DIGITAL SIGNAL PROCESSING

Chapter 5
FIR Filter Design

FIR Filter design

- Aims
  - To explain type of filter, FIR filter specifications and FIR filter design steps.

- Expected Outcomes

  - At the end of this course, students should be able to design digital FIR filter based on the filter specifications and steps.

Definition of Filter

- Filter is required in the digital signal processing to filter the raw input signals to the desired frequency and suppress noise in signal processing.
- Filter consists of Finite Impulse Response (FIR) and Infinite Impulse Response Filter (IIR).
- There are four type filter such as Low -pass, High-pass, Band-pass and Band -stop filter.

FIR Filter

- It is a system where the output of the system only depend on the input signals.
- The system only has zeros and no poles.
- The system has no feedback.
- The system always stable.
- Example of the difference equation that can describe the system;

➔ $y(n) = x(n) + x(n-2) - 2x(n+1)$

FIR Filter Specifications

- The process of filter design begins with filter specifications which include the filter characteristics (Low-pass, high-pass, band-pass, band-stop filter), Filter Type (**FIR** or **IIR**), passband frequency, stopband frequency, transition width frequency, cut-off frequency, sampling frequency, filter order (N), stopband attenuation and passband ripple.

- The second step is to calculate filter frequency Response, **$H(e^{j\omega})|$.**

- The third step is to find the filter coefficient or filter impulse response.

- The last step is to implement filter coefficient and choose appropriate filter structure for implementation.

FIR Filter Specifications

- Before designing the FIR Filter, the set of the specification must be defined.

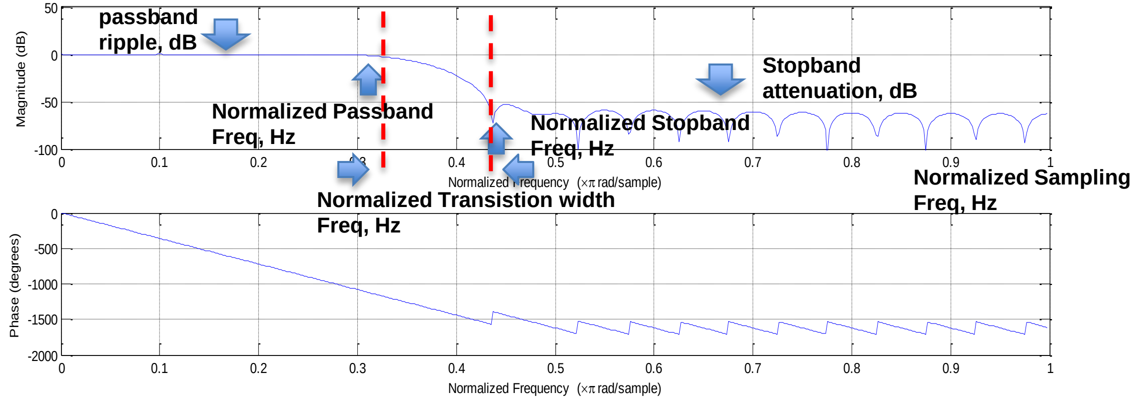
- In order to do that, need to determine the cut-off frequency, ω_c ($2\pi f_c$) for the filter.

- The Frequency Response for the ideal Low-Pass Filter is:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega}, & |\omega| \leq \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases}$$

FIR Filter Specifications

- The FIR Low-Pass Filter is illustrated by diagram below:



Amplitude Parameters;

- δ_p = Passband Ripple, $R_p = -20\log[(1-\delta_p)/(1+\delta_p)]$ in dB
- Peak ripple, $\alpha_p = -20\log[(1-\delta_p)]$ in dB
- δ_s = Stopband Ripple/Attenuation,
- $A_s = -20\log[(\delta_s / 1+\delta_p)]$ in dB
- Minimum stopband attenuation, $\alpha_s = -20\log[(\delta_s)]$ in dB

Frequency Parameters;

- Sampling frequency = F_s
- Passband frequency, $\omega_p = 2\pi f_p$, Normalized Passband frequency = ω_p / F_s
- Stop-band frequency, $\omega_s = 2\pi f_s$, Normalized Passband frequency = ω_s / F_s
- Transition width, $\Delta\omega = \omega_s - \omega_p$
- Cut-off frequency, $\omega_c = (\omega_s + \omega_p) / 2$

