Universiti Malaysia PAHANG
Engineering • Technology • Creativity

Principles of Communications System

Chapter 2 (part 1): Amplitude Modulation

Learning Outcomes

By the end of this chapter, students should be able to:

- Write the time domain equation for an AM relates it to the signal itself
- Solve problems involving amplitude modulation

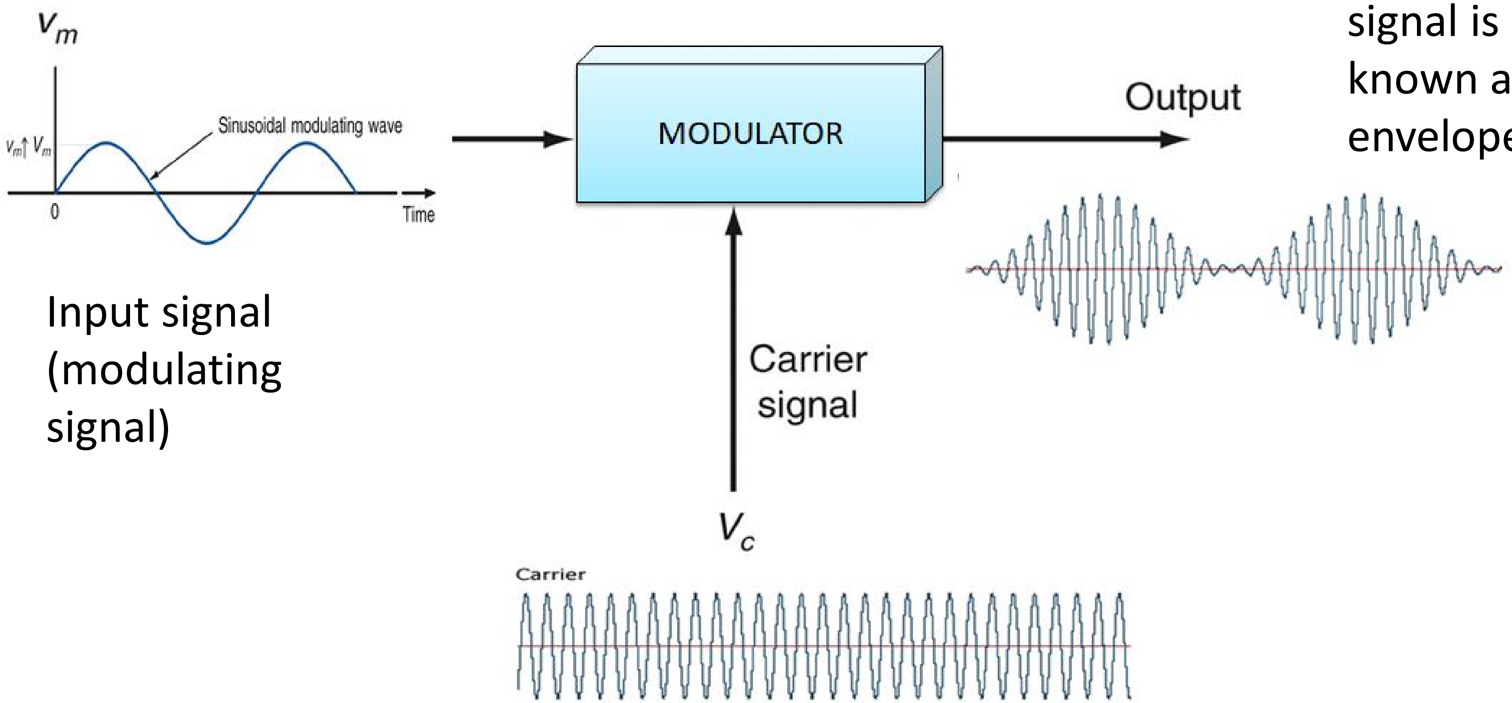
Introduction

Å Amplitude Modulation is a modulation technique where the amplitude of a carrier signal (high frequency) is modified by a lower frequency modulating signal (also known as Information or input signal)

Å It is the earliest and simplest form of analog modulation

AM Concepts

- Am modulator block diagram, showing the input and output signals



Input signal (modulating signal)

