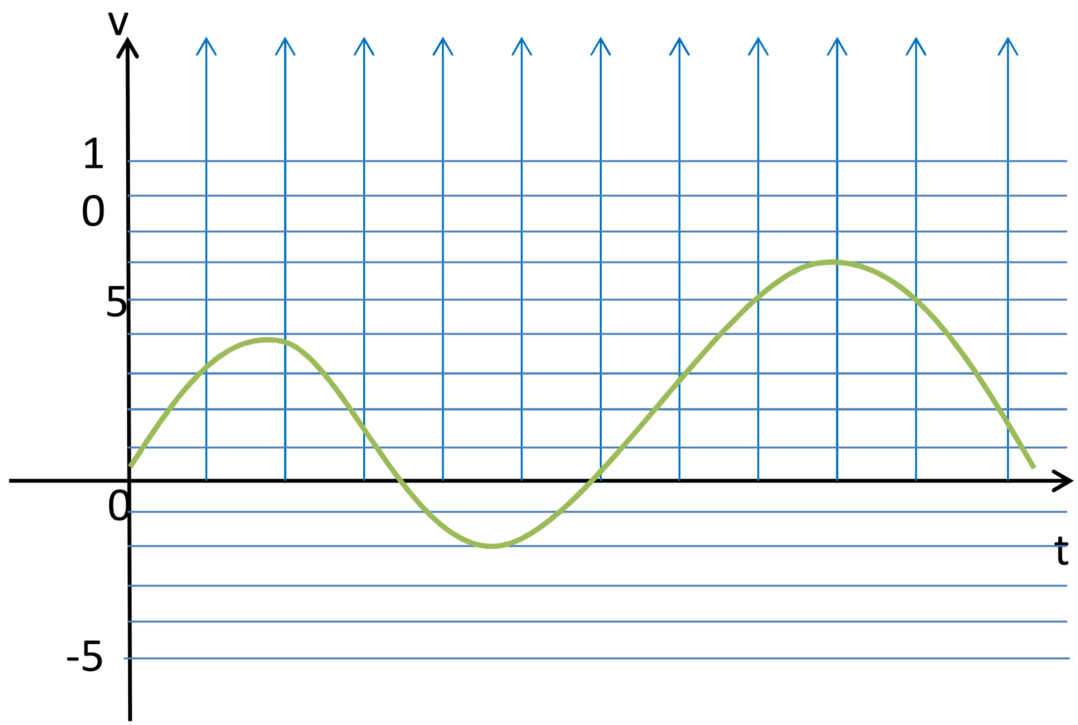
analog compression is done to the input signal before sampling and quantization at the transmitter

Expansion is done at the receiver

COMPANDING (compression and expanding)

Example 2

- Consider an analog signal shown in the figure. Let's say we want to encode the signal using 3- bit PCM system



Example 2

- Thus, the number of levels, L = 2^3 = 8
- And by examining the signal, we can see the m_{max} = 6V and m_{min} = -2V
- Thus the quantization interval is

$$\Delta V = \frac{m_{\max} - m_{\min}}{L} = \frac{6-(-2)}{8} = 1V$$

- And the middle quantized value for level 0 should be:

$$V_0 = m_{\min} + \frac{\Delta V}{2} = -2 + \frac{1}{2} = -1.5$$

- So the middle value for level 1 should be:

$$V_1 = -1.5 + 1 = \text{-}0.5$$

Exercise 2

- We can then build up a table to show the levels and corresponding voltage values. Complete the table below:

LEVEL	Quantized value (V)	VOLTAGE RANGE (V)
7	5.5	5 to 6
6	4.5	4 to 5
5	3.5	3 to 4
4	2.5	2 to 3
3	1.5	1 to 2
2	0.5	0 to1
1	-0.5	-1 to 0
0	-1.5	-2 to -1

Encoding

- The next step in PCM is to determine the binary code for each PAM signal based on their levels.
- Let's look at example 2 before, where we are trying to build a 3-bit PCM system.
- Thus the lowest binary number should be 000 while the highest is 111
- We can then set the binary coding for each level

