DIGITAL SIGNAL PROCESSING

Chapter 1
Introduction to Discrete-Time Signals & Sampling

by
Dr. Norizam Sulaiman
Faculty of Electrical & Electronics Engineering
norizam@ump.edu.my

Introduction to Discrete-time Signal

- Aims
  - To explain background of discrete-time signals, sampling and re-sampling process.

- Expected Outcomes
  - At the end of this chapter, students should be able to differentiate the different between continuous and discrete-time signal, obtain good discrete-time signal by avoiding aliasing and finally, able to understand sampling parameters and able to perform sampling and re-sampling process of the discrete-time signals.

Definition of Digital Signal Processing

- Digital Signal Processing is process to convert continuous signal into discrete form signal.

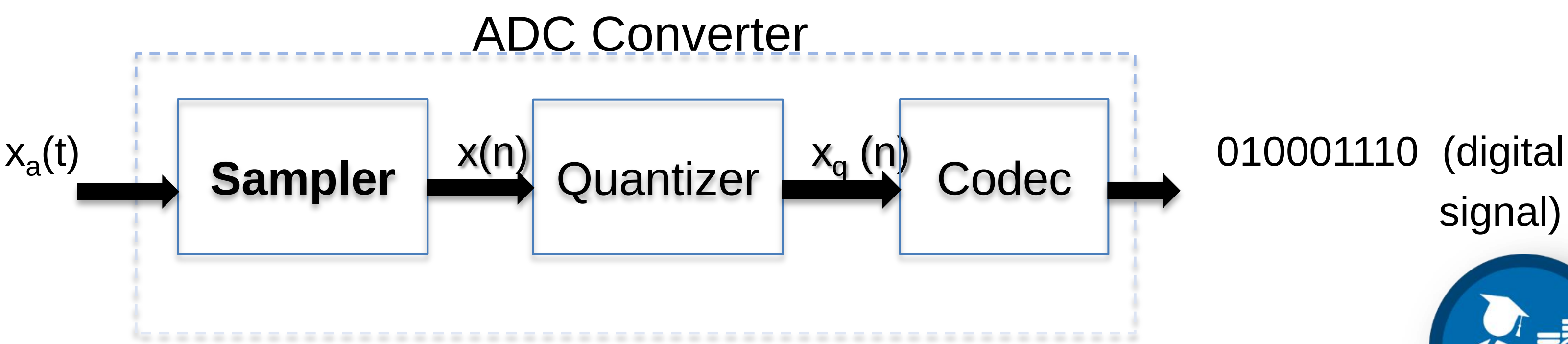
- Among the continuous signals are human voice, electrical signals, radio wave, optical signals, and audio signals.

- The signals and systems must come together. The systems are required to operate the signals. For example, Thermometer is used to measure Temperature, Microphone to carry out analog signal (human voice) and convert it to electrical signal, Charge-Couple Device (CCD) used in in Camera or Digital Camera to convert image to picture and so on. In general, the system is characterized by the type of operation that it performs on the signal.

- Analog signal must be converted into Discrete before DSP techniques can be applied. The analog signal is basically denoted as $\boldsymbol{x}$[t] or $\boldsymbol{x_a}$[t] because it varied by time. The analog signal comes in form of sinusoid (sine or cosine wave).

$$x_a[t] = A\cos(\Omega t + \Phi)$$

Sampling Process

- The Analog or continous signal is digitized by using Integrated Electronic Circuit device called an Analog-to-Digital Converter (ADC). The output of ADC will be in the form of binary number that represents the analog signal such as electrical voltage.
- ADC is basically consists of Sampler, Quantizer and Coder. All this elements are built up by CMOS Switched-Capacitor (for Sampling), Op-Amp (Signal Amplification) & Comparator (Quantizer).
- The coder in ADC will convert the output of the Quantizer to b-bit binary sequence that can be read by DSPs (Digital Signal Processors).
- 

Sampling Parameters

- The sampling process involved several notations as shown below;
- Sampling Period : T_S
- Sampling Frequency : $F_s = 1 / T_S$
- Digital (Discrete) Frequency : $f = F / F_S$
- Normalized Digital Frequency : $\omega = \Omega T_S$, $\omega = 2\pi f$
- Continuous signal frequency : $\Omega = 2\pi F$
- The Maximum Input Frequency : $F_B = F_s / 2$
- Cut-off Frequency : F_C

Aliasing in Sampling

- ***Aliasing*** is an error in signal when the sampling frequency less than twice the maximum input signal bandwidth as defined below :

$$F_s < 2F_B$$

- It happens due to the overlap of the input signal with its sampled signal. When this occurred, the original shape of input signal is lost and cannot be reconstructed.

