BEE1133 Circuit Analysis

Chapter 5A
First Order Circuit

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Chapter Description

Aims

This chapter is aimed to:

1. Introduce the source free RC and RL circuit
2. Explain the equation related for both circuit

Expected Outcomes

Student should be able to

1. Determine the source free of both RC and RL circuit

References

1. C. Alexander and M. Sadiku, “Fundamentals of Electric Circuits”, 4th ed., McGraw-Hill, 2008.
2. J. Nilsson and S. Riedel, “Electric Circuits”, 8th ed., Prentice Hall, 2008.

BASIC CONCEPT

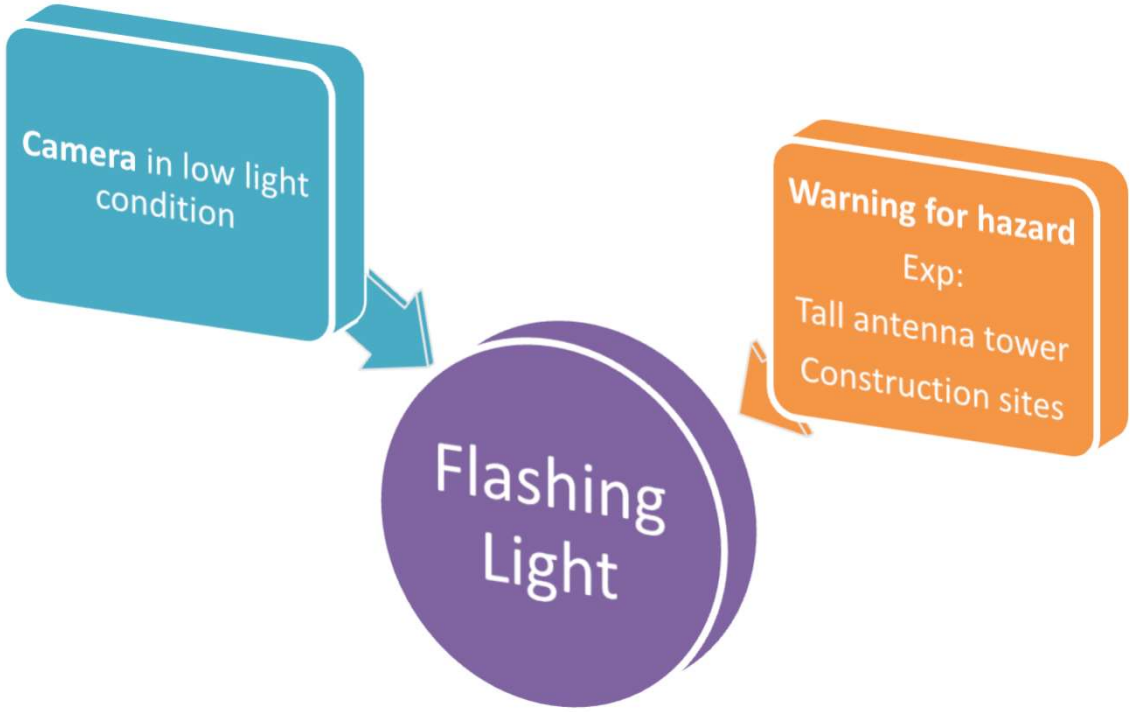
9.1 Introduction to energy storage elements

9.2 The source-free RC circuit

9.3 The source-free RL circuit

9.4 Unit step function

Practical Application



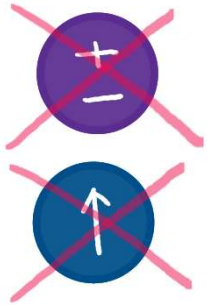
What does First Order mean?

