Universiti Malaysia PAHANG

# DIGITAL SIGNAL PROCESSING

# Chapter 2
# Discrete-Time Signals & Systems

by
Dr. Norizam Sulaiman
Faculty of Electrical & Electronics Engineering
norizam@ump.edu.my

# Introduction to Discrete-time Signal

- Aims
  - To explain the characteristics of discrete-time signals (DTS), classification of DTS, Linear-Time Invariant System (LTI) system and convolution of the discrete-time signals in time-domain.

- Expected Outcomes
  - At the end of this course, students should be able to understand DTS & LTI characteristics, and classify the system and perform convolution of discrete-ti

# Discrete-time Signal and System

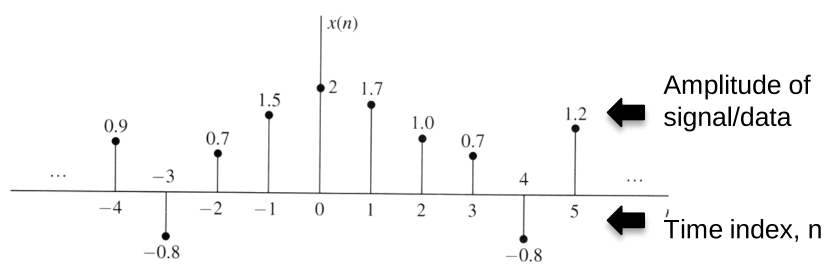
- Discrete-Time Signals are represented mathematically as sequences of numbers.
- The sequences of numbers **x** with nth number in the sequence is denoted as ***x***[n] where n being integer in the range of  such as n={…,-3,-2,-1,0,1,2,3,…}.

  It is also called time index.
- The components of DTS are listed below;
  - A delay  y[n] = *x*[n-1]
  - An advance y[n] = *x*[n+1]
  - Scaling y[n] = a*x*[n]
  - Downsampling (Decimation) y[n] = *x*[nM]
  - Upsampling (Interpolation) y[n] = *x*[n/L]
  - Nonlinear y[n] = tanh[a*x*(n)]
- Graphical representation of the DTS system is shown below.

$$x(n) = \{\ldots,0,\underset{\uparrow}{0},1,4,1,0,0,\ldots\}$$

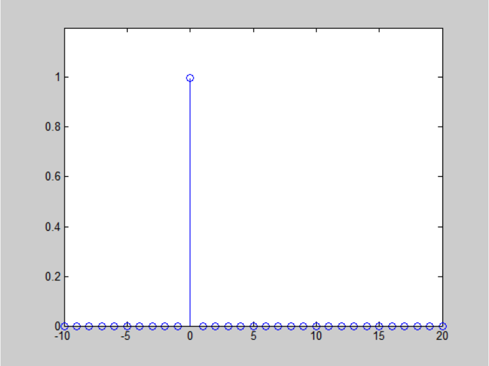
indicate the time origin
($n$ = 0)

indicate infinite sequence

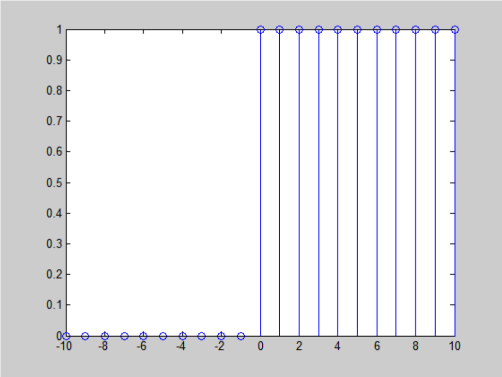


# Sequence Representation

- The discrete-time signal sequence representation consists of unit impulse, unit step, exponential and sinusoidal is shown in diagrams below;



