# DIGITAL SIGNAL PROCESSING

# Chapter 8
# IIR Filter Structure

# IIR Filter Structure

- Aims
  - To explain type of II filter structure, components and methods to realize the IIR filter structure.

- Expected Outcomes
  - By completing the chapter, students should be able to develop the appropriate filter structure based on the type and characteristics of the IIR filter.

# FIR Filter Structure

- As described in the previous chapter on the LTI system, those system can be modeled using :
  1. A Difference/Differential equation, y(n) = $x[n] + x[n-1] + \dots$
  2. Impulse Response, h(n)
  3. Transfer Function, H(z)

- Hence, the systems that described by the difference equations can be represented by structures consisting of an interconnection of the basic operations of addition, multiplication by a constant or signal multiplication, delay and advance.

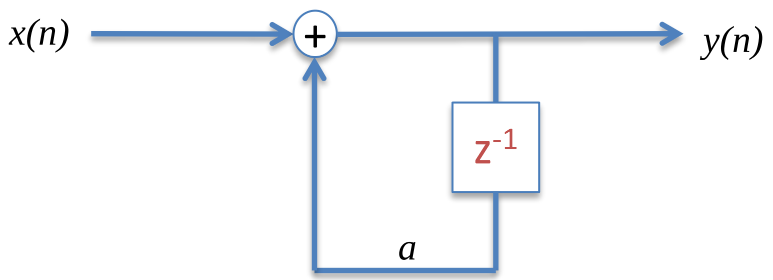
# IIR Filter Structure

- Infinite Impulse Response (IIR) of LTI can be realized by the following difference equation & block diagram:

  **$y(n) - ay(n-1) = x(n)$,** a is constant

  Thus, The Transfer Function, **$H(z) = 1 / (1 - az^{-1})$** and

  Block diagram:




- Example of the difference equation that describe the IIR system:

  ➔ **$y[n] = ½ y[n-1] + 2x[n] + x[n-1]$**

- The system transfer function to describe IIR system:

  ➔ **$H(z) = B(z) / A(z) = \Sigma(b(k)z^{-k}) / (1 + \Sigma(a(k)z^{-k}))$**

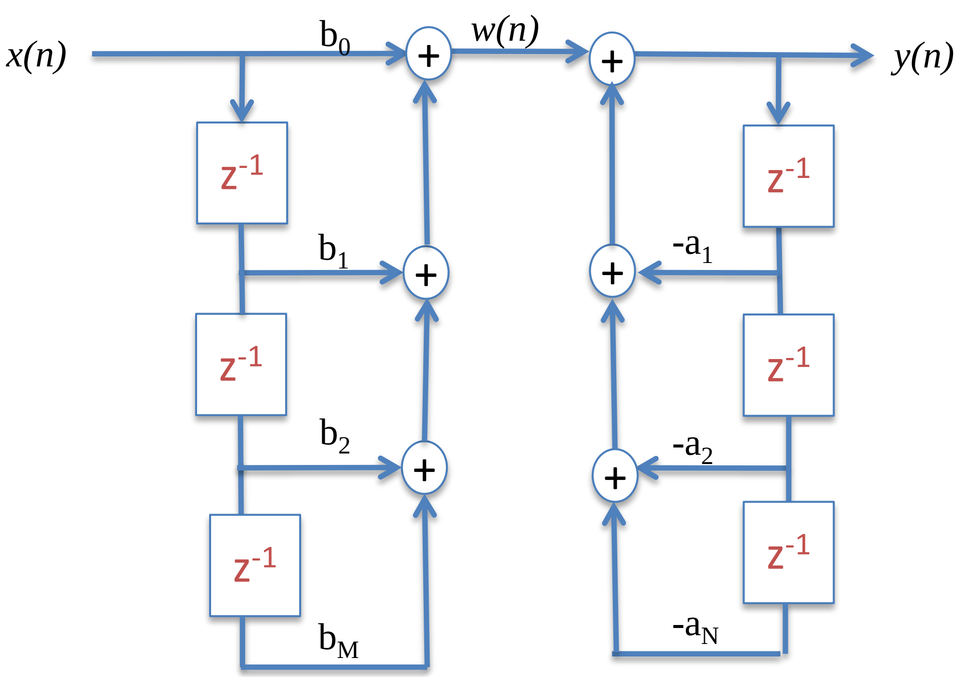
# Type of IIR Filter Structure

- IIR system/filter can be realized in several structures:

