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BEE1133 Circuit Analysis

Chapter 5B
First Order Circuit

by

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Chapter Description

Aims

This chapter is aimed to:

1. Introduce the step response RC and RL circuit
2. Explain the equation related for both circuit

Expected Outcomes

Student should be able to

1. Determine the step response of both RC and RL circuit

References

1. C. Alexander and M. Sadiku, “Fundamentals of Electric Circuits”, 4th ed., McGraw-Hill, 2008.
2. J. Nilsson and S. Riedel, “Electric Circuits”, 8th ed., Prentice Hall, 2008.

BASIC CONCEPT

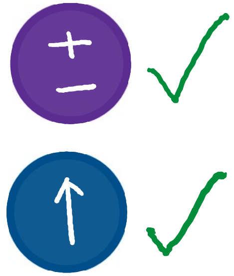
10.1 Step response of an RC circuit

10.2 Step response of an RL circuit

Step Response?

Step Response

- DC source EXIST in the circuit (t>0)
- The energy stored in the capacitor or inductor RELEASED to R or CHARGED



Initial Value

- This is the **ONLY** value that is guaranteed to remain constant before and after the switch changes.
- Assume circuit has remained in same state for a long time leading up to time of switch change.

(Capacitor -> open, Inductor->short)

- Compute V_C or I_L using simplified circuit at t=0

Final Value

- Assume circuit has remained in same state for a long time after switch change.

(Capacitor -> open, Inductor->short)

- Compute $V_C(\infty)$ or I_L (∞)using simplified circuit at $t=\infty$

RC CIRCUIT

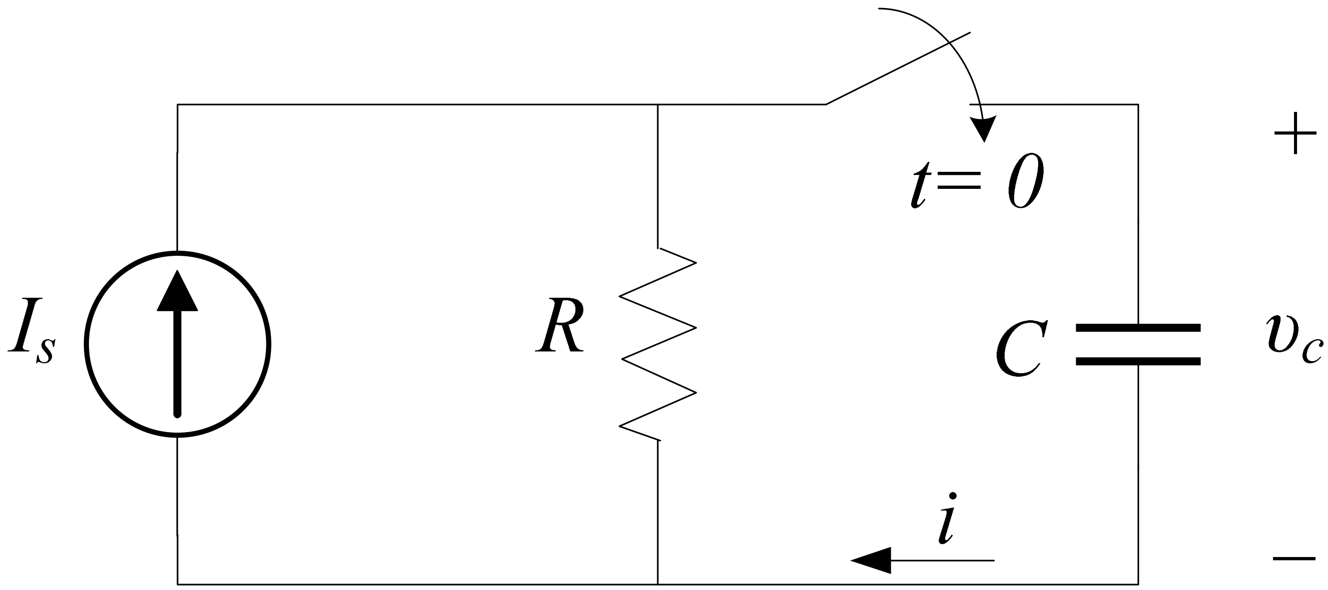
Thing's To Remember for Capacitor

- If the V = Constant or DC, I across terminal C = 0.

(C is **OPEN CIRCUIT**)

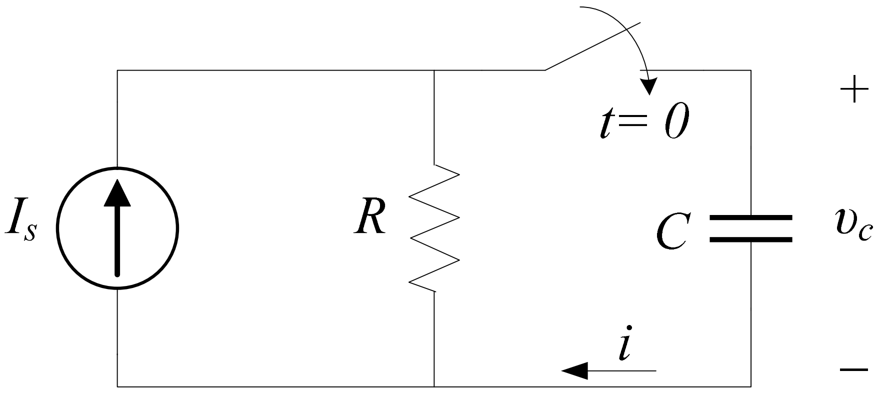
- V cannot change instantaneously across capacitor; that is, such a change would produce infinite voltage.