FIR Filter Coefficient (Impulse response)

- The Impulse Response of the Filter is defined as:

$$h_d(n) = \frac{\sin(n - \alpha)\omega_c}{\pi(n - \alpha)} \quad \text{where } \alpha = N/2$$

- The FIR Filter Specification is defined as below;

$$1 - \delta_p < |H(e^{j\omega})| \leq 1 + \delta_p \quad , 0 < |\omega| \leq \omega_p$$

$$|H(e^{j\omega})| \leq \delta_s \quad , \quad \omega_s \leq |\omega| < \pi$$

FIR Filter Design by Window Method

- The Filter is designed by ***windowing*** the impulse response:

$$\boldsymbol{h(n) = h_d(n)w(n)}$$

➢ w(n) is a finite-length window that is equal to zero outside the interval of $0 \leq n \leq N$

FIR Filter Design by Window Method

- Basically, there are 4 type of window :

➢ **Rectangular**

$w(n) = 1, \quad 0 \le n \le N$

$\qquad 0, \quad \text{elsewhere}$

➢ **Hanning**

$w(n) = 0.5 - 0.5\cos(2\pi n/N), \quad 0 \le n \le N$

$\qquad 0, \quad \text{elsewhere}$

FIR Filter Design by Window Method

- **Hamming**

$$w(n) = \begin{cases} 0.54 - 0.46\cos(2\pi n/N), & 0 \le n \le N \\ 0, & \text{elsewhere} \end{cases}$$

- **Blackman**

$$w(n) = \begin{cases} 0.42 - 0.5\cos(2\pi n/N) + 0.08\cos(4\pi n/N), & 0 \le n \le N \\ 0, & \text{elsewhere} \end{cases}$$

FIR Filter Design by Window Method

- The relationship between the length of window, N and Filter Transition width is shown below:

$$N\Delta f = c$$

- c is a parameter of the window.

FIR Filter Design by Window Method

- The window parameter, c is shown below:

1. **Rectangular**
   $N\Delta \mathbf{f} = 0.9$, $\alpha_s = -21$ dB

2. **Hanning**
   $N\Delta \mathbf{f} = 3.1$, $\alpha_s = -44$ dB

3. **Hamming**
   $N\Delta \mathbf{f} = 3.3$, $\alpha_s = -53$ dB

4. **Blackman**
   $N\Delta \mathbf{f} = 5.5$, $\alpha_s = -74$ dB

FIR Filter Design by Window Method: Example

Design the FIR Filter to meet the following specification by using window method.

$$0.99 \le |H(e^{j\omega})| \le 1.01, \quad 0 < |\omega| \le 0.19\pi$$
$$|H(e^{j\omega})| \le 0.01, \quad 0.21\pi \le |\omega| \le \pi$$

Solution:

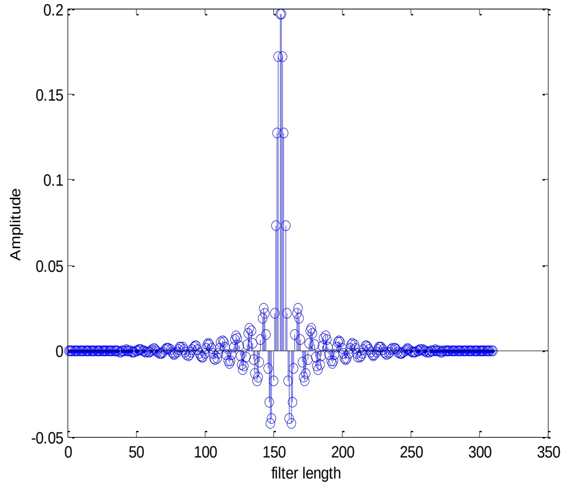
1. From the spec given, $\delta_s = 0.01$, thus α_s = **-40 dB.**
2. From the window parameter, the stopband attenuation is close to **Hanning** Window, thus it is preferable to use **Hanning** Window for this design.