1. DIRECT FORM I
2. DIRECT FORM II (CANONIC)
3. CASCADE FORM
4. PARALLEL FORM

# IIR Filter Structure : Direct Form I

- The example IIR filter structure for Direct Form I is shown in diagram below:



# IIR Filter Structure : Direct Form I

- **Example:**

Determine the ***Direct Form I*** structure of the IIR Filter described by the following difference equation;

$$y(n) = 0.5y(n-1) - 0.76y(n-2) + 0.63y(n-3) + x(n) + 0.875x(n-1)$$

## Solution:

1. Compute the Transfer Function, H(z).

$$Y(z) = 0.5z^{-1}Y(z) - 0.76z^{-2}Y(z) + 0.63z^{-3}Y(z) + X(z) + 0.875z^{-1} X(z)$$

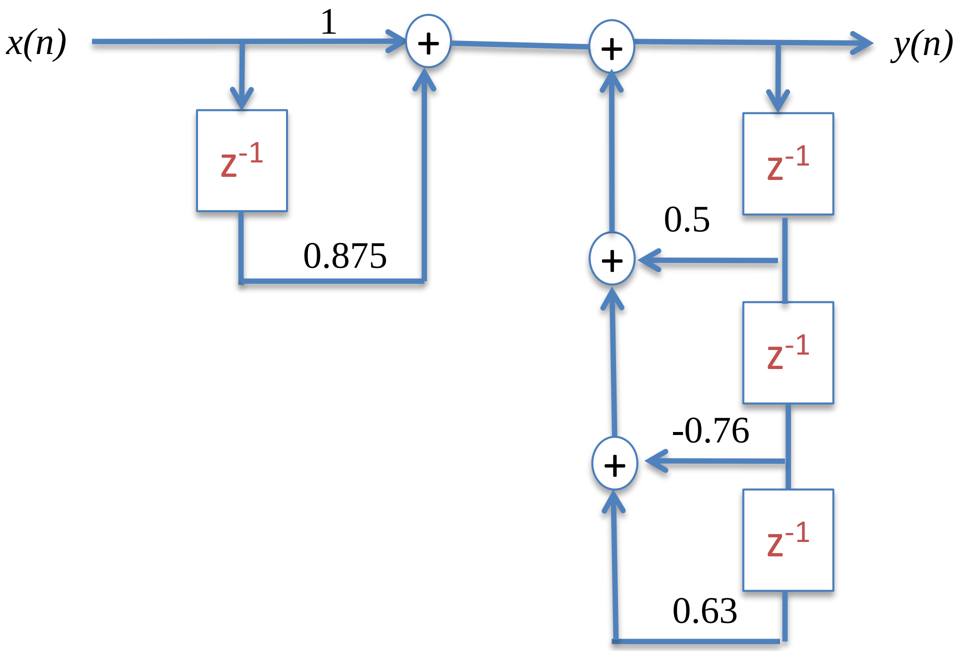
$$H(z) = Y(z) / X(z) = (1 + 0.875z^{-1}) / (1 - 0.5z^{-1} + 0.76z^{-2} - 0.63z^{-3})$$