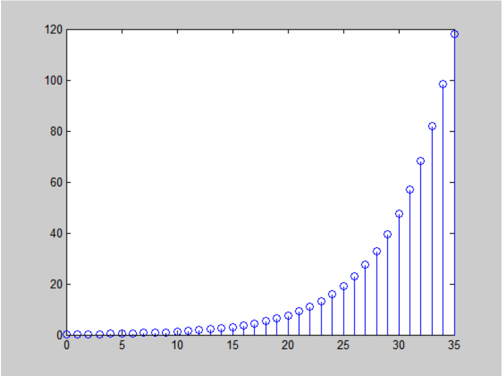

Unit Impulse => $\delta[n] = \{1, n = 0; \quad 0, n \neq 0\}$



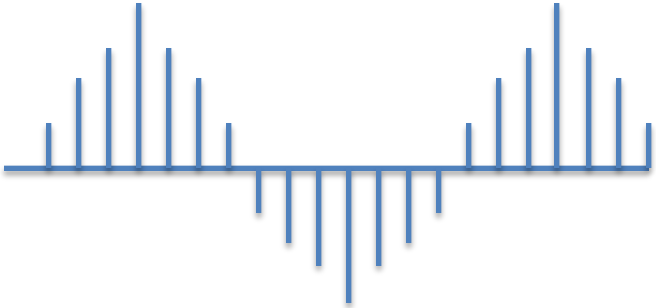

Unit step => $\delta[n] = \{1, n \geq 0; \quad 0, n < 0\}$

# Sequence Representation

- The discrete-time signal sequence representation consists of unit impulse, unit step, exponential and sinusoidal is shown in diagrams below;




Exponential => $x[n] = Aa^n$



Sinusoid => $x[n] = A\cos(\omega t + \Phi)$

# Sequence Representation

- A signal can be shifted in time by replacing the variable *n* by *n-k* including the unit impulse and unit step.

- When *k* is positive, the sample will be shifted to the right of its original location => $x(n-k)$

- When *k* is negative, the sample will be shifted to the left of its original location => $x(n+k)$

# Linear Time-Invariant (LTI) System

- The LTI system characteristics are shown below;

**1. Dynamic Systems**

=>Systems use ***Delay*** called Dynamic.

Example: y[n] = *x*[n-1]

2. **Static (Memory less) Systems**

=> Systems that ***do not use Delay*** called Static such as

Example: y[n] = *x*[n]

The Order of the systems is defined as the number of the delay elements in the systems.

# Linear Time-Invariant (LTI) System

3. **Causality**

=> The systems that **do not use** advance element.

Example: $y[n] = x[n]-x[n-1]$,

$y[n] = ax[n]$

4. **Non-causality Systems**

=> The systems that **use** advance element such as

$y[n] = x[n+1]-x[n]$

Example: $y[n] = x[n+1]-x[n]$

$y[n] = x[n^2]$

# Linear Time-Invariant (LTI) System

5. **Time-Invariant Systems**

=> A systems are called ***Time-Invariant Systems*** if the delay of the input sequence cause a shift to the output sequence. If the input sequence is $x[n]=x[n-n_o]$, then the output sequence is $y[n] = y[n-n_o]$

6. **Time Varying Systems**

=> The systems that will be affected by varying of time.

If $y[n] = x[An]= x[An-n_o]$, where A is an integer, then

$y[n-n_o] = x[A(n-n_o)] \neq y[n]$

# Linear Time-Invariant (LTI) System

- To test the system is a time-invariant or time-variant based on:

$$y(n,k) = \tau[x(n-k)]$$

- Test will be perform separately for the left-hand side (LHS) and right-hand side (RHS)
- If y(n,k)=y(n-k) → Time-invariant

# Linear Time-Invariant (LTI) System : Example

Determine the characteristic of the LTI system below;

a) $y(n) = x(n) - x(n-1)$

b) $y(n) = nx(n)$

c) $y(n) = x(-n)$

d) $y(n) = x(n)\cos \omega_0 n$

# Linear Time-Invariant (LTI) System : Example

Testing the system with:

$y(n,k) = \tau[x(n-k)]$ where $y(n,k) = y(n-k)$

or it can be written as $y(n-k) = \tau[x(n-k)]$

a) Examining the right hand side of the system:

$$\tau[x(n-k)] = x((n)-k) - x((n-1)-k)$$
$$= x(n-k) - x(n-k-1)$$

put $-k$ at every input stated in the system

# Linear Time-Invariant (LTI) System : Example

For the left hand side:

$$y(n-k) = x((n-k)) - x((n-k)-1)$$
$$= x(n-k) - x(n-k-1)$$

put $-k$ at every n of the input

From the observation, it is clear that the result in the left hand side is the same as in the right hand side. It is concluded that the system is time-invariant