- Circuits that contain capacitors and inductors can be defined by differential equations
- circuits with **ONLY ONE** capacitor ***OR*** **ONLY ONE** inductor can be defined by a first order differential equation

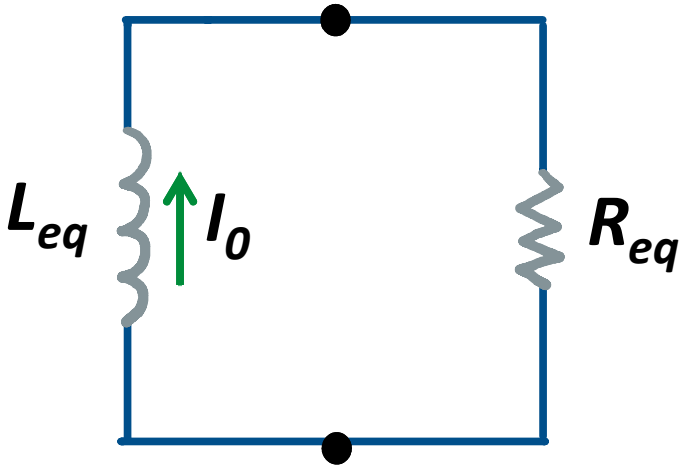
Source FREE?

Source Free

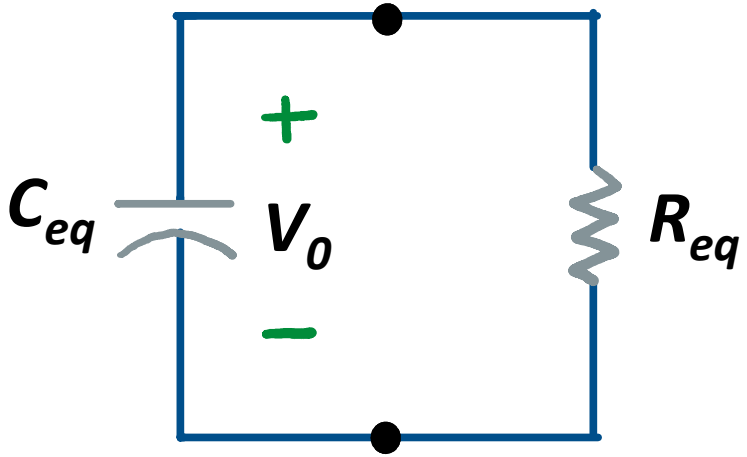
- DC source is suddenly **DISCONNECTED** (No source in the circuit)
- The energy stored in the capacitor or inductor RELEASED to R (DISCHARGED)



Source FREE?



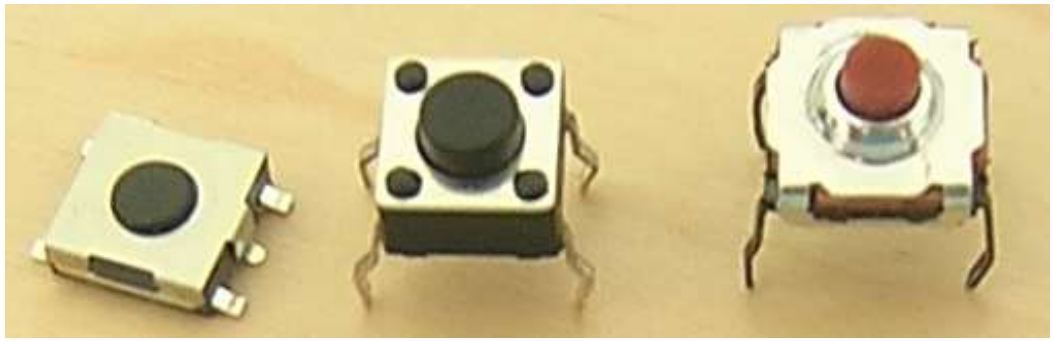
RL Circuit



RC Circuit

Switch

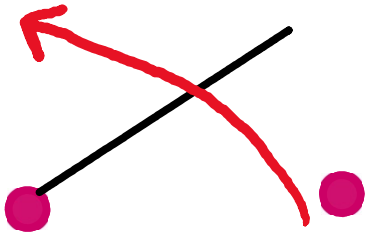
Source:
https://electronics.stackexchang