2. Now, draw Direct Form I structure based on the Transfer Function, H(z).

# IIR Filter Structure : Direct Form I

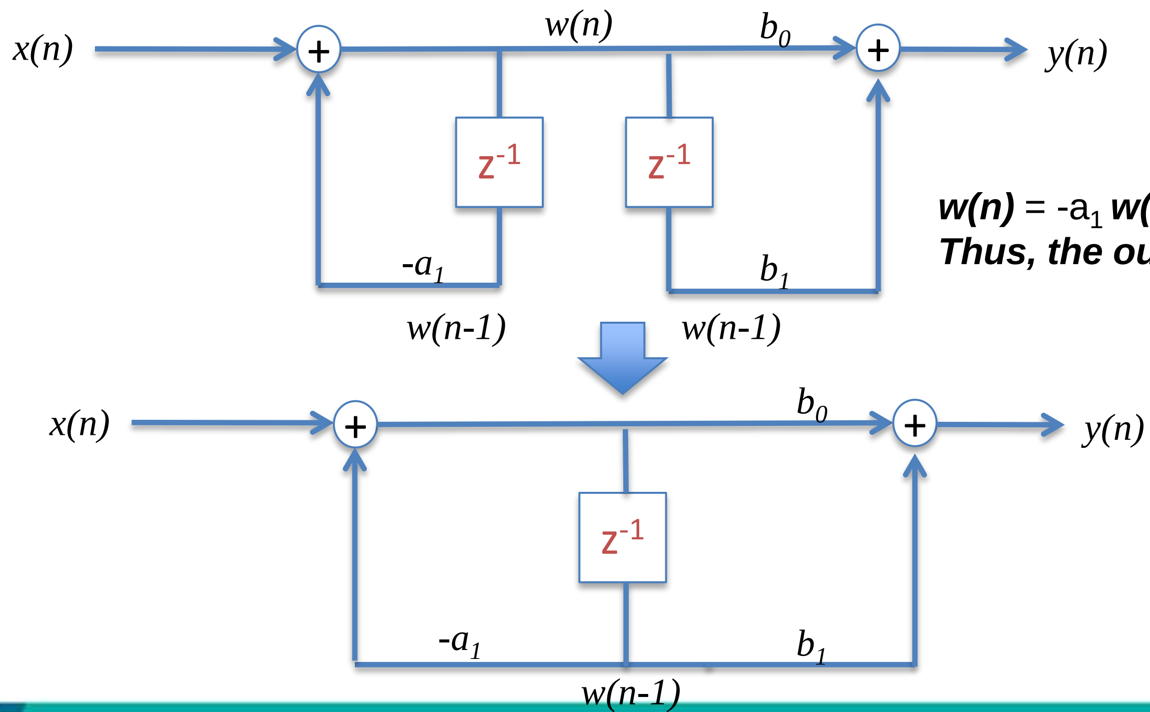
- **Example:**

3. The Direct Form I IIR Filter Structure is shown below:



# IIR Filter Structure : Direct Form II (Canonic)

- In order for the system to have efficiency in the memory requirement, the delays of the Direct Form I structure can be delayed to become Direct Form II or Canonic form as shown below:




$\boldsymbol{w(n)} = -a_1 \boldsymbol{w(n-1)} + \boldsymbol{x(n)}$

***Thus, the output,*** $\boldsymbol{y(n)} = b_0 \boldsymbol{w(n)} + b_1 \boldsymbol{w(n-1)}$

# IIR Filter Structure : Direct Form II (Canonic)

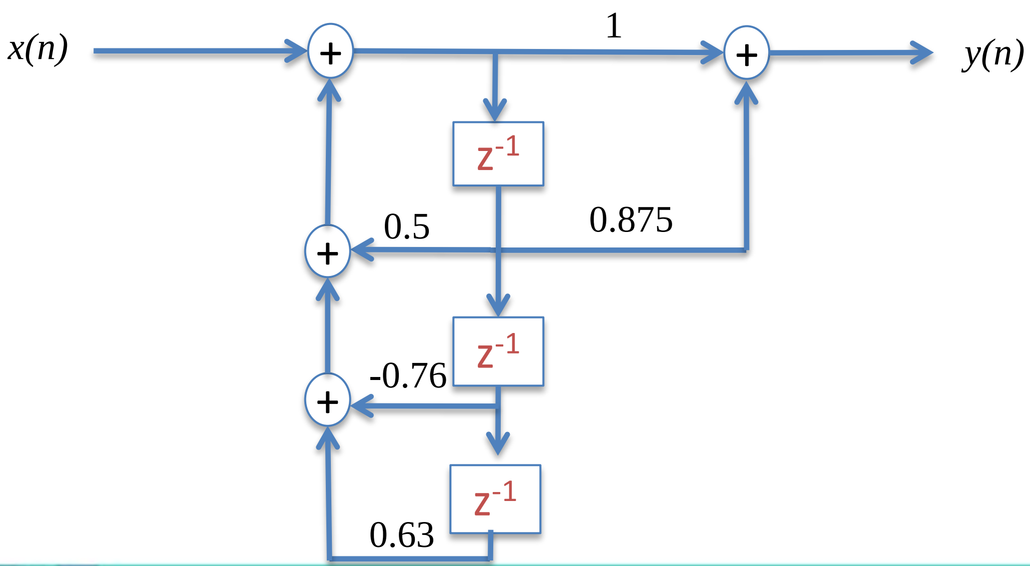
- **Example:**

Determine the ***Direct Form II (Canonic)*** structure of the IIR Filter described by the following difference equation;

$$y(n) = 0.5y(n-1) - 0.76y(n-2) + 0.63y(n-3) + x(n) + 0.875x(n-1)$$

**Solution:**

Draw the structure based on the transfer function obtain from previous example.



# IIR Filter Structure : Cascade Form

- To produce Cascade form of the IIR filter structure, the numerator and denominator of the factorized system will be paired other such as;

  $H(z) = H_1(z).H_2(z)$

- The Cascade form can be described in term of the mathematical formula as stated below;

  $H(z) = p_0 \prod^{k} (1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}) / (1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2})$

# IIR Filter Structure : Cascade Form

- **Example:**

  Determine the ***Cascade Form*** structure of the IIR Filter described by the following difference equation;

  $$y(n) = -\tfrac{1}{4}y(n-1) + 3/8y(n-2) + x(n) + 2x(n-1) + x(n-2)$$

  **Solution:**

  1. Obtain the transfer function as below;

  $$Y(z) = -\tfrac{1}{4}z^{-1}Y(z) + 3/8\,z^{-2}Y(z) + X(z) + 2z^{-1}X(z) + z^{-2}X(z)$$

  $$H(z) = Y(z) / X(z) = (1 + 2z^{-1} + z^{-2}) / (1 + \tfrac{1}{4}z^{-1} - 3/8z^{-2})$$

  2. Factorize the transfer function as below;

  $$H(z) = [(1 + z^{-1})(1 + z^{-1}) / [(1 - \tfrac{3}{4}z^{-1})(1 + \tfrac{1}{2}z^{-1})]$$