Modulated signal is also known as AM envelope

AM Envelope

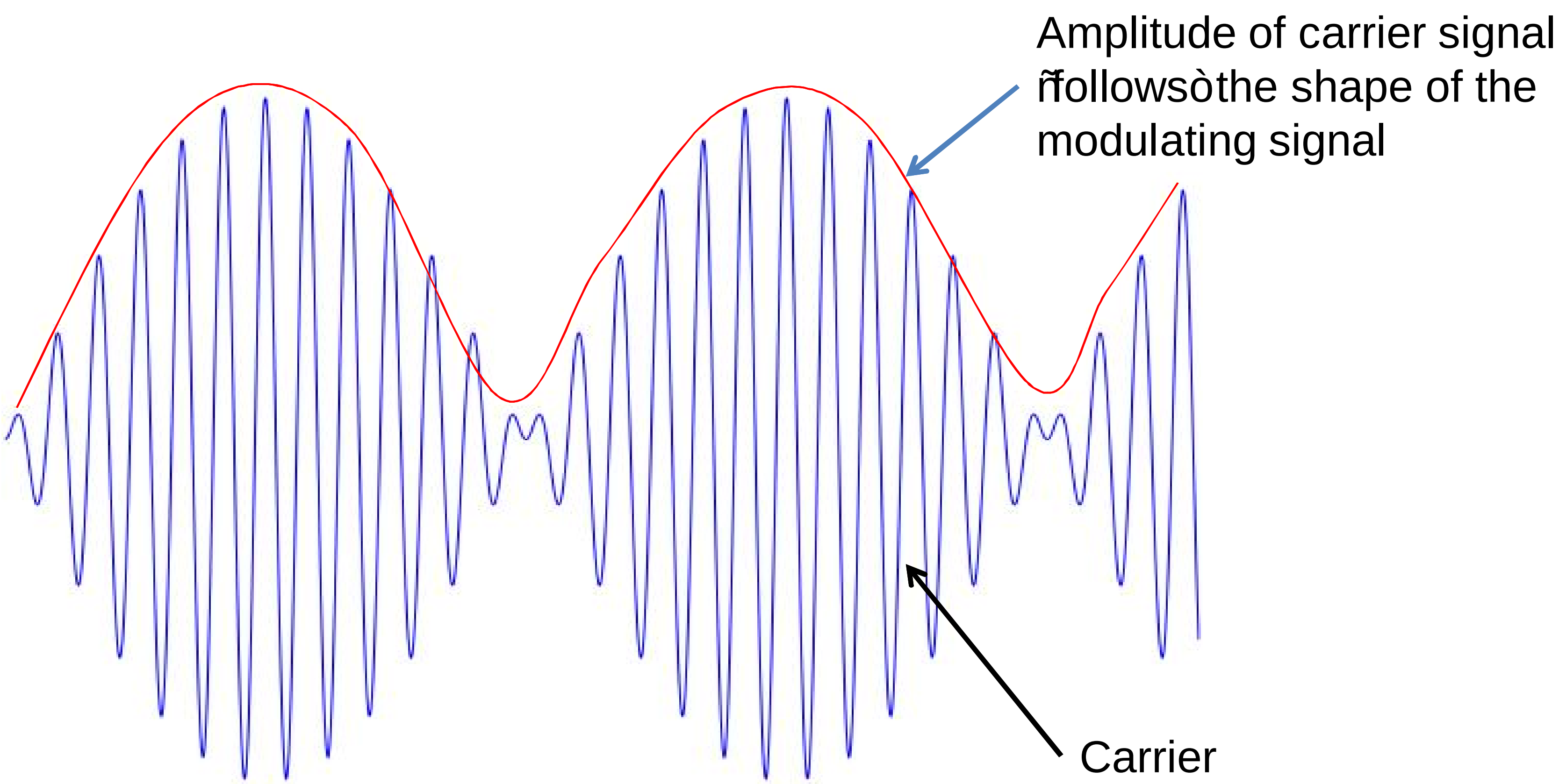


Image source: https://en.wikipedia.org/wiki/File:Amplitude_modulation.svg

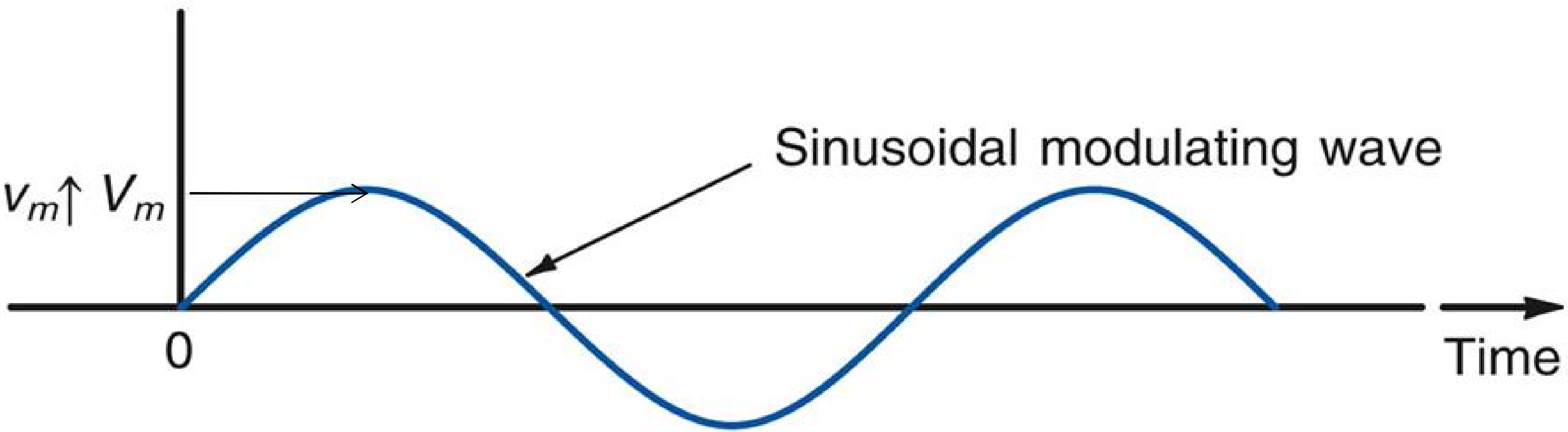
AM wave equation

Let's say the modulating signal (input signal) is a sinusoid signal :

$$v_m\,(t) = E_m \sin\,(2\pi f_m t)$$

Or $$v_m\,(t) = V_m \sin\,(2\pi f_m t)$$

E_m = V_m= peak modulating signal amplitude(volts)



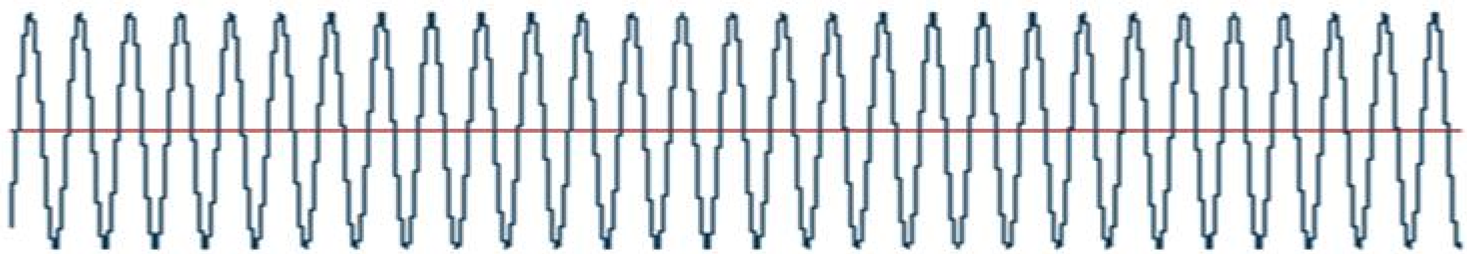
AM wave equation

Carrier signal is always a sinusoid signal. The equation is given by:

$$v_c\,(t) = E_c \sin\,(2\pi f_c t)$$

Or $$v_c\,(t) = V_c \sin\,(2\pi f_c t)$$

$E_c = V_c$ = *peak carrier amplitude (volts)*



AM wave equation

AM envelope can be expressed as:

$$V_{am}(t) = [E_c + E_m \sin(2\pi f_m t)] (\sin 2\pi f_c t) \ldots\ldots\ldots(1)$$

WHERE:

E_c = Amplitude of the carrier signal

E_m = Amplitude of the modulating signal

f_m = frequency of the modulating signal

AM wave equation

Expanding eq (1) we get:

$$V_{am} = E_c \sin(2\pi f_c t) + E_m \sin(2\pi f_m t)\left[\sin(2\pi f_c t)\right] \ldots\ldots.(2)$$

Carrier signal

Modulating signal

Later we will see how this equation can be further improved to make it more meaningful

AM Wave Equation

- Peak amplitude of modulating signal must be less than the peak amplitude of the carrier:

$$E_m < E_c$$

- **Otherwise, distortion** will occurs

Example 1:

An AM modulator has two input. The first input is a sine wave with a frequency of 500 Hz and amplitude of 1V RMS. The second input is a carrier signal with RMS voltage of 2 V and frequency of 1.5 MHz. Determine the equation for the modulated signal.