RC Circuit



RC Circuit

t > 0



$$-I_s + I_R + i = 0$$

$$I_s = \frac{v_c}{R} + C\frac{dv_c}{dt}$$

$$\vdots$$

$$v_c(t) = I_sR + (V_o - I_sR)e^{-\frac{t}{RC}}$$

$$v_c(t) = I_sR + (V_o - I_sR)e^{-\frac{t}{\tau}}$$

General Approach

For $0 < t < \infty$

Unknown = Final + (Initial – Final)$e^{-t/\tau}$

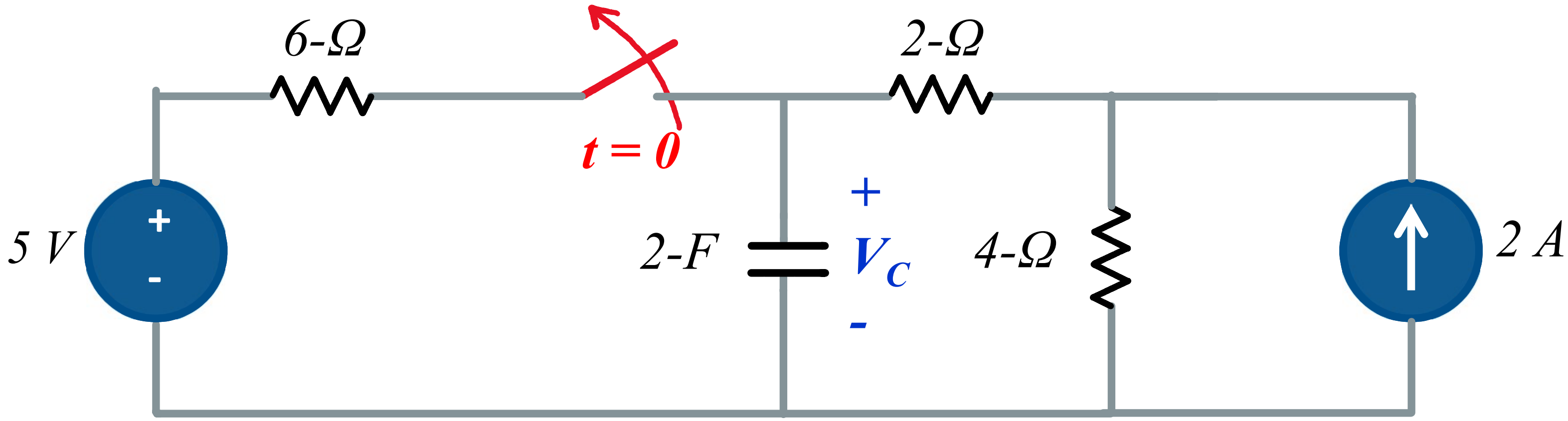
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

Time Constant, τ

$$\tau = R_{th}C$$

Thevenin resistance across the capacitor terminal

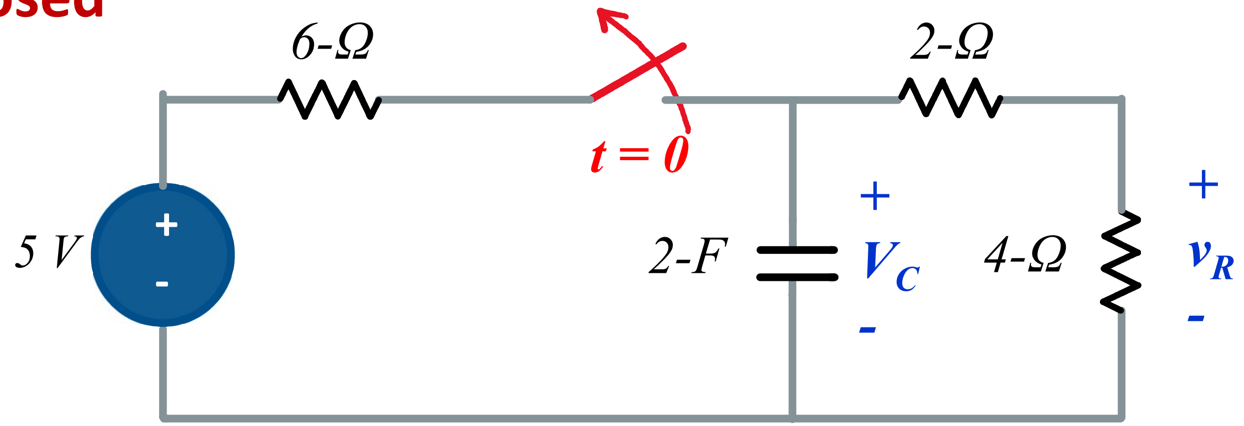
RC Circuit



The switch in the circuit has been closed for a long time before it open at t=0. Find $V_c(t)$ for $t \geq 0$.