Source:
http://en.wikipedia.org/wiki/file:tactile_switches.jpg

Please refer to this website for more information regarding the type of electronic switch
http://tech.txdi.org/switchtypes

Circuit Symbol



CLOSED for a long time before it **OPEN** at t=0

$t = 0^-$

the time just prior to switching

$t = 0$

the instant when the switch is opened

$t = 0^+$

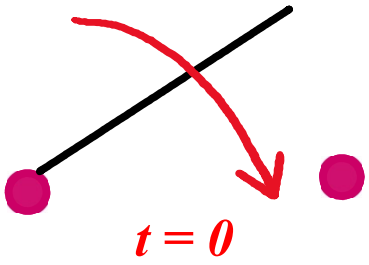
the time immediately following switching

$t(0)$

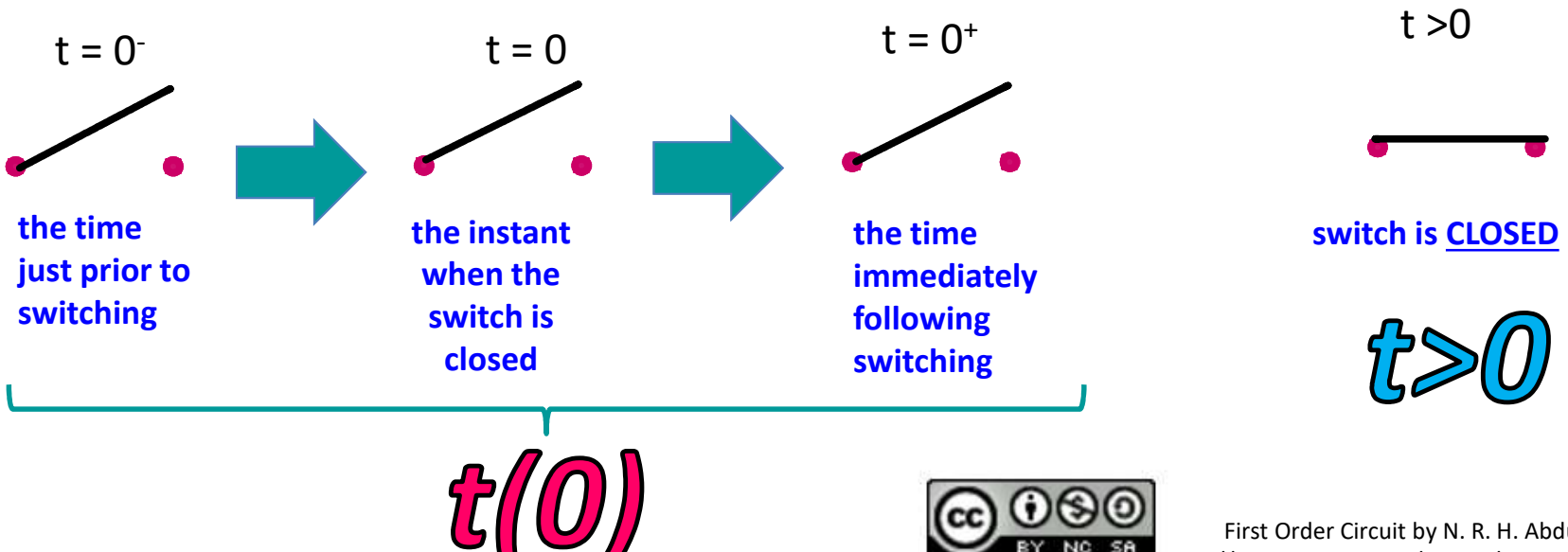
switch is **OPENED**

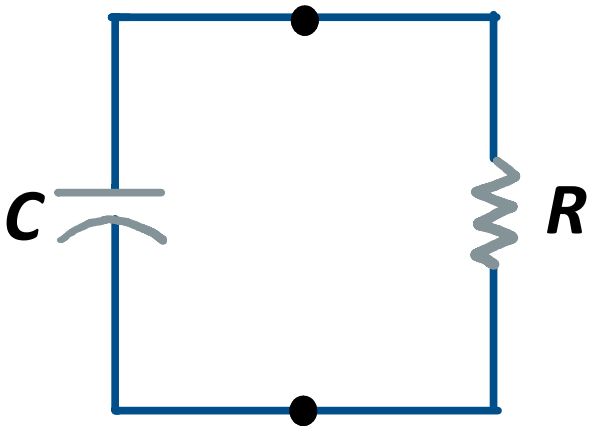
$t>0$