Solution:

$$\square_{\square} \; \square \; \sqrt{2} \; \square \; 2 \; \square \; 2.83\square$$

$$\square_{\square} \; \square \; \sqrt{2} \; \square \; 1 \; \square \; 1.41\square$$

$V_{am}(t) = [E_c + E_m \sin(2\pi f_m t)] (\sin 2\pi f_c t)$

$= [2.83 + 1.41 \sin(2\pi x 500t)] \sin(2\pi x 1.5 x 10^6 t)$

$= [2.83 + 1.41 \sin(3.14 x 10^3 t)] \sin(9.42 x 1.5 x 10^6 t)$ V

Modulation Index

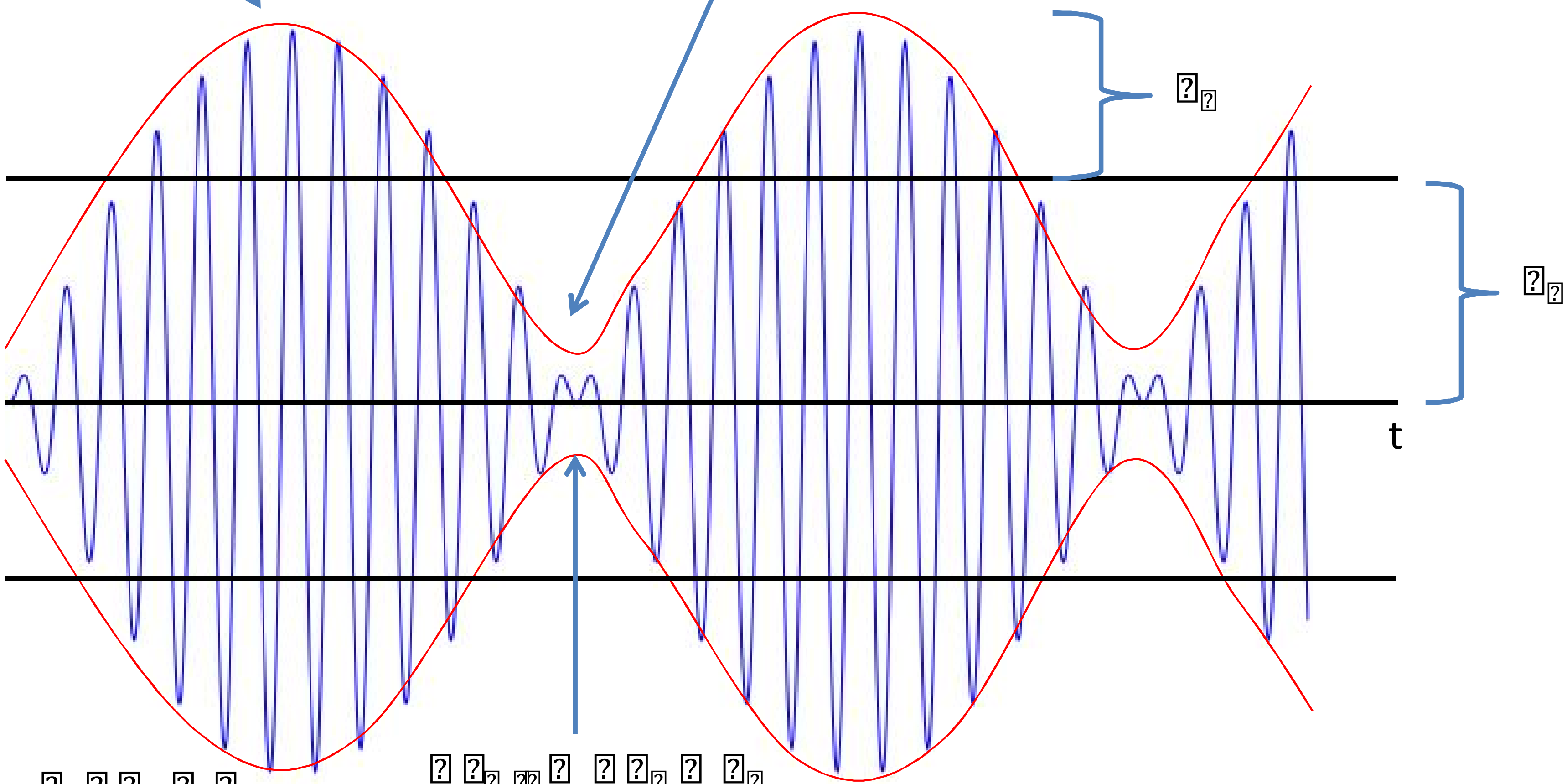
- **modulation index (*m*)** is a ratio between the amplitude of the modulating signal to the amplitude of the carrier signal.
- It shows the depth of the modulation