Example 2

LEVEL	QUANTIZED VALUE (V)	VOLTAGE RANGE (V)	CODE
7	5.5	5 to 6	111
6	4.5	4 to 5	110
5	3.5	3 to 4	101
4	2.5	2 to 3	100
3	1.5	1 to 2	011
2	0.5	0 to1	010
1	-0.5	-1 to 0	001
0	-1.5	-2 to -1	000

Exercise 3

- Based on the PCM coding table for Example 2, Determine, what is the quantized value and PCM code for the following analog values:

(a) 2.35V
(b) -1.4 V
(c) 5.78 V
(d) -0.9 V
(e) 1.6 V

Bit Rate

According to the Nyquist theorem, the sampling rate at least 2 times the highest frequency contained in the signal.

$$f_s = 2f_m$$

So we can say that **sampling rate, fs = 1/Ts**
Where Ts is sampling interval

Bit rate is used to describe digital signals.
The bit rate is number of bits sent in 1s, expressed in bits per second (bps)

$$f_b = f_s \times n$$

f_b = bitrate (bps)
f_s = sampling rate (sample/second)
n = number of bits per sample (bits/sample)

Example 3

- Refer back to Example 2 PCM system. Let's say the highest frequency on the analog signal, f_m is 200Hz.
- Thus $f_s = 2f_m = 400\ Hz$

- And since the system is a 3-bit PCM,

$$f_b = f_s \times n = 400(3) = 1200 \text{ bps}$$

Exercise 3

Find the sampling rate for PCM system describe previously, if the number of bit/sample for the PCM system is changed to:

(a) 2 bits

(b) 4 bits

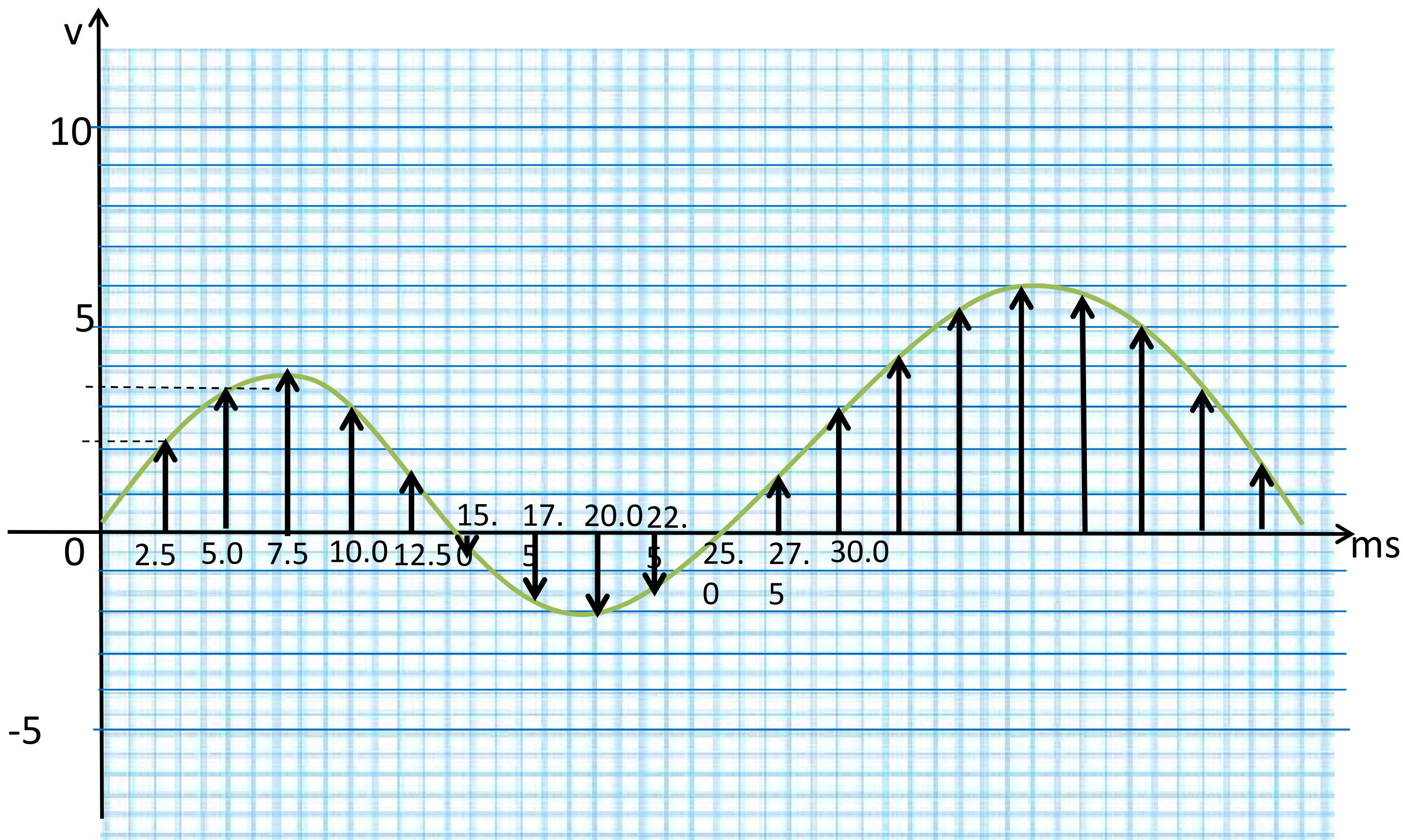
(c) 8 bits

Which system is better in terms of bps?

Example 4

- Refer back to example 2 & 3, let's now try to find out what are the quantized values for each sampled signal
- Sampling rate, $f_s = 400Hz$
- Sampling interval, Ts = 1/f_s = 1/400 = 2.5ms
- Thus at every 2.5ms, the original analog signal is sampled.