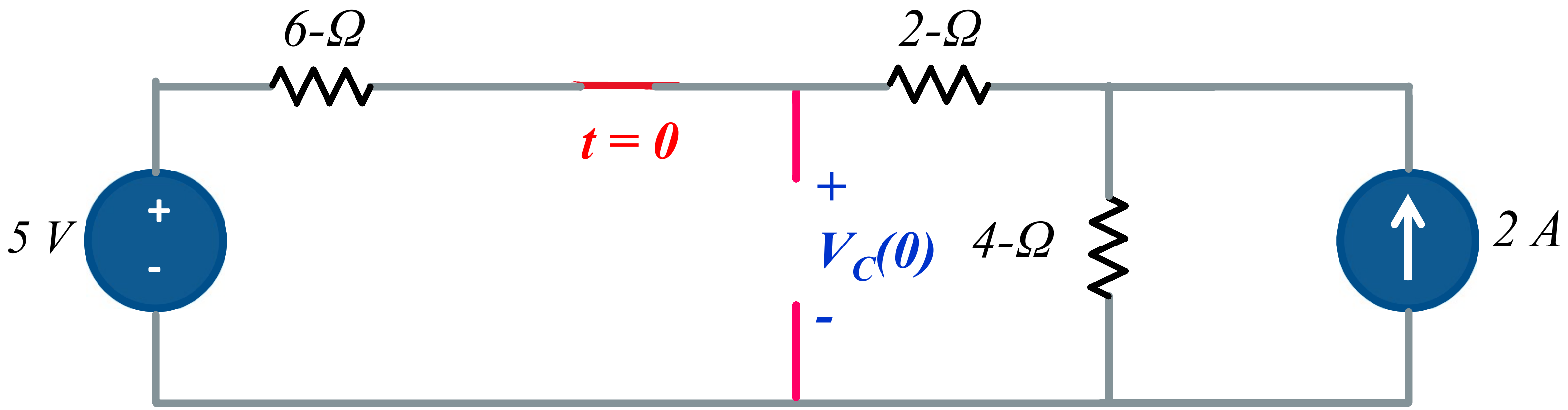
STEP 1: Find the initial voltage, $V_C(0)$ across the capacitor, t = 0 (C open circuit)

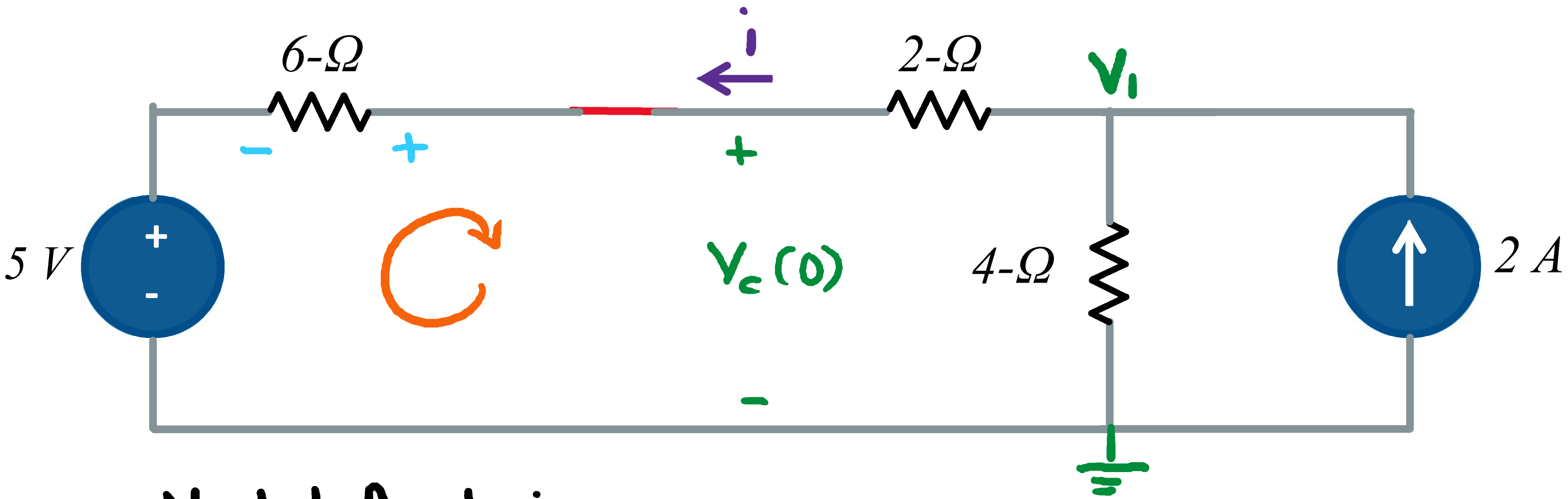
Switch Closed



- Assume, 5 V =constant current/dc & switch closed for a long time
- C appears O/C prior to release of the stored energy