$$m = \frac{E_m}{E_c}$$

- **Percentage of modulation.**

$$M = \frac{E_m}{E_c} \times 100$$

AM Envelope



Modulation Index

Å **modulation index (*m*) can also calculate it using**

$$m = \frac{\frac{1}{2}\left(V_{\max} - V_{\min}\right)}{\frac{1}{2}\left(V_{\max} + V_{\min}\right)} = \frac{\left(V_{\max} - V_{\min}\right)}{\left(V_{\max} + V_{\min}\right)}$$

where
$$V_{\max} = E_c + E_m$$
$$V_{\min} = E_c - E_m$$

Modulation Index

Å Substituting *m* into AM modulated signal gives:

$$V_{am}(t) = [E_c + E_m \sin(2\pi f_m t)] (\sin 2\pi f_c t)$$

$$m = \frac{E_m}{E_c}$$

$$V_{am}(t) = E_c [1 + m \sin(2\pi f_m t)] (\sin 2\pi f_c t)$$

Exercise

For an AM DSBFC envelope with $+V_{max}$ = 40V and $+V_{min}$ = 10V, determine:

(i) Peak change in amplitude of the modulated wave (E_m)
(ii) Unmodulated carrier amplitude (E_c)
(iii) Modulation index & percent modulation

Modulation Index for Multiple Modulating Frequencies

When there are more than one sine waves (with different frequencies) modulating a single carrier, the total resultant modulation index is:

$$m = \sqrt{m_1^2 + m_2^2 + \bullet\bullet\bullet}$$

m = total resultant modulation index
m_1, m_2,é = modulation index due to individual modulating component

Over modulation and Distortion

- AM modulation index, m must be between 0 and 1.
- if *m* is greater than 1 (over modulation) distortion will occur
- Distortion causes intelligence signal to become unintelligible.
- Over modulation may cause the signal to occupy a larger bandwidth than normal

