DIGITAL SIGNAL PROCESSING

Chapter 7
FIR Filter Structure

FIR Filter Structure

- Aims
  - To explain the type of FIR filter structure and the methods to construct the structure.

- Expected Outcomes

  - By completing the chapter, students should be able to develop the appropriate filter structure based on the type and characteristics of the FIR filter.

FIR Filter Structure

- As described in the previous chapter on the LTI system, those system can be modeled using :
  1. A Difference/Differential equation, y(n) = *x*[n] + *x*[n-1] + …
  2. Impulse Response, h(n)
  3. Transfer Function, H(z)

- Hence, the systems that described by the difference equations can be represented by structures consisting of an interconnection of the basic operations of addition, multiplication by a constant or signal multiplication, delay and advance.

FIR Filter Specifications

- Example of the difference equation that can describe the FIR system;

  ➔ **$y(n) = x(n) + x(n-2) – 2x(n+1)$**

- The system transfer function to describe the FIR system;

  ➔ **$H(z) = \sum h(n)z^{-n}$**

FIR Filter Structure

- The implementation of LTI system can be realized in term of **Block Diagram** and **Signal Flow Graph**.
- The LTI system can be represented in 2 manner :

a. Block Diagram

b. Signal Flow Graph

FIR Filter Structure Components

- The component of structures are;

➢ 1. Adder

➢ 2. Constant Multiplier

➢ 3. Signal Multiplier

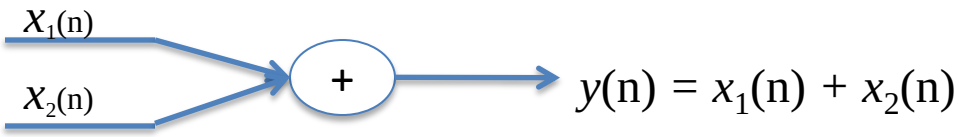
➢ 4. Delay

➢ 5. Advance

FIR Filter Structure Block Diagram

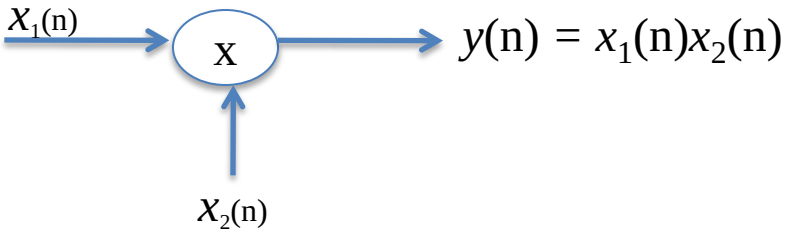
- The component of structures are;

➢ 1. Adder

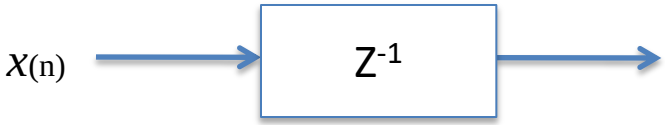


➢ 2. Constant Multiplier

➢ 3. Signal Multiplier



➢ 4. Delay



➢ 5. Advance

Type of FIR Filter Structure

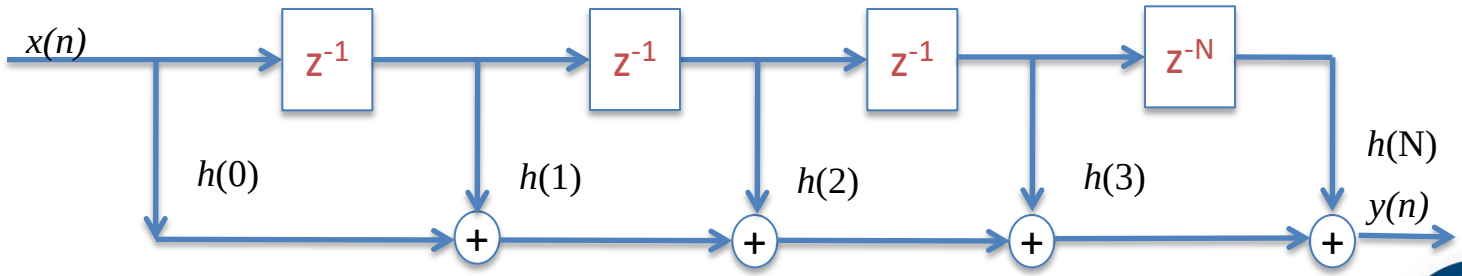
There are several different realization block diagram of FIR system.