STEP 1: Find the initial voltage, $V_C(0)$ through the capacitor, t = 0 (C open circuit)





Nodal Analysis

$$\frac{V_1 - 5}{8} + \frac{V_1}{4} - 2 = 0$$

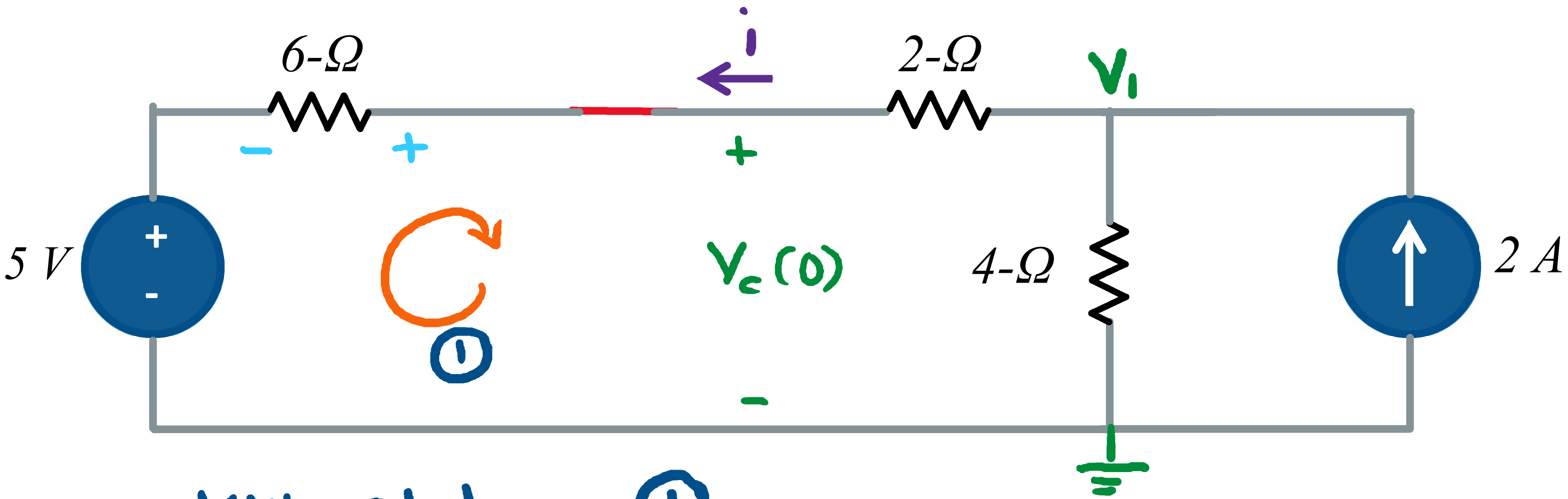
$$V_1\left(\frac{1}{8} + \frac{1}{4}\right) = \frac{5}{8} + 2$$

$$V_1\left(\frac{3}{8}\right) = \frac{21}{8}$$

$$V_1 = \frac{\left(\frac{21}{8}\right)}{\left(\frac{3}{8}\right)} = 7\ V$$

$$i = \frac{V_1}{8} = \frac{7}{8}$$

$$= 0.875\ A$$



KVL at Loop ①

$$-5 - 6(i) + V_c(0) = 0$$

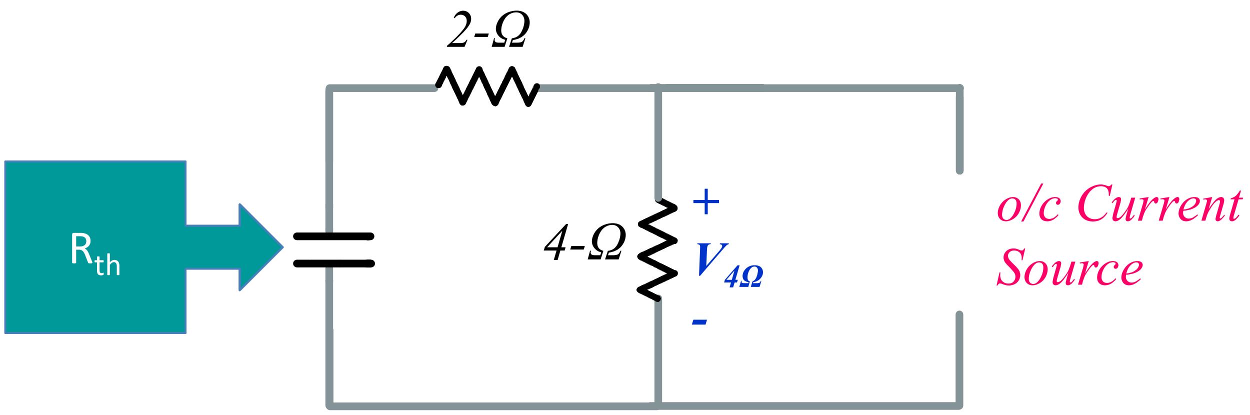
$$V_c(0) = 5 + 6i$$

$$= 5 + 6(0.875)$$

$$= 10.25\ V$$

STEP 2: Find the time constant of the circuit, τ (t>0)