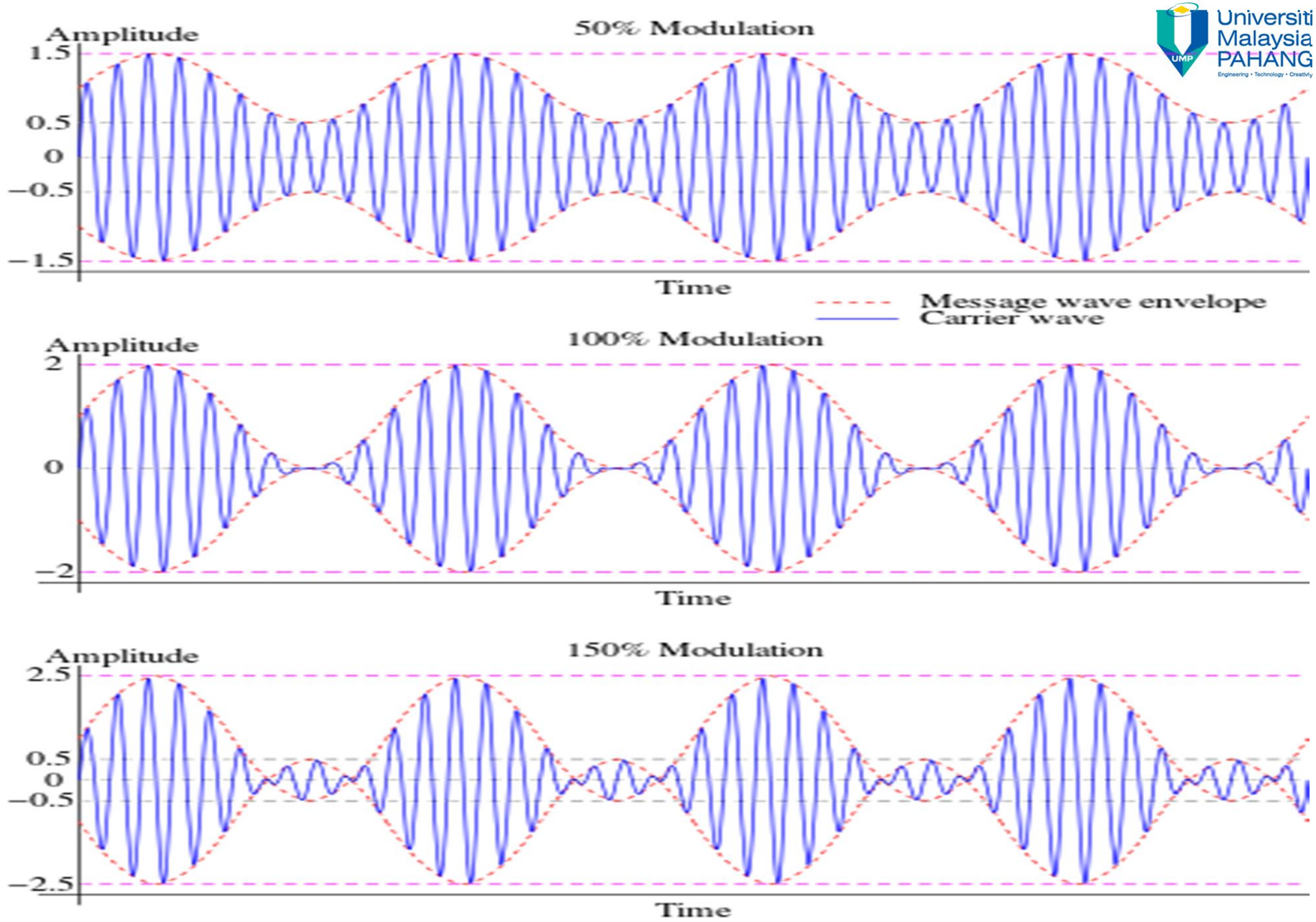


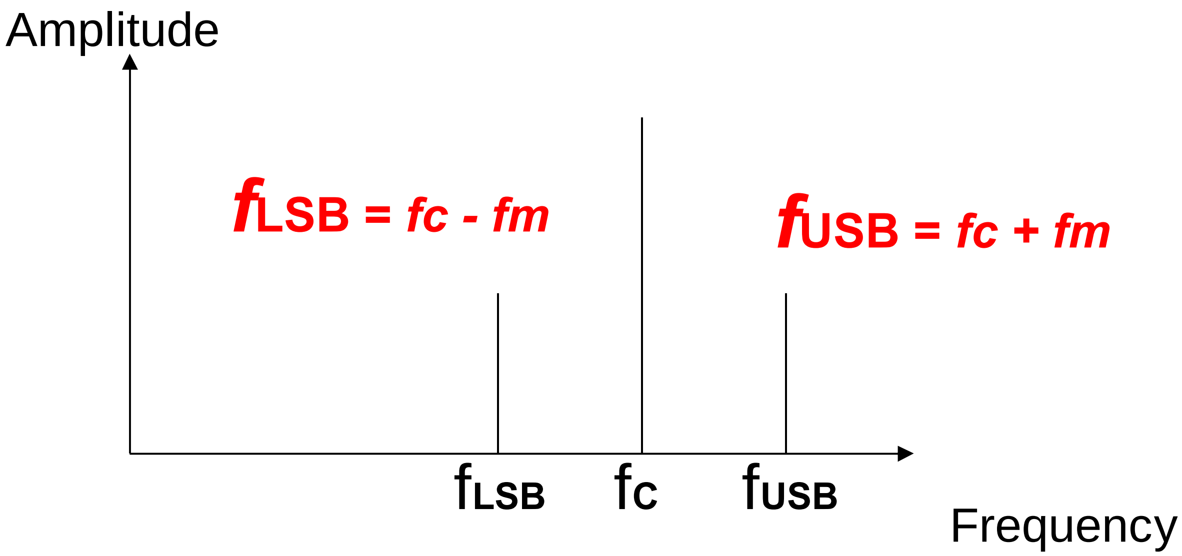
Image source: https://commons.wikimedia.org/wiki/File:Amplitude-Modulated_Wave-hm-64.svg

Sidebands

- Amplitude modulation process produces **sidebands**
- Modulation by a sine-wave will generates two sidebands
- A more complex wave (e.g. voice or video)with multiple frequencies modulation will generates a range of sidebands.
- To see the sidebands, AM signal is viewed in frequency domain.