1. DIRECT FORM

The difference equation of Direct Form structure can be described as below;

$$y[n] = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(N-1)z^{-(N-1)} + h(N)z^{-N}$$

This structure realization uses separate delays (memory) for both input & output signal samples and the signal flow is from left to right (1 direction) as shown below;



Type of FIR Filter Structure

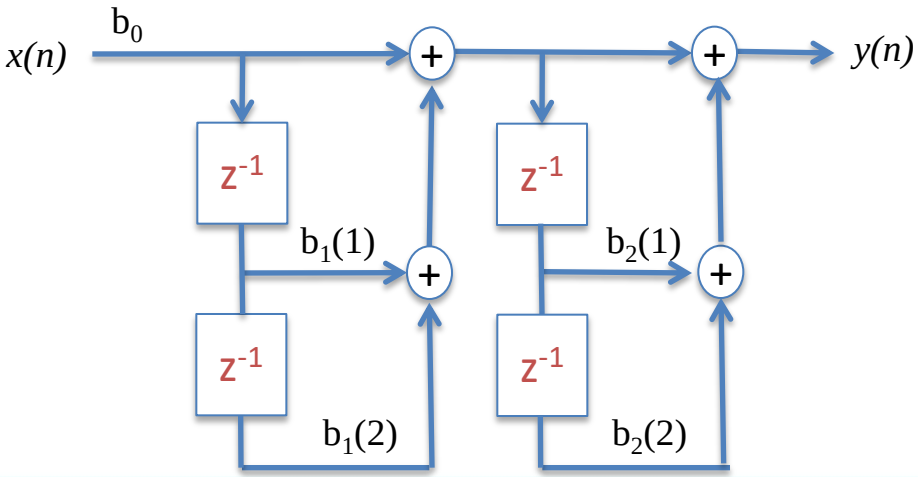
2. CASCADE FORM

For the higher order of FIR filter, the FIR filter structure can be realized by implementing Cascade form for the lower order of the filter as described by the polynomial equation below;

$$H(z) = \Sigma h(n)z^{-n} = \prod(b_0(k) + b_1(k)z^{-1} + b_2(k)z^{-2}$$

The structure realization for 2nd order system using Cascade Form is shown below;

$$H_k[z] = b_k(0) + b_k(1)z^{-1} + b_k(2)z^{-2}$$



FIR Filter Structure : Example

EXAMPLE 1

The system of LTI is described by the following difference equation:

$$y[n] = 0.9x[n] + x[n-1] + 0.5x[n-2] - 2.5x[n-3] - 0.2x[n-4]$$

Draw a **Direct Form structure** realization for the system described by this difference equation.

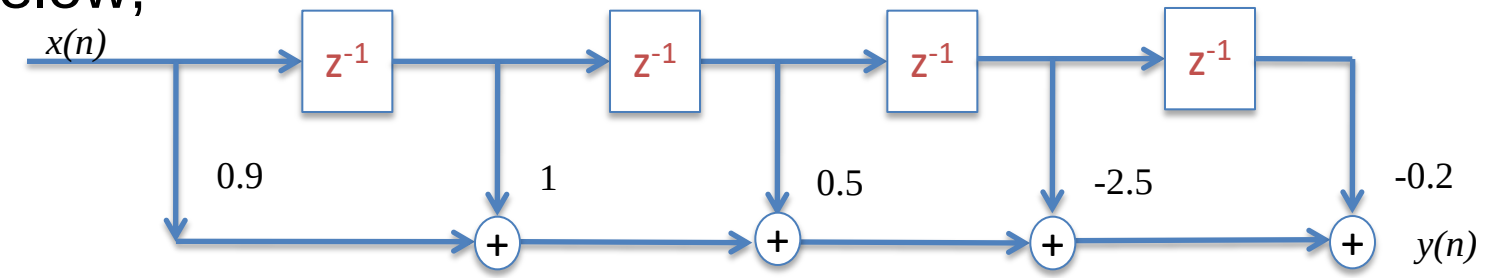
SOLUTION:

1. This is FIR system & thus, convert it to z-transform;
   $Y(z) = 0.9X(z) + z^{-1}X(z) + 0.5z^{-2}X(z) - 2.5z^{-3}X(z) - 0.2z^{-4}X(z)$
2. Obtain the Transfer function, H(z);
   $H(z) = Y(z) / X(z) = 0.9 + z^{-1} + 0.5z^{-2} - 2.5z^{-3} - 0.2z^{-4}$,
   since, $\mathbf{H(z)} = \Sigma h[n]z^{-n}$,
   thus, h(0) = 0.9, h(1) = 1, h(2) = 0.5, h(3) = -2.5, h(4) = -0.2

FIR Filter Structure : Example

SOLUTION:

3. Construct Direct Form structure for the system as shown below;



FIR Filter Structure : Example

EXAMPLE 2

The LTI system is described by the transfer function as shown below;

$$H(z) = (1 + \tfrac{1}{2} z^{-1} - 2z^{-2})(1 + \tfrac{1}{4} z^{-1} + z^{-2})$$

Draw a **Cascade structure** realization for the system described by this difference equation.

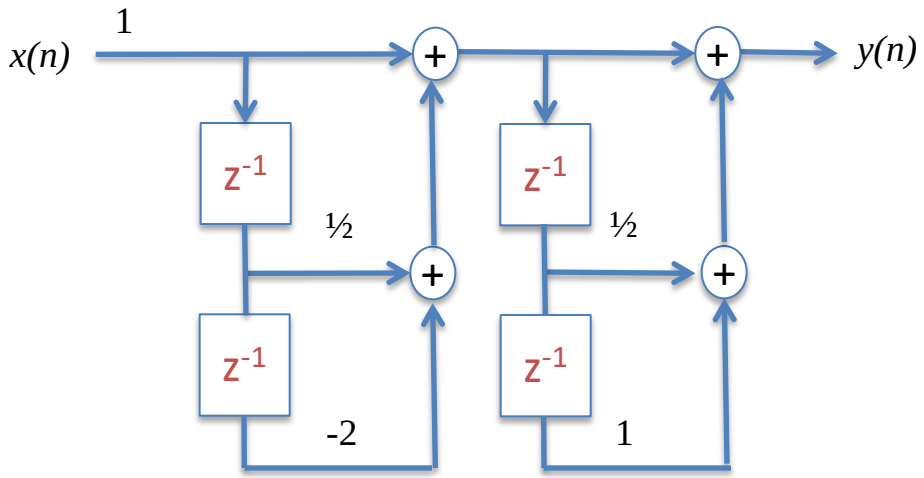
1. The FIR of 2 Transfer function is cascade form;

2. Thus, the transfer functions are;
   $H_1(z) = 1 + \frac{1}{2} z^{-1} - 2z^{-2}$, $H_2(z) = (1 + \frac{1}{4} z^{-1} + z^{-2})$

FIR Filter Structure : Example

EXAMPLE 2

3. Now, draw the structure of the system;



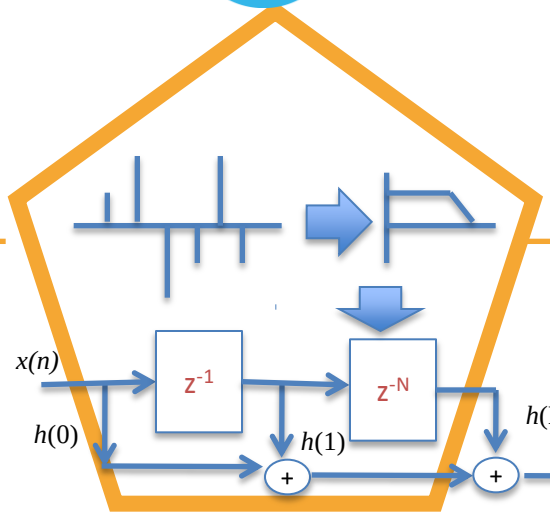
$H_1(z)$ $H_2(z)$

FIR FILTER STRUCTURE

5 To construct FIR filter structure

4 To construct the block diagram of the filter.

1 To identify difference equation of the system, y(n)



3 To identify the structure components

2 To convert the difference equation into Transfer Function, H(z)

Conclusion of The Chapter

- Able to identify the FIR structure components.
- Able to differentiate the type of FIR structure.
- Able to construct the FIR structure from the difference equation and transfer function of the LTI system.

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