# Linear Time-Invariant (LTI) System : Example

b) Examining the right hand side of the system:

$$\tau[x(n-k)] = nx((n)-k)$$
$$= nx(n-k)$$

For the left hand side:

$$y(n-k) = ((n)-k)x((n)-k)$$
$$= (n-k)x(n-k)$$

It is shown that the left hand side ≠ the right hand side. Hence, the system is time-variant

# Linear Time-Invariant (LTI) System : Example

c) The system is: $y(n) = x(-n)$

RHS: $\tau[x(n-k)] = x((-n)-k) = x(-n-k)$

LHS: $y(n-k) = x(-(n-k)) = x(-n+k)$

$$y(n-k) \neq \tau[x(n-k)]$$

$$\therefore \text{The system is time - variant}$$

# Linear Time-Invariant (LTI) System : Example

d) The system is: $y(n) = x(n)\cos\omega_0 n$

RHS: $\tau[x(n-k)] = x((n)-k)\cos\omega_0 n$

LHS: $y(n-k) = x((n)-k)\cos\omega_0((n)-k)$

$$= x(n-k)\cos\omega_0(n-k)$$

$$y(n-k) \neq \tau[x(n-k)]$$

$$\therefore \text{The system is time-variant}$$

# Linear Time-Invariant (LTI) System

- **Linear Systems**

  The system is said to be linear if the **superposition** holds either in delay, advance, scaling and adder operation.

  For example, If input signal to the system is a$x$[n] + b$x$[n], then Output signal is ay[n] + by[n].

  For example, If input signal to the system is $x$[2n], then Output signal is y[2n].

# Linear Time-Invariant (LTI) System

- **Testing Linearity of the Systems**

  Is the system described below linear or not ?

$$y[n] = x[n] + x[n-1]$$

**Steps** :

a. Now, applying superposition by considering input as :
   $x[n] = ax[n] + bx[n]$
b. Substitute the equation above with equation in (a), become
   $y[n] = (ax[n] + bx[n]) + (ax[n-1] + bx[n-1])$
c. Rearrange the equation above become :-
   $y[n] = a(x[n] + x[n-1]) + b(x[n] + x[n-1]) \Rightarrow ay[n] + by[n]$
d. Since the output equation follows an input signal, the system is **Linear** since superposition is hold.

# Linear Time-Invariant (LTI) System

- **Testing Linearity of the Systems**

$$y[n] = 2(x[n])^2$$

**Step** :

a. Now, applying superposition by considering input as :
$x[n] = ax[n] + bx[n]$

b. Substitute the equation above with equation in (a), become
$y[n] = 2(ax[n] + bx[n])^2 = 2(ax[n])^2 + 2(bx[n])^2 + 2\, ax[n][bx[n]$

c. Since the output equation is not same as original equation or did not follow the input equation, the system is **Non-linear** since superposition is not hold.

# Convolution

- The convolution of signals can be implemented in discrete-time domain to convolve 2 signals using 2 techniques;

  1. Graphical method

     (a) Folding

     Perform time-reversal at y-axis such $x(k) = x(-k)$

     (b) Shifting

     Perform time-shifting such as $x(n-k)$

     (c) Multiplication

     Multiply the shifting signal with the other signals such as $v(k) = h(k)x(n-k)$

     (d) Summation

  2. Mathematical method (Sliding rule)

# Convolution : Graphical Method

- Convolve the following signals;