Sidebands

- Sidebands with frequency higher than carrier frequency is called upper sidebands (f_{USB})
- Sidebands with frequency lower than carrier frequency is called lower sidebands (f_{LSB})



AM Equation with sidebands

$$v(t) = \underbrace{E_c \sin \omega_c t}_{} + \underbrace{mE_c \sin \omega_m t \sin \omega_c t}_{}$$

The first term is carrier

The second term can be expand using trigonometric identities

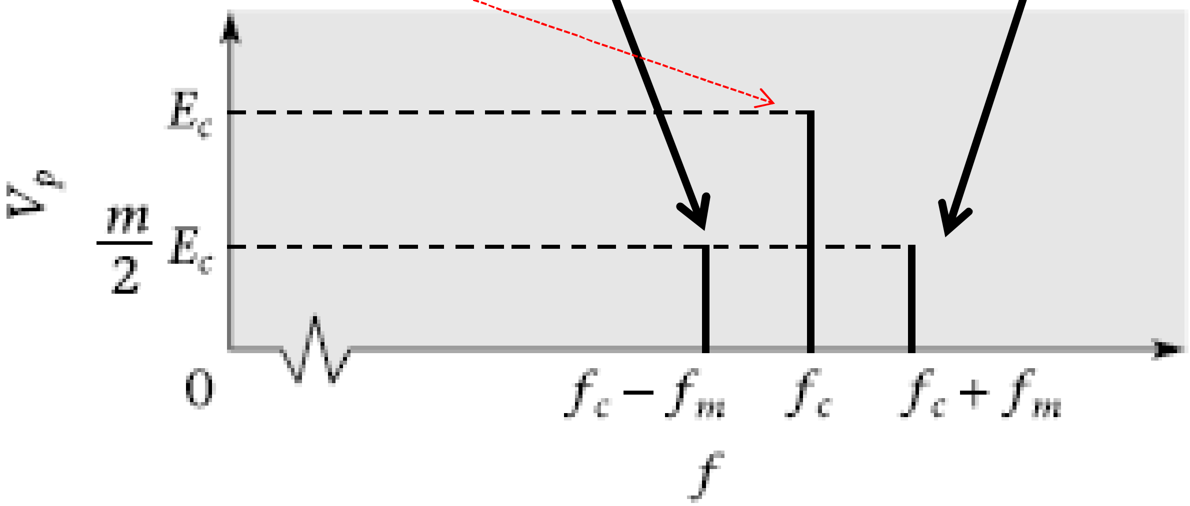
$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

and $\cos A = \cos(-A)$, *to give;*

$$v(t) = E_c \sin \omega_c t + \frac{mE_c[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]}{2}$$

Full carrier AM: Frequency Domain

$$v(t) = \underbrace{E_c \sin \omega_c t}_{\text{carrier}} + \underbrace{\frac{mE_c}{2} \cos(\omega_c - \omega_m)t}_{\text{Lower side bands}} - \underbrace{\frac{mE_c}{2} \cos(\omega_c + \omega_m)t}_{\text{Upper side bands}}$$



Bandwidth

- Bandwidth for an AM signal is calculated by:

$$B = 2f_m$$

B = bandwidth in Hertz
F_m = highest modulating frequency in Hertz

- Bandwidth is also the difference between the upper and lower sideband frequencies.

$$B = f_{USB} - f_{LSB}$$

Example 3

An AM modulator has maximum modulating frequencies of 5 kHz. If the carrier signal has a frequency of 980 kHz, find the sidebands frequencies and total bandwidth. Sketch the frequency spectrum.