Switch Open



$$R_{th} = R_{2Ω,4Ω} = 6\text{-}Ω$$

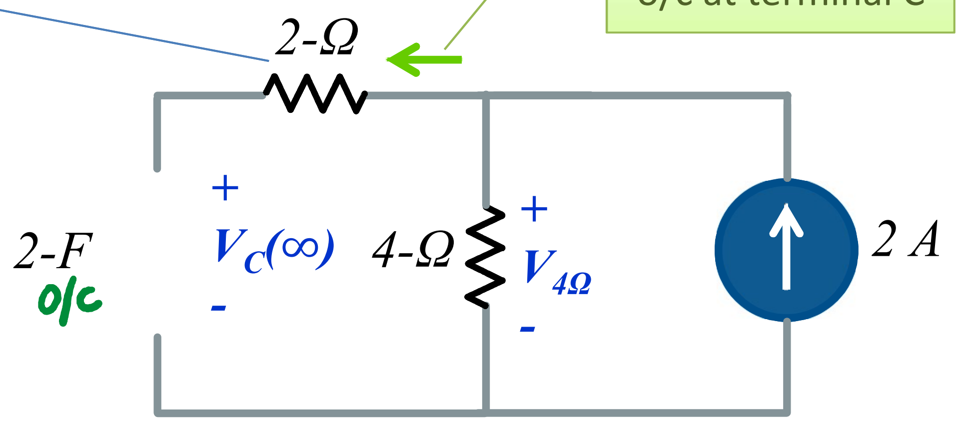
$$\tau = R_{th}C = 6\ (2) = 12$$

STEP 3: Find the final voltage, $V_C(\infty)$ through the capacitor, t = ∞ (C open circuit)

Switch Open

$V_{2\Omega}$= 0-V
s/c

i=0, because of
o/c at terminal C



$$V_c(\infty) = V_{4\Omega}$$
$$= I_{2A}R_{4\Omega}$$
$$= 2(4)$$
$$= 8\ V$$

$V_C(t)$

i. $V_c(t)$ for $t \geq 0$.

t = 0 { $V_C(0) = 10.25$ V

t > 0 { $\tau = 12$

t = ∞ { $V_C(\infty) = 8$ V

Substitute →

$$V_c(t) = v(\infty) + [v(0) - v(\infty)]e^{-\frac{t}{\tau}}$$

$$= 8 + [10.25 - 8]e^{-\frac{t}{12}} \text{ V}$$

$$= 8 + 2.25e^{-\frac{t}{12}} \text{ V}$$

RL CIRCUIT

Thing's To Remember for Inductor

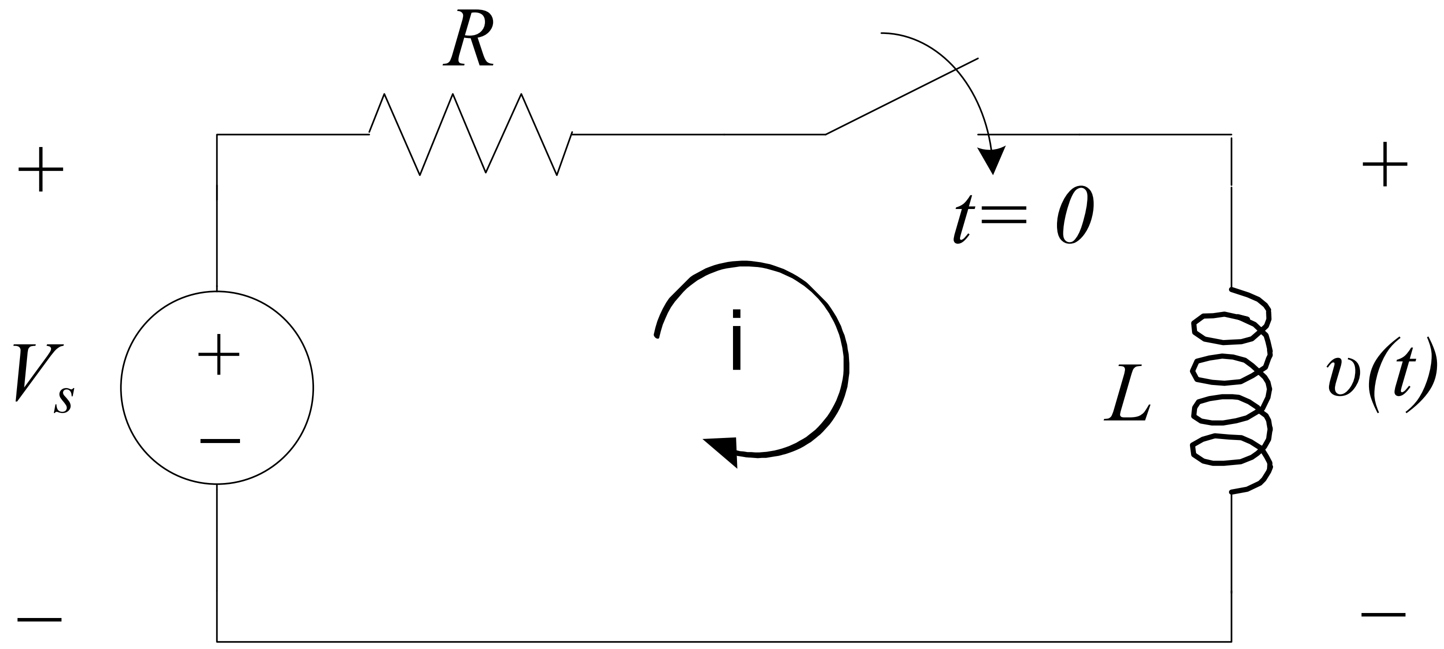
- If the *I* = Constant or DC, V across ideal L = 0.