Circuit Symbol



OPEN for a long time before it **CLOSED** at t=0





RC CIRCUIT

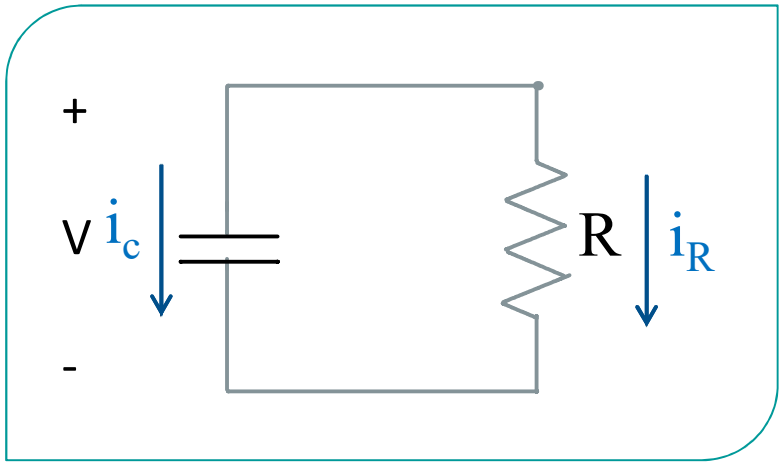
Thing's To Remember for Capacitor

- If the V = Constant or DC, I across terminal C = 0.

(C is **OPEN CIRCUIT**)

- V cannot change instantaneously across capacitor; that is, such a change would produce infinite voltage.

t > 0



$$I_c + I_R = 0 \qquad I_C = C\frac{dV}{dt}$$

$$C\frac{dV}{dt} + \frac{V}{R} = 0$$

$$\vdots$$

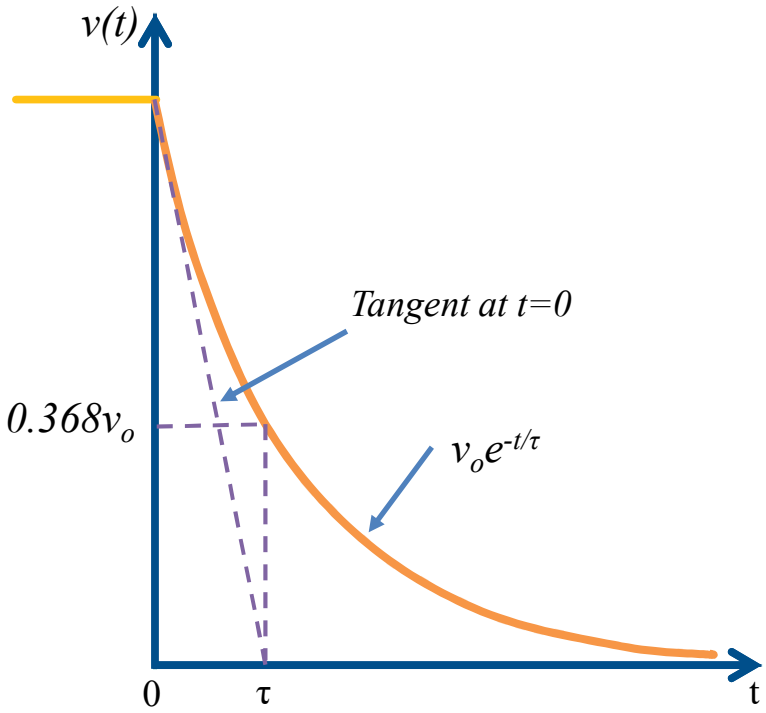
$$V(t) = V(0)e^{-\frac{t}{RC}}$$

$V(0) = V_o$

$\tau = RC$ time constant (s)