  3. Split the transfer function to become;

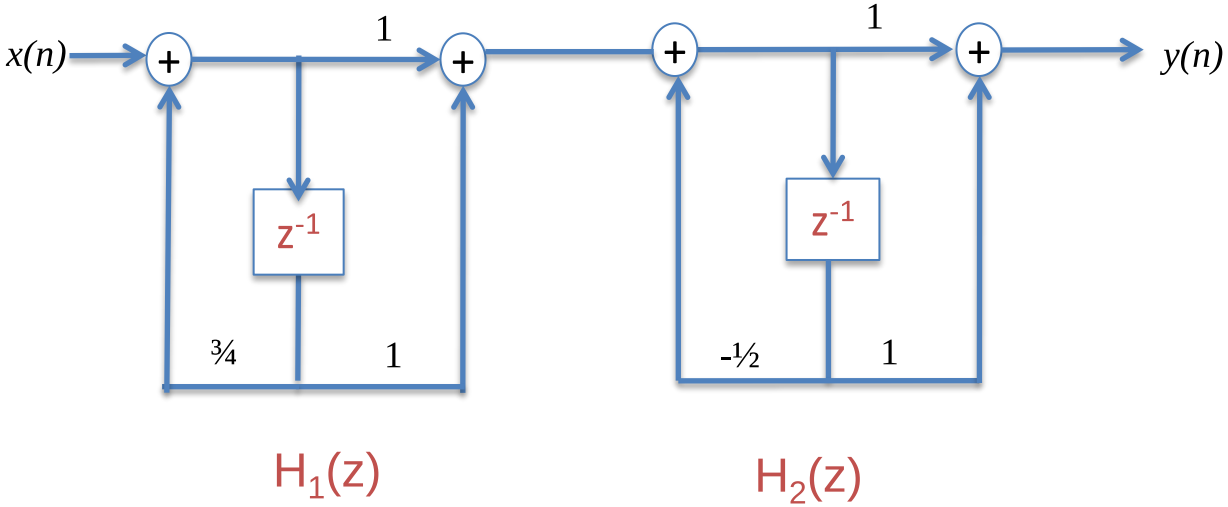
  $$H(z) = H_1(z).H_2(z) = [(1 + z^{-1}) / (1 - \tfrac{3}{4}z^{-1})].[(1 + z^{-1}) / (1 + \tfrac{1}{2}z^{-1})]$$

# IIR Filter Structure : Cascade Form

- **Solution:**

4. Construct the Cascade structure based on the transfer function;

$H(z) = H_1(z).H_2(z) = [(1 + z^{-1}) / (1 - ¾ z^{-1})].[(1 + z^{-1}) / (1 + ½ z^{-1})]$



# IIR Filter Structure : Parallel Form

- To produce Parallel form of the IIR filter structure, the numerator and denominator of the factorized system will be split into summation form such as;

  $H(z) = H_1(z) + H_2(z)$

- Partial Fraction Expansion (PFE) method can be employed to obtain the split of the transfer function.

- Then, the structure will be constructed from each transfer function.

# IIR Filter Structure : Parallel Form

- **Example:**

  Determine the ***Parallel Form*** structure of the IIR Filter described by the following difference equation;

  $$y(n) = -\tfrac{1}{4}\, y(n-1) + 3/8\, y(n-2) + x(n) + 2x(n-1) + x(n-2)$$

  **Solution:**

  1. Obtain the transfer function as below;

     $$Y(z) = -\tfrac{1}{4}\, z^{-1}Y(z) + 3/8\, z^{-2}Y(z) + X(z) + 2z^{-1}X(z) + z^{-2}X(z)$$

     $$H(z) = Y(z) / X(z) = (1 + 2z^{-1} + z^{-2}) / (1 + \tfrac{1}{4}\, z^{-1} - 3/8 z^{-2})$$

  2. Factorize the transfer function as below;

     $$H(z) = [(1 + z^{-1})(1 + z^{-1}) / [(1 - \tfrac{3}{4}\, z^{-1})(1 + \tfrac{1}{2}\, z^{-1})]$$