  (L is **SHORT CIRCUIT**)

- *I* cannot change instantaneously in an inductor; that is, the current cannot change by a finite amount in zero time.

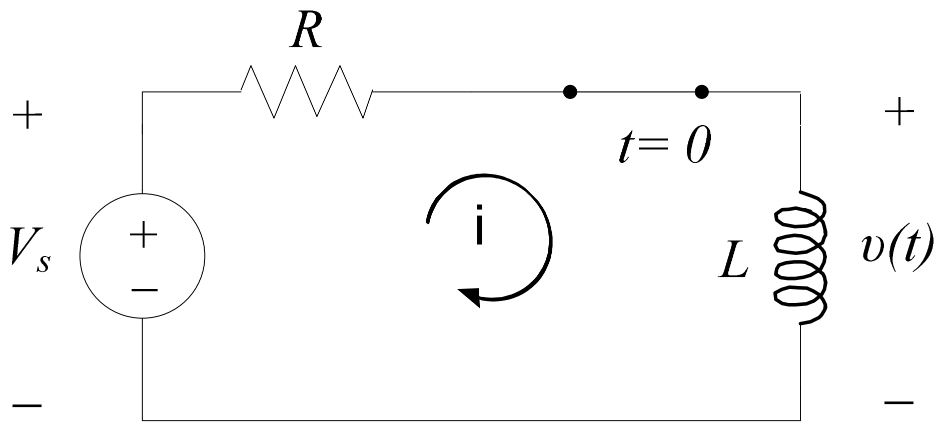
  Example: When someone opens the switch on an inductive circuit in an actual system, the current initially continues to flow in the air across the switch.

RL Circuit



RL Circuit

t > 0



$$-V_s + V_R + \upsilon(t) = 0$$

$$V_s = Ri + L\frac{di}{dt}$$

$$\vdots$$

$$i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-\left(\frac{R}{L}\right)t}$$

$$i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-\frac{t}{\tau}}$$

General Approach

For $0 < t < \infty$

Steady State | Transient

Unknown = Final + (Initial – Final)$e^{-t/\tau}$

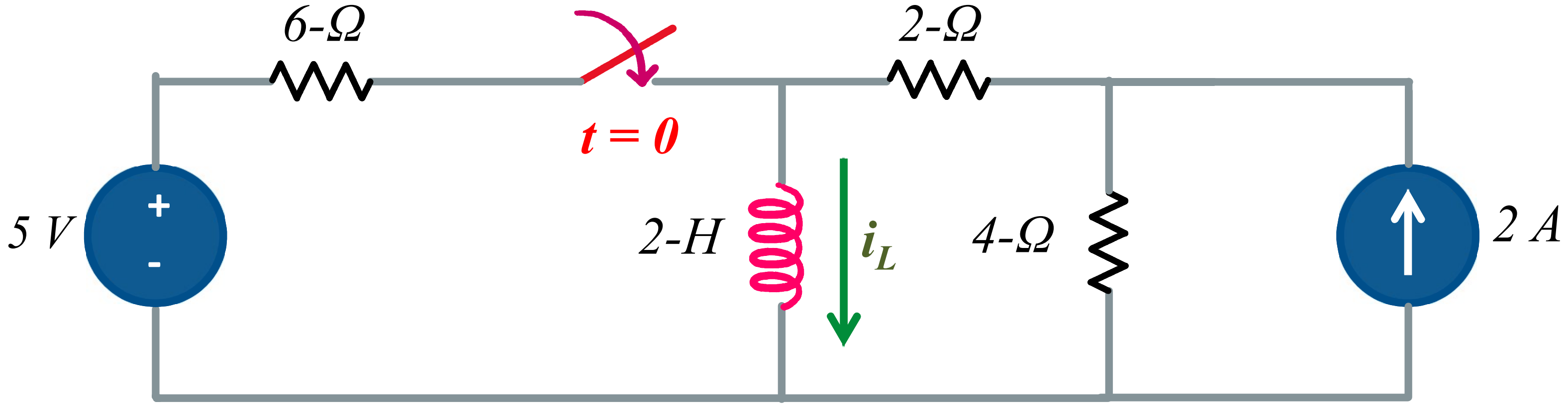
$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