$$\therefore V(t) = V_o e^{-\frac{t}{\tau}}$$

Voltage Response



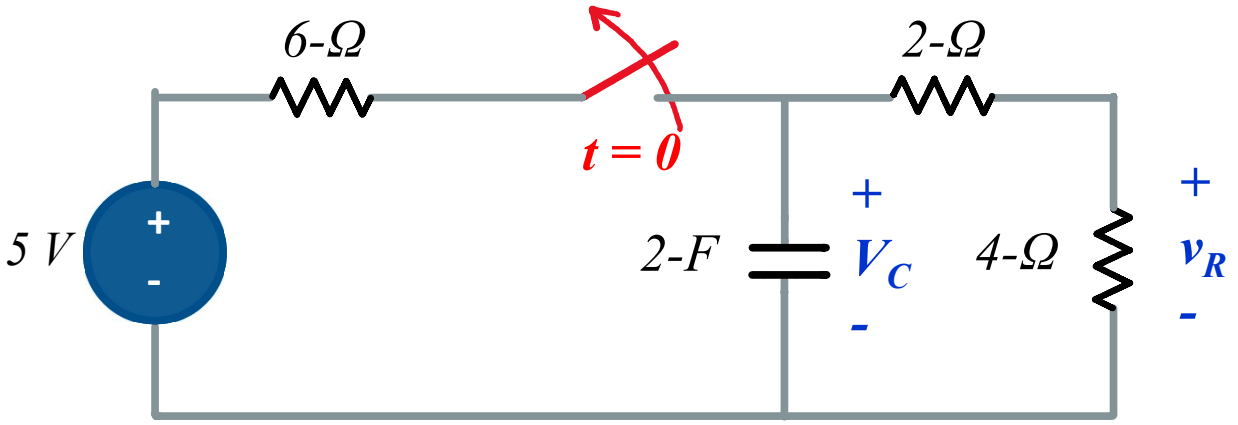
RC Circuit

$$V(t) = V_o e^{-\frac{t}{\tau}}$$

t=0	t>0
Find the voltage across C (C is open circuit)	Calculate $\tau=1/RC$ (no source exist in the circuit when the switch started to operate)

First Order Circuit by N. R. H. Abdullah
http://ocw.ump.edu.my/course/view.php?id=251

RC Circuit



The switch in the circuit has been closed for a long time before it open at t=0. Find

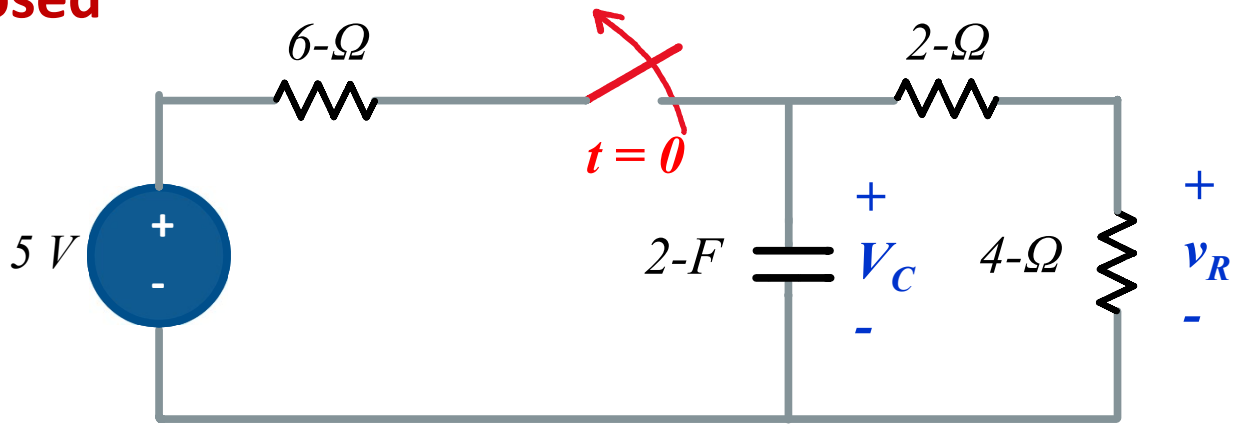
i. $i_L(t)$ for t ≥ 0.