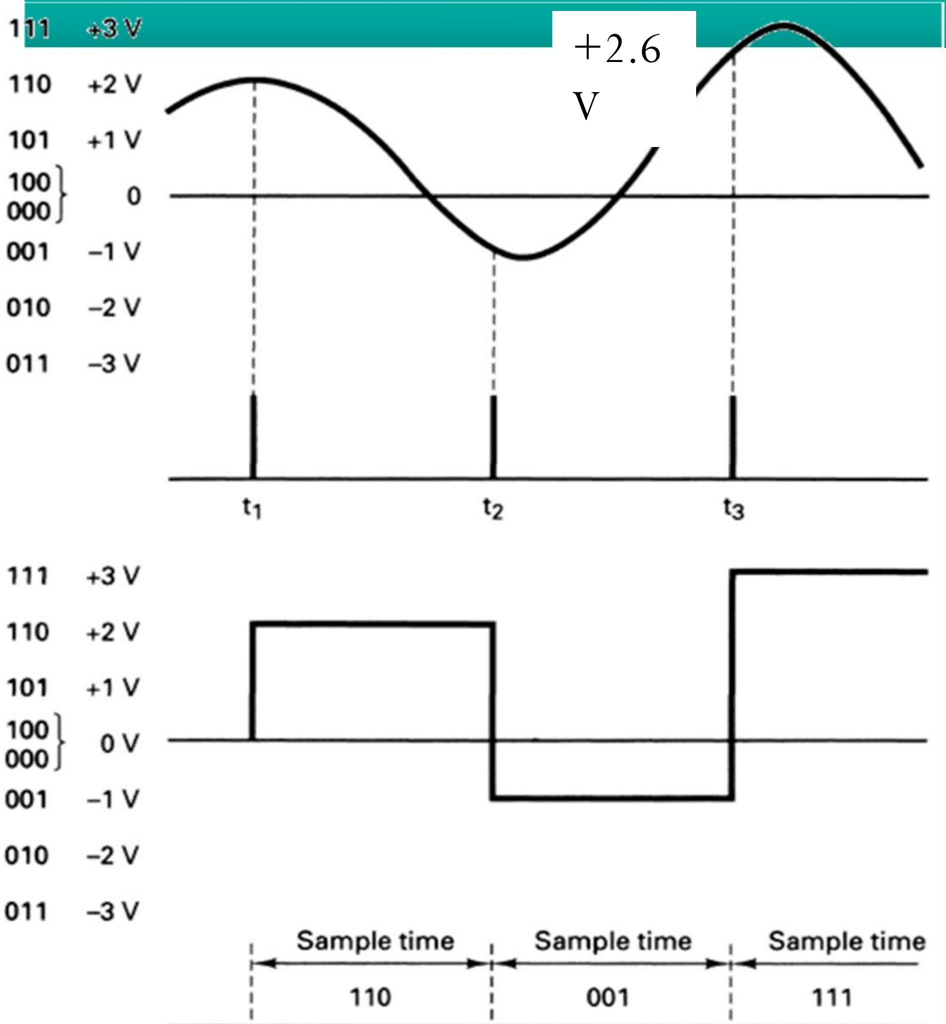
Example 4



Example 2

LEVEL	Quantized value (V)	VOLTAGE RANGE (V)	CODE
7	5.5	5 to 6	111
6	4.5	4 to 5	110
5	3.5	3 to 4	101
4	2.5	2 to 3	100
3	1.5	1 to 2	011
2	0.5	0 to1	010
1	-0.5	-1 to 0	001
0	-1.5	-2 to -1	000

Example 5: Folded binary code



Input analog signal

Sampling pulse

PAM signal

PCM code

What is the PCM code for +2.6 V??

Example 4

- The folded binary code table for this example is shown below:

Level	Decimal value	Quantization range (V)	CODE	
			Sign	Magnitude
7	+3	+2.5 to +3.5	1	1 1
6	+2	+1.5 to +2.5	1	10
5	+1	+0.5 to 1.5	1	01
4	+0	0 to +0.5	1	00
3	-0	0 to -0.5	0	00
2	-1	-0.5 to -1.5	0	01
1	-2	-1.5 to -2.5	0	10
0	-3	-2.5 to -3.5	0	11

0 V codes each have an input range equal to only one half a quantum

Example 4

- Thus for 2.6V, the PCM code would be 111.
- while -2.6 will have PCM code of 011
- Exercise: Find the PCM codes for the following sampled voltage:

(a) 1.8V

(b) -1.8V

(c) -2.3 V

(d) 2.3V

Dynamic Range for Folded Binary Code

- Because of the smaller step size in the middle of the levels, dynamic range is changed to:

$$DR = \frac{V_{\max}}{V_{\min}} = \frac{V_{\max}}{\text{resolution}} = 2^n - 1$$

$$DR_{(dB)} = 20\log\left(2^n - 1\right)$$

Coding Efficiency

Coding efficiency is a numerical indication of how efficiently a PCM code is utilized

$$\text{coding efficiency} = \frac{\text{minimum number of bits}}{\text{actual number of bits}} \times 100$$

Example 6

A folded binary PCM systems has the following specification:

Maximum Analog Input Frequency = 4 kHz

Maximum decoded voltage at the receiver = $\pm$2.55 V

The dynamic range = 46 dB

Determine the following :

(a) Minimum Sampling Rate

(b) Minimum number of bits used in PCM code

(c) Resolution

(d) Quantization Error

(e) Coding Efficiency

Solution

(a) The minimum sampling rate:

$f_s = 2f_a = 2(4\text{ kHz}) = 8\text{ kHz}$

(b) Calculate the Dynamic range :

$46 = 20\log(V_{max} / V_{min})$

$V_{max} / V_{min} = \text{antilog}(46/20) = 199.5$

Thus, the minimum number of bit used:

$n = \log(199.5 + 1) / \text{Log } 2 = 7.63$

(c) Resolution is defined as:

$V_{max} / 2^n - 1 = 0.01\text{ V}$

(d) Quantization Error :

$Q = \text{resolution} / 2 = 0.01\text{ V} / 2 = 0.005\text{V}$

(e) Coding Efficiency

Coding efficiency

$= (8.63/9)(100)$

$= 95.89\%$

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