Time Constant, τ

$$\tau = \frac{L}{R_{th}}$$

Thevenin resistance across the inductor terminal

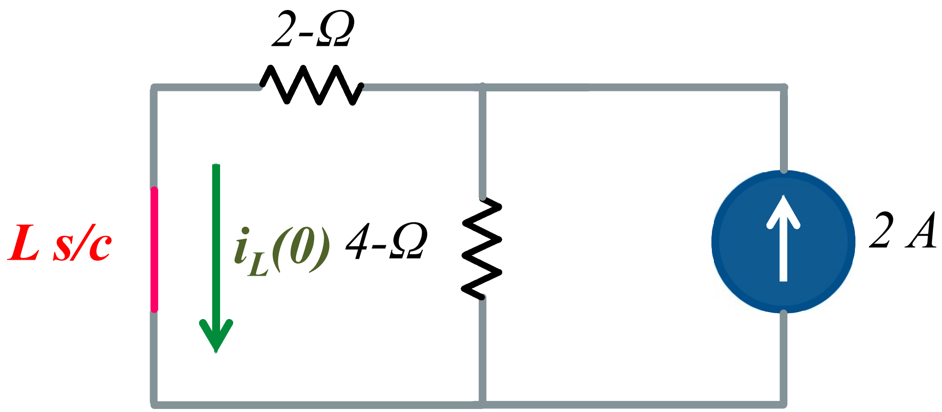
RL Circuit



The switch in the circuit has been opened for a long time before it close at t=0. Find $i_L(t)$ for t ≥ 0.

STEP 1: Find the initial current, $i_L(0)$ through the inductor, t = 0 (L short circuit)

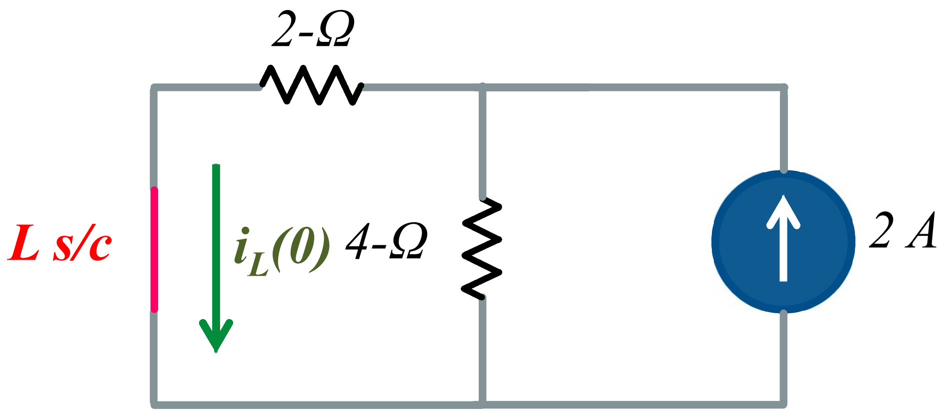
Switch Open



- Assume, 2 A =constant current/dc & switch opened for a long time
- L appears S/C prior to release of the stored energy

Cont.