- In order to avoid aliasing and be able to reconstruct the input continuous signal from its sampled signal, the sampling frequency must be greater than or equal to twice the highest frequency in the continuous signal. Thus, the filter cut-off frequency be must selected as in the range below;

$$F_B < F_c < F_s - F_B$$

Sampling : Example

The input continuous signal which have frequency of 2kHz enter the DTS System and being sampled at every 0.1ms. Calculate the digital and normalized frequency of the signal in Hz and rad.

Solution :

1. Calculate the Sampling Rate :
   $F_s = 1 / T_s = 1 / (0.1ms) = 10$ kHz.

2. Now, calculate the normalized digital frequency.
   $f = F / F_s = 2$ kHz / 10 kHz $= 0.2$ Hz.

3. The digital frequency in radian,
   $\omega = 2\pi f = 2\pi\ (0.2) = 0.4\pi$ rad.

4. The normalized digital frequency (angular) in radian,
   $\omega = \ = \Omega T_s = 2\pi F T_s = 2\pi(2kHz)(0.1ms) = 0.4$.

Sampling : Example

The analog signal that enters the DTS is in the form of
$x_a[t] = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$

a. Determine the input signal bandwidth.
b. Determine the ***Nyquist*** rate for the signal.
c. Determine the minimum sampling rate required to avoid aliasing.
d. Determine the digital (discrete) frequency after being sampled at sampling rate determined from c.
e. Determine the discrete signal obtained after DTS.

Solutions :

a. The frequencies existing in the signals are :

$F_1 = \Omega_1 / 2\pi = 50\pi / 2\pi = 25$ Hz.
$F_2 = \Omega_2 / 2\pi = 300\pi / 2\pi = 150$ Hz.
$F_3 = \Omega_3 / 2\pi = 100\pi / 2\pi = 50$ Hz.
F_B = Maximum input frequency = **150 Hz**.

Sampling : Example

Solutions :

b. The Nyquist rate is defined as ;

$F_N = F_s = 2F_B = 2(150\ \text{Hz}) = \mathbf{300\ Hz}$.

c. The minimum sampling rate required to avoid aliasing ;

is $F_s \geq 2F_B = \mathbf{300\ Hz}$.

d. The discrete-signals frequencies are described below;

$f_1 = F_1 / F_s = 25\ \text{Hz} / 300\ \text{Hz} = \mathbf{1/12}$

$f_2 = F_2 / F_s = 150\ \text{Hz} / 300\ \text{Hz} = \mathbf{1/2}$

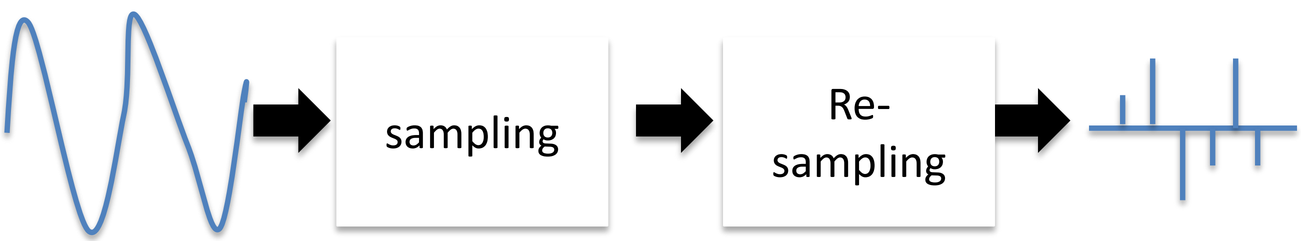
$f_3 = F_3 / F_s = 50\ \text{Hz} / 300\ \text{Hz} = \mathbf{1/6}$

e. The equation of discrete-time signal after sampling process is described by the equation below;

$$x[n] = x_a[nT_s] = 3\cos[2\pi n(1/12)] + 10\sin[2\pi n(1/2)] - \cos[2\pi n(1/6)]$$

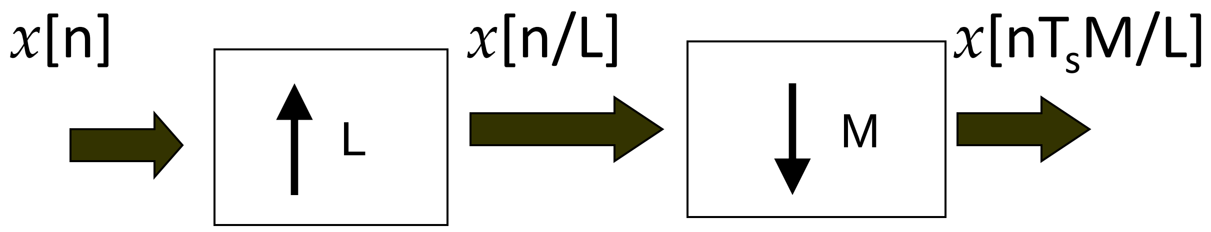
Re-Sampling Process

- Re-sampling or multi-rate required if the same signals are used at different technologies of signal application.
- Re-sampling involved the down-sampling (decimation) process and up-sampling (interpolation) process. The decimation factor denoted by “D” or “M”. Meanwhile, the up-sampling factor is denoted by “I” or “L”. The factor must be greater than 1.