  3. Split the transfer function using PFE technique to obtain;

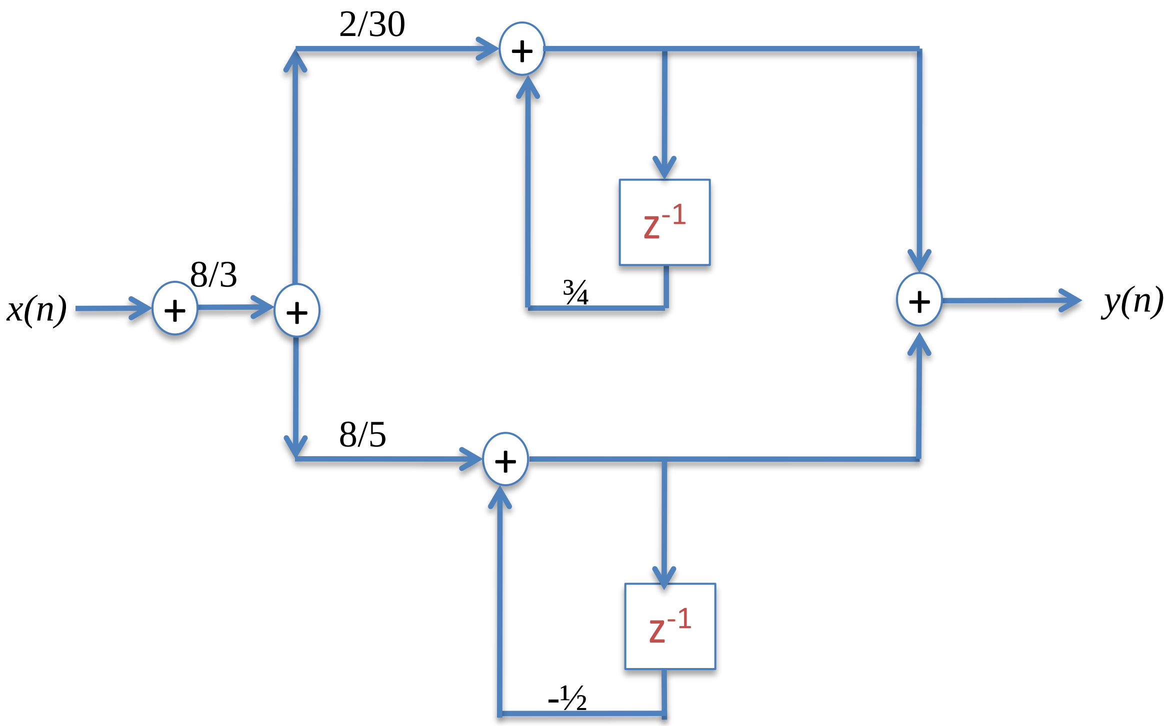
     $$H(z) = H_1(z) + H_2(z) = 8/3 + [(2/30) / (1 - \tfrac{3}{4}\, z^{-1})] + [(18/5) / (1 + \tfrac{1}{2}\, z^{-1})]$$

# IIR Filter Structure : Parallel Form

- **Solution:**

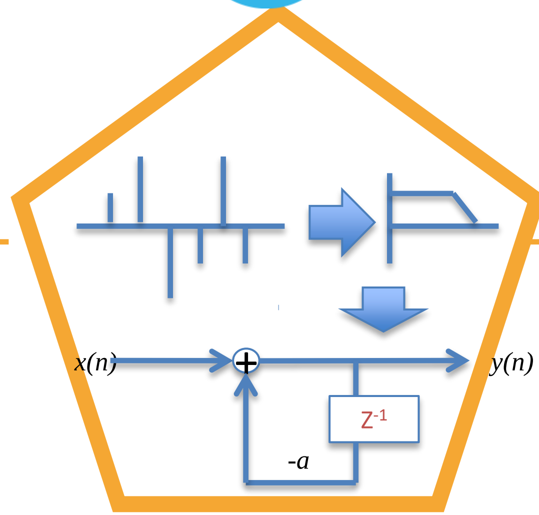
  4. Construct the structure according to the transfer function below;

  $H(z) = H_1(z) + H_2(z) = 8/3 + [(2/30) / (1 - ¾ z^{-1})] + [(8/5) / (1 + ½ z^{-1})]$



# IIR FILTER STRUCTURE

5 — To construct IIR filter structure




4 — To construct the block diagram of the filter.

1 — To identify difference equation of the system, y(n)

3 — To identify the structure components

2 — To convert the difference equation into Transfer Function, H(z)

# Conclusion

- Able to identify the IIR structure components.
- Able to differentiate the type of IIR structure.
- Able to construct the IIR structure from the difference equation and transfer function of the LTI system.

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