Switch Open

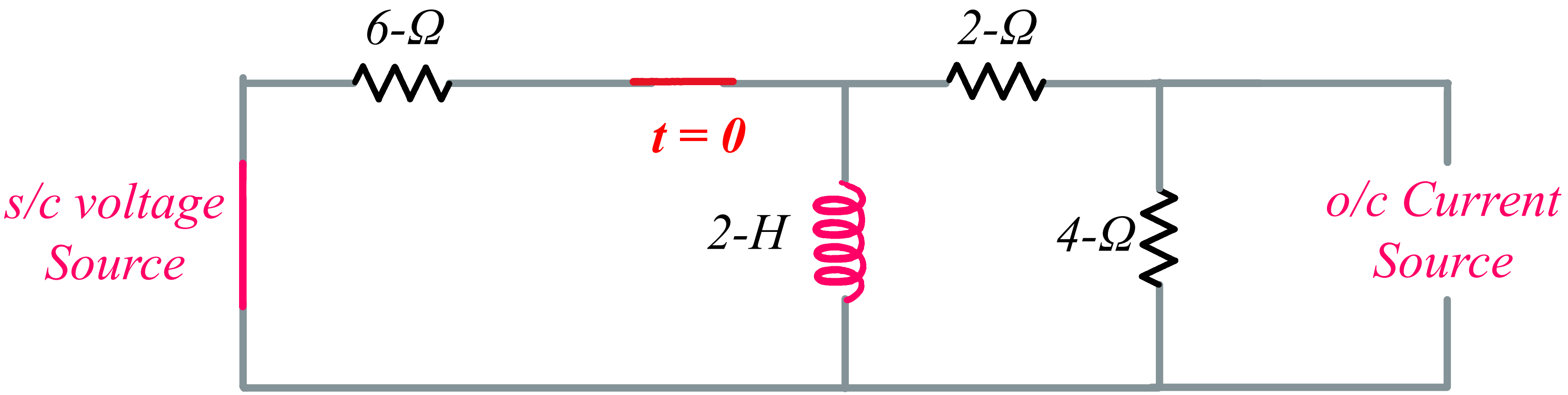


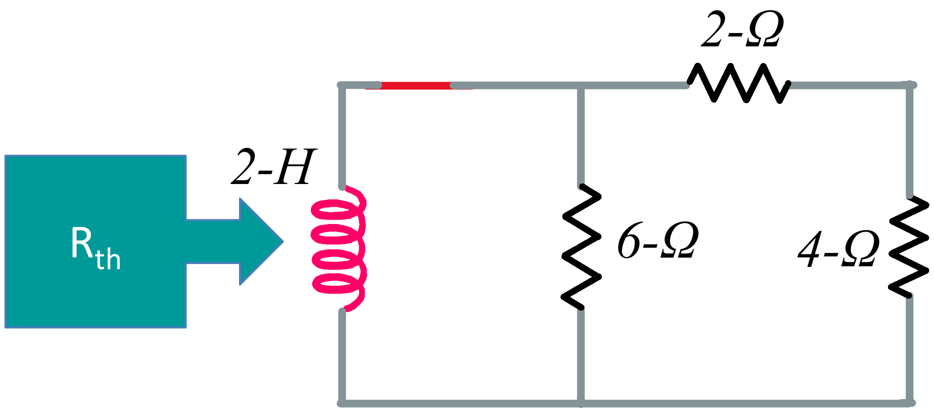
Using Current Divider

$$i_L(0) = \frac{4}{4+2}(2) = 1.33\ A.$$

STEP 2: Find the time constant of the circuit, τ (t>0)

Switch Closed





$$
\begin{aligned}
R_{th} &= \left(R_{4\Omega,2\Omega}\right)//R_{6\Omega} \\
&= (4+2)//6 \\
&= 6//6 \\
&= 3\ \Omega
\end{aligned}
$$

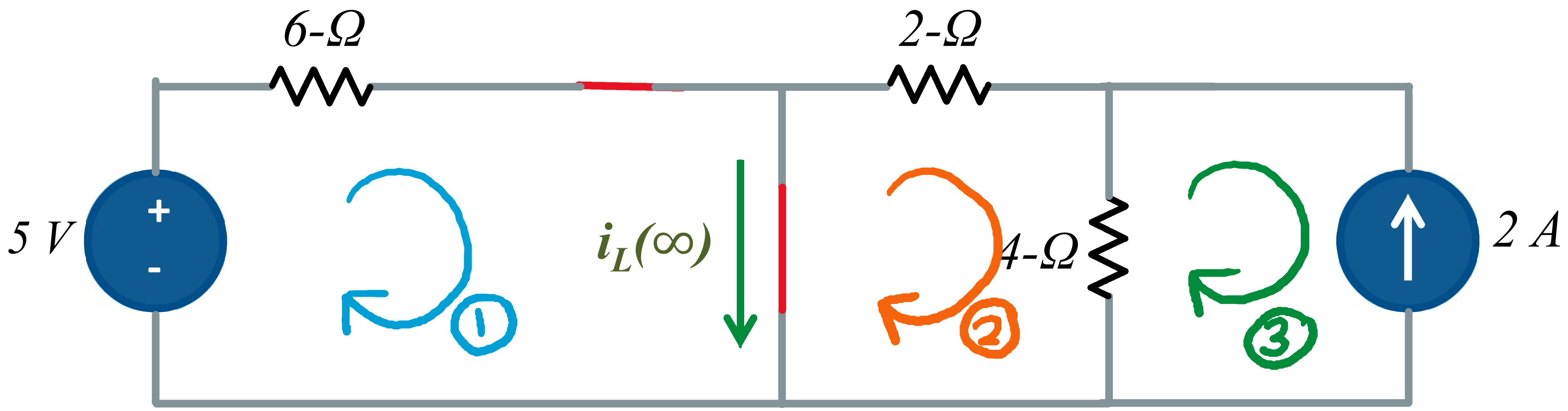
Cont.

$$R_{th} = 3\ \Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{2}{3}$$

STEP 3: Find the final current, $i_L(\infty)$ through the inductor, t = ∞ (L short circuit)

Switch Closed



Using Mesh Analysis

KVL at Loop 1

$-5 + 6i_1 = 0$

$i_1 = \frac{5}{6}$ …………………(1)

KVL at Loop 2

$2i_2 + 4(i_2 - i_3) = 0$

$6i_2 - 4i_3 = 0$…………(2)

At Loop 3

$i_3 = -2$………(3)

Subs. (3) – (2)

$6i_2 - 4(-2) = 0$

$6i_2 = -8$

$i_2 = -\frac{8}{6} = -\frac{4}{3}$ A

Therefore,

$i_L(\infty) = i_1 - i_2$

$= \frac{5}{6} - \left(-\frac{4}{3}\right)$

= **2.17 A**

$i_L(t)$

t = 0 — $i_L(0) = 1.33$ A

t > 0 — $\tau = 2/3$

t = ∞ — $i_L(\infty) = 2.17$ A

Substitute

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-\frac{t}{\tau}}$$

$$= 2.17 + [1.33 - 2.17]e^{-\frac{t}{\left(\frac{2}{3}\right)}} \text{ V}$$

$$= 2.17 - 0.84e^{-\frac{3}{2}t} \text{ V}$$

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