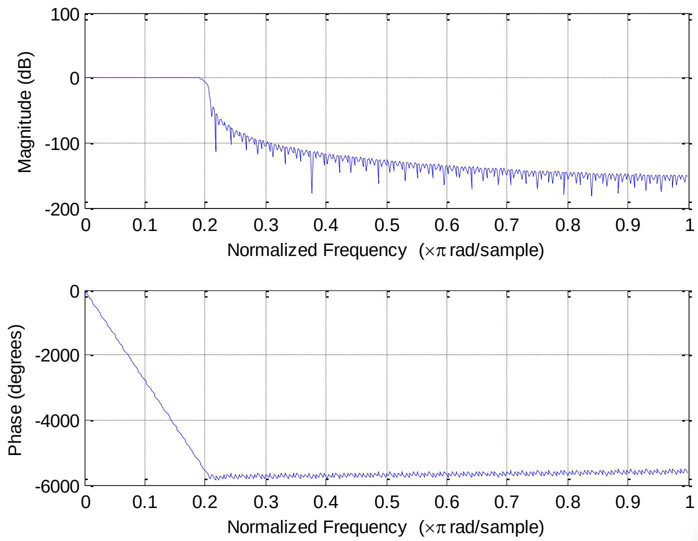
FIR Filter Design by Window Method: Example 1

3. Now, calculate the Transition Band or Width, $\Delta \mathbf{f}$:
   From the given spec, $\Delta\boldsymbol{\omega} = \omega_s - \omega_p = 0.02\pi$,
   Thus, $\Delta \mathbf{f} = \Delta\boldsymbol{\omega} / 2 = \mathbf{0.01}$
4. By using Hanning Window,
   $N\Delta \mathbf{f} = 3.1$
   Thus, $N = 3.1 / \Delta \mathbf{f} = 3.1 / 0.01 = \mathbf{310}$
5. Next, determine the Cut-Off Frequency, ω_c
   $\omega_c = \omega_s + \omega_p / 2 = 0.21\pi + 0.19\pi = 0.40\pi / 2 = \mathbf{0.2\pi}$
6. Calculate the delay, α
   $\alpha = N/2 = 310 / 2 = \mathbf{155}$
7. Finally, the Impulse Response of the FIR Filter that meet the spec is:
   $\mathbf{h_d(n) = \sin [0.2\pi(n-155)]/(n-155)\pi}$

FIR Filter Design by Window Method: Example

- A plot of FIR filter coefficient using Window method based on example 1;
- FIR Filter Coefficients,   FIR Filter response





FIR Filter Design by Optimal Method: Example 2

Design the Bandpass Filter to meet the following specification:

passband frequency = 900-1100 Hz

passband ripple = < 0.87 dB

stopband attenuation > 30 dB

sampling frequency = 15 kHz

Transition frequency = 450 Hz

Use Optimal Method to find suitable Filter Coefficients.

FIR Filter Design by Optimal Method: Example

Solution:

1. Normalized all the frequencies by dividing the passband and stopband frequencies with sampling frequency.

   450 -> 450/15000 = 0.03

   900 -> 900/15000 = 0.06

   1100 -> 1100/15000 = 0.073

   1550 -> 1550/15000 = 0.1033

   7500 -> 7500/15000 = 0.5

2. Obtain the Passband Ripple & Stopband Attenuation,

   $\delta_p = 0.10535$, $\delta_s = 0.031623$

FIR Filter Design by Optimal Method: Example

The value of N is determined by:

$$N = \frac{-20\log_{10}(\sqrt{\delta_p \delta_s}) - 13}{14.6\Delta f/Fs} + 1$$

Thus, $N = \frac{-20\log_{10}(0.057719) - 13}{14.6\,(0.03)} + 1$

- The suitable Filter length,

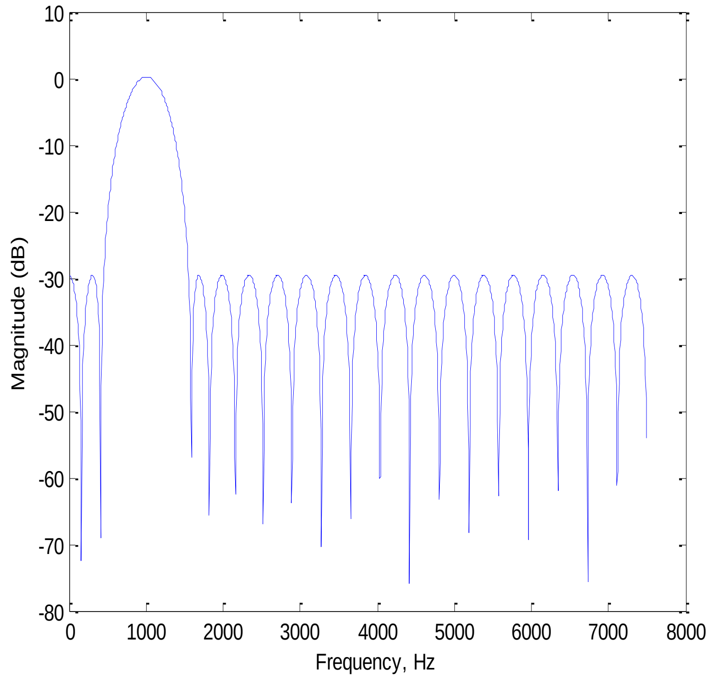
  N = 28

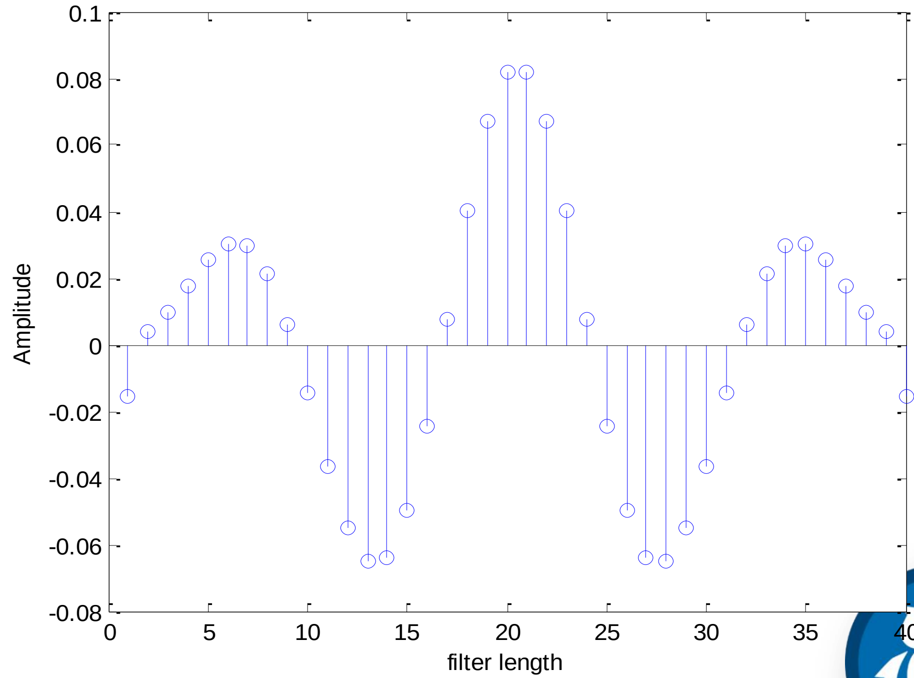
- The Filter Coefficient can be determined by:

  $h_d(n) = \sin[0.015\pi(n-14)]/(n-14)\pi, \quad 0 \le n \le N-1$

FIR Filter Design by Optimal Method: Example

- Now, plot the filter coefficients based using Optimal method based on example 2;
- Filter response; Filter coefficients





FIR Filter Design by Optimal Method

- Optimal Method provides an easy and efficient way of computing FIR Filter Coefficients.
- For most application, optimal method will yield filters with a good amplitude response characteristics for reasonable values of N.

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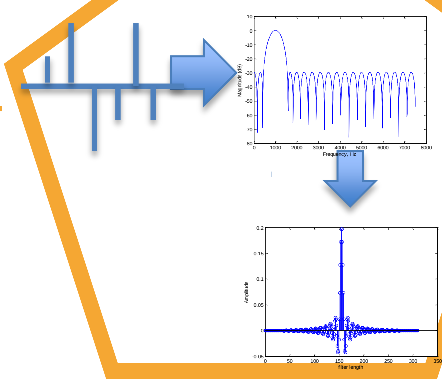
FIR FILTER DESIGN

5 To design FIR filter based on the filter specifications

4 To improve filter design based on filter specifications and filter order or length



1 To identify type of filter and filter specifications

3 To calculate filter coefficients using window or optimal method

2 To convert filter specifications into the desired filter

Conclusion

- Able to understand the type and characteristics of FIR filter design.

- Able to design the FIR filter using window and optimal method.

- Able to calculate the filter coefficients or filter impulse response from the filter specifications.

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