ii. $i_R(t)$ for t ≥ 0.

iii. $v_R(t)$ for t ≥ 0.

STEP 1: Find the initial voltage, $V_C(0)$ across the capacitor, t = 0 (C open circuit)

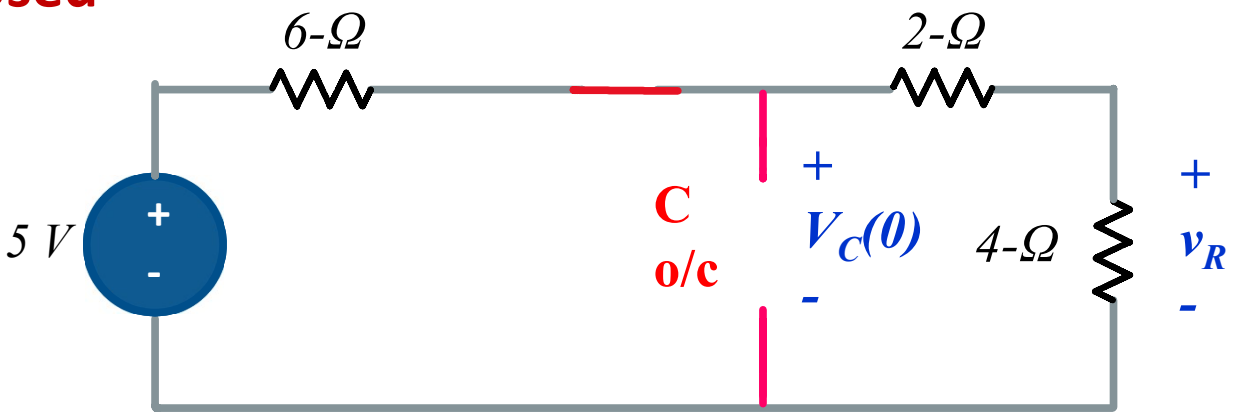
Switch Closed



- Assume, 5 V =constant current/dc & switch closed for a long time
- C appears O/C prior to release of the stored energy

Cont.

Switch Closed

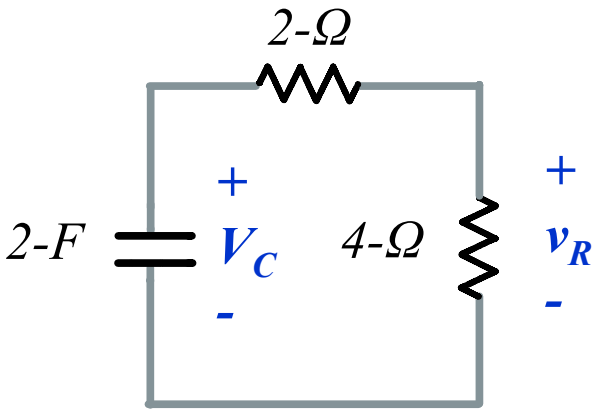


Find $V_c(0)$ using voltage divider,

$$V_C(t) = \frac{2+4}{2+4+6}(5) = 2.5\ V$$

STEP 2: Find the time constant of the circuit, τ (t>0)

Switch Open



$$\tau = RC = (6)(2) = 12$$

$V_C(t)$

i. $V_c(t)$ for $t \geq 0$.