$x(n) = \{1, 2, 3, 1\}$, $h(n) = \{1, 2, 1, -1\}$

$h(-k) = \{-1, 1, 2, 1\}$

**$Y(0) = \Sigma h(-k)x(k) = 2(1)+1(2) = 4$**

$V_1(k) = \{1, 4, 3\}$

**$Y(1) = \Sigma h(1-k)x(k) = 1(1)+2(2)+1(3)= 8$**

$V_2(k) = \{-1, 2, 6, 1\}$

**$Y(2) = \Sigma h(2-k)x(k) = -1(1)+1(2)+2(3)+1(1)=8$**

$V_3(k) = \{0, -2, 3, 2\}$

**$Y(3) = \Sigma h(3-k)x(k) = 0(1)-1(2)+1(3)+2(1)=3$**

$V_4(k) = \{0, 0, -3, 1\}$

**$Y(4) = \Sigma h(4-k)x(k) = 0(0)+0(2)-1(3)+1(1)=-2$**

$V_5(k) = \{0, 0, -1, 0\}$

**$Y(5) = \Sigma h(5-k)x(k) = 0(0)+0(2)-1(1)+1(0)=-1$**

$V_{-1}(k) = \{0, 0, 0, 1\}$

**$Y(5) = \Sigma h(5-k)x(k) = 0(0)+0(0)-0(0)+1(1)= 1$**

**$Y(n) = \{ 1, 4, 8, 8, 3, -2, -1\}$**

# Convolution: Mathematical /Sliding rule

- Fold h(k);

x(k) : 1 2 3 1

h(k) : 1 2 1 -1 ➔ h(-k): -1 1 2 1

- Slide h(k) and through x(k), at k=0;

1 2 3 1

-1 1 2 1

**Y(0)= -1(0) + 1(0) + 2(1) + 1(2) = 4**

Slide h(k) and through x(k), at k=1

1 2 3 1

-1 1 2 1

- **Y(1)= -1(0) + 1(1) + 2(2) + 1(3) + 0(1) = 8**
- Slide h(k) and through x(k), at k=2

1 2 3 1

-1 1 2 1

**Y(2)= -1(1) + 1(2) + 2(3) + 1(1) = 8**

# Convolution: Mathematical /Sliding rule

- Slide h(k) and through x(k), at k=3

  1 2 3 1

  -1 1 2 1

**Y(3)= 0(1) - 1(2) + 1(3) + 2(1) + 1(0) = 3**

- Slide h(k) and through x(k), at k=4

  1 2 3 1

  -1 1 2 1

**Y(4)= 0(1) + 0(2) -1 (3) + 1(1) + 2(0) + 1(0) = 2**

- Slide h(k) and through x(k), at k=5

  1 2 3 1

  - 1 1 2 1

**Y(5)= 0(1) + 0(2) +0 (3) - 1(1) + 1(0) + 2(0) + 1(0) = -1**

- Slide h(k) and through x(k), at k=-1

  1 2 3 1

  -1 1 2 1

**Y(-1)= -1(0) + 1(0) +2(0) +1(1) + 0(2) + 0(3) + 0(1) = 1**

**Y(n) = { 1, 4, 8, 8, 3, -2, -1}**

# INTRODUCTION TO DISCRETE-TIME SIGNAL

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To obtain discrete-time signals and system by inspecting their DTS & LTI characteristics and to obtain system output in time-domain using convolution process

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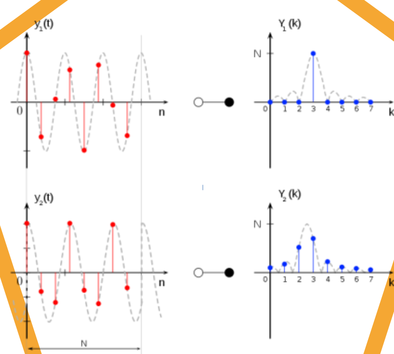

1

To identify discrete-time system (DTS), DTS classification, LTI system

2

To test DTS and LTI systems

3

To inspect DTS and LTI characteristic and to perform convolution

4

To improve signal characteristics, systems and convolution process

# Conclusion of The Chapter

- Able to understand the type of signals, systems and signal characteristics.
- Able to understand the different between continuous signal with discrete-time signal.
- Able to understand the process to convert the continuous signal to discrete-time signal using sampling & re-sampling technique.
- Able to perform signal convolution in time domain.

Teaching slides prepared by
***Dr. Norizam Sulaiman***,
Senior Lecturer,
Applied Electronics and Computer Engineering,
Faculty of Electrical & Electronics Engineering, Universiti Malaysia Pahang,
Pekan Campus, Pekan, Pahang, Malaysia