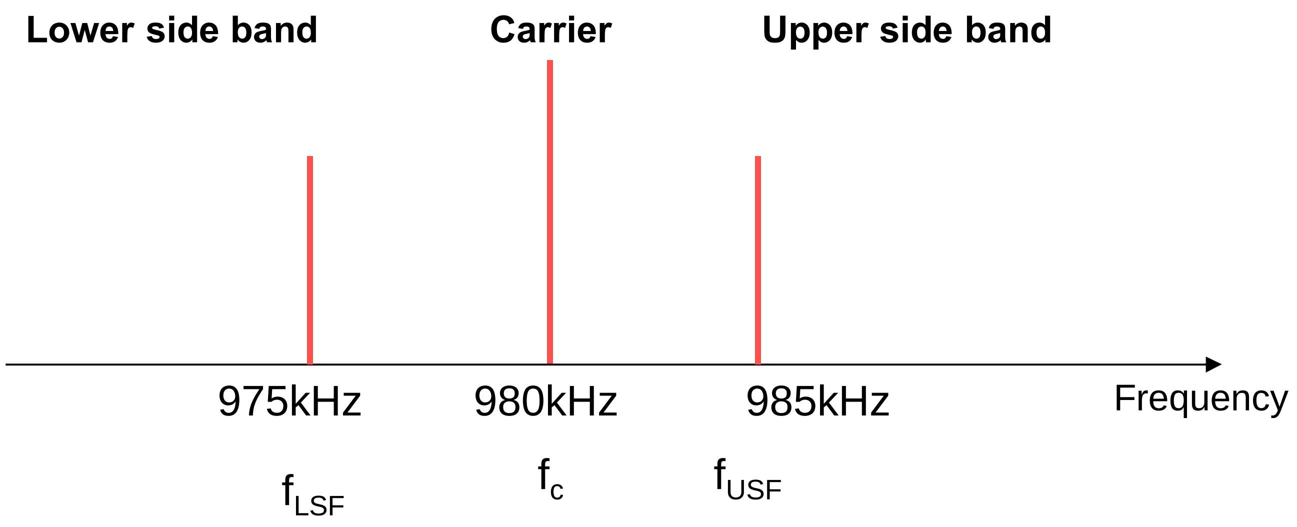
Solution

$f_{USB} = f_c + f_m$ =980 + 5 = 985 kHz

$f_{LSB} = f_c - f_m$ = 980 – 5 = 975 kHz

BW = $f_{USB} - f_{LSB}$ = 985 – 975 = 10 kHz
Or
BW = 2 (5 kHz) = 10 kHz

Solution:



Lower side band
Carrier
Upper side band
975kHz
980kHz
985kHz
Frequency
f_{LSF}
f_c
f_{USF}

Power Relationship : Carrier

Å Power developed when carrier signal appear across resistance R is:

$$P_c = \frac{\left(E_c/\sqrt[2]{2}\right)^2}{R}$$

$$= \frac{{E_c}^2}{2R}$$

Å Where Ec is peak carrier voltage and R is resistance

Power relationship

- AM signal is a composite of the carrier and sideband signal voltages.
- Total transmitted power (P_T) is the sum of carrier power (P_c) and power of the two sidebands (P_{USB} and P_{LSB}).

$$P_T = P_c + P_{USB} + P_{LSB}$$

Power Relationship

$$P_T = P_c + P_{USB} + P_{LSB}$$

$$P_c = \frac{E_c{}^2}{2R}$$

$$P_{usb} = \frac{E_{usb}{}^2}{2R}$$

$$P_{lsb} = \frac{E_{lsb}{}^2}{2R}$$

$$P_T = P_{AM} = P_c\left(1 + \frac{m^2}{2}\right)$$

Power relationship: Sidebands

Å Power in the sidebands can be found by:

$$P_{lsb} = \frac{E_{lsb}^2}{2R}$$

$$= \frac{\left(\frac{mE_c}{2}\right)^2}{2R}$$

$$= \frac{m^2 E_c^2}{4 \times 2R}$$

$$= \frac{m^2}{4} \times \frac{E_c^2}{2R}$$

$$P_{lsb} = P_{usb} = \frac{m^2}{4} P_c$$

Total AM Power

Å The total power in the whole signal is just the sum of the power in the carrier and the sidebands

$$P_T = P_{AM} = P_c\left(1 + \frac{m^2}{2}\right)$$

Transmission efficiency, η for AM: $$\eta = \frac{P_{SB}}{P_T} \times 100\%$$

where P_{SB} is the total sidebands signal power that contains information

$$\eta = \frac{\frac{m^2 P_c}{2}}{P_c\left(1+\frac{m^2}{2}\right)} = \frac{m^2}{2\left(1+\frac{m^2}{2}\right)} = \frac{m^2}{2+m^2}$$

If $m = 1$ (100% modulation), the average power, $P_{SB} = 50\% \ P_c = P_c/2$

It shows that the P_{SB} is dependent on m.

AM power efficiency

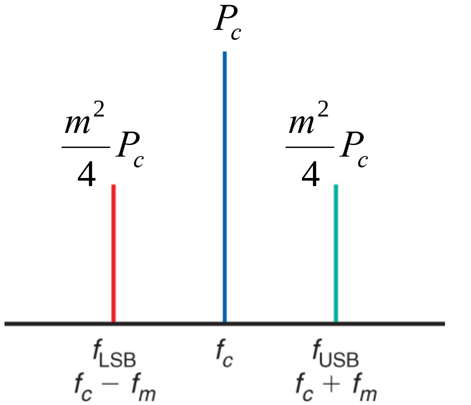
- AM power efficiency is :

$$\eta = \frac{m^2}{2 + m^2} \times 100\ \%$$

- the efficiency of AM modulation increases as the modulation index m, increases.

AM power efficiency

- In AM, information is carried by the sidebands.
- However, at maximum modulation, the sideband power is at most 33% of the total transmitted power.
- Which means that 67% of the power is wasted in the carrier.
- To maximize the efficiency of AM we need to
  - Suppress the carrier
  - Eliminate one of the sidebands



$$P_T = P_c\left(1 + \frac{m^2}{2}\right)$$

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