Re-Sampling Process

- The Downsampling and Upsampling process can be combined as shown in diagram below. Here, the new sampling factor will be determined by the ratio of sampling factor.



$$T_s' = T_s M/L$$

$$F_s' = F_s L/M$$

Downsampling or Decimation
Example

The discrete-time signal with a sequence of x[n] of {0, 1, 2, 3, 2, 1, 0} is downsampled by a factor 2 with sampling frequency of 3 kHz.

$$x[nT_sM]$$

$$x[n]=\{0,1,2,3,2,1,0\} \Rightarrow \boxed{\downarrow 2} \Rightarrow x'[n]=\{0,2,2,0\}$$

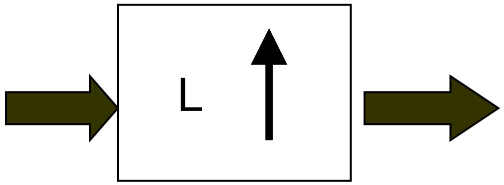
$$F_s' = F_s/M = 3000/2 = 1.5 \text{ kHz.}$$

Upsampling or Interpolation
Example

The discrete-time signal with a sequence of x[n] of {0, 1, 2, 3, 2, 1, 0} is upsampled by a factor 2 with sampling frequency of 3 kHz.

$x[n/L]$]={0, 0, 1, 0, 2,0, 3, 0, 2, 0, 1, 0, 0}

$x[n]$={0,1,2,3,2,1, 0}



$F_s' = F_s L = (3000)(2) = 6$ kHz.

Combination of Downsampling & Upsampling

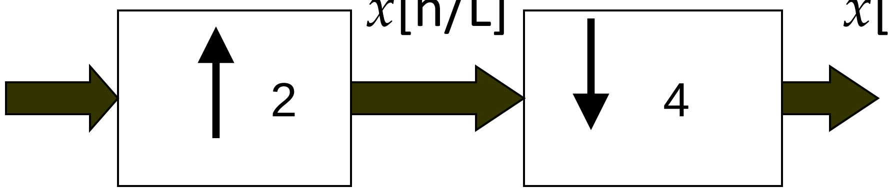
Example

- The example of implementing the combination of Downsampling with factor of 4 and Upsampling with factor of 2 and sampling frequency of 3 kHz are shown below.,

$x[n] = \{\ldots,-2, -1, 3, 5, 4, 9, 7, 11, 13\ldots\}$

$x[n/L]$ $x[nT_sM/L]$

$x(n)$



$x(n) = \{\ldots, 0, -2, 0, 0, 3, 0, 0, 4, 0, 0, 7, 0, 0, 13, \ldots\}$

$T_s' = T_sM/L$

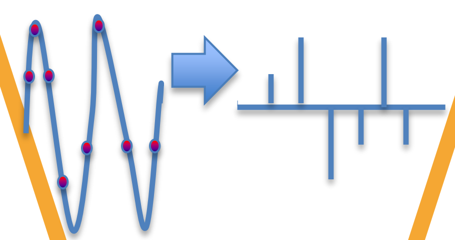
$F_s' = F_sL/M = (3000)(2/4) = 1.5$ kHz.

INTRODUCTION TO DISCRETE-TIME SIGNALS & SAMPLING

5 To understand characteristics of continuous signals and discrete-time signals, conversion process and sampling & re-sampling process

4 To improve signals sampling technique to avoid aliasing

1 To identify and differentiate continuous & discrete-time signals



3 To perform sampling & re-sampling process of discrete-time signals

2 To convert continuous signals to discrete-time signals

Conclusion

- Able to understand the type of signals, systems and signal characteristics.
- Able to understand the different between continuous signal with discrete-time signal.
- Able to understand the process to convert the continuous signal to discrete-time signal using sampling & re-sampling technique.

Teaching slides prepared by
Dr. Norizam Sulaiman,
Senior Lecturer,
Applied Electronics and Computer Engineering,
Faculty of Electrical & Electronics Engineering, Universiti Malaysia Pahang,
Pekan Campus, Pekan, Pahang, Malaysia