$t = 0$ { $V_C(0) = 2.5$ V

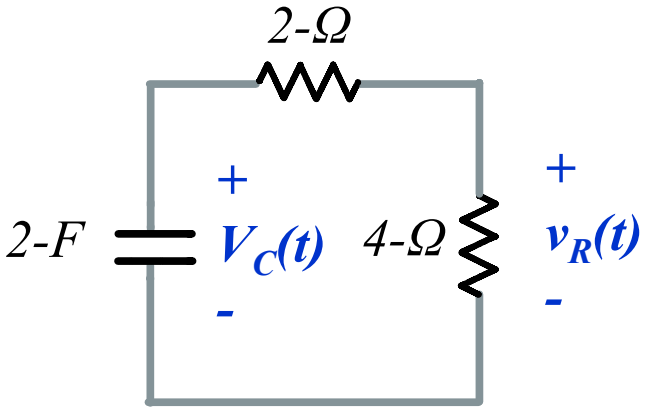
$t > 0$ { $\tau = 12$

Substitute

$$V_c(t) = V_C(0)e^{-\frac{t}{\tau}}$$
$$= 2.5\, e^{-\frac{t}{12}} \text{ V}$$

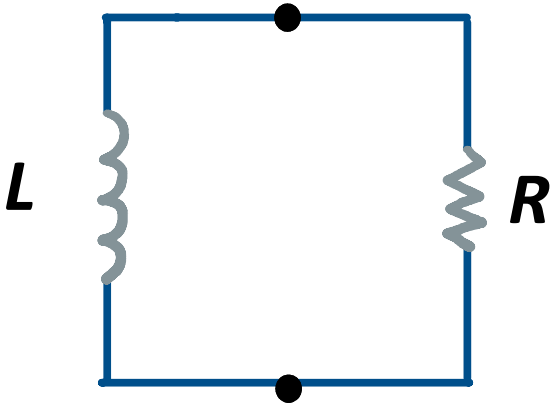
$v_R(t)$

ii. $v_R(t)$ for t ≥ 0.



Using Voltage Divider,

$$v_R(t) = \frac{4}{2+4}\left(V_c(t)\right) = \frac{4}{6}(2.5\, e^{-\frac{t}{12}}) = 1.67e^{-\frac{t}{12}}\ V$$



RL CIRCUIT

Thing's To Remember for Inductor

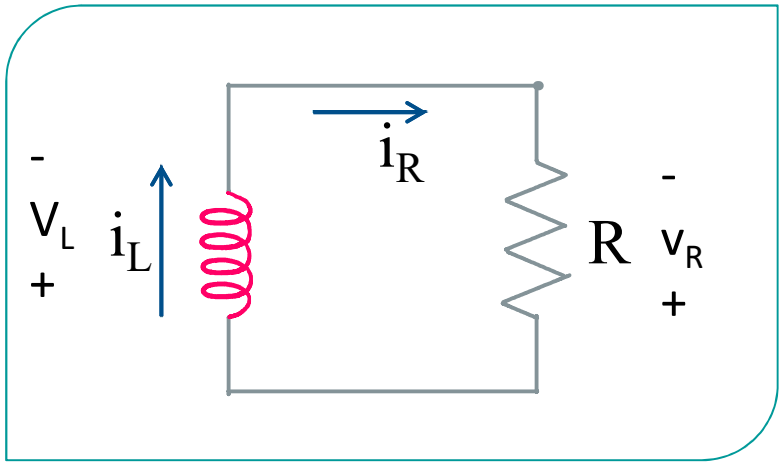
- If the *I* = Constant or DC, V across ideal L = 0.

  (L is **SHORT CIRCUIT**)

- *I* cannot change instantaneously in an inductor; that is, the current cannot change by a finite amount in zero time.

  Example: When someone opens the switch on an inductive circuit in an actual system, the current initially continues to flow in the air across the switch.

t > 0



$$V_L + V_R = 0 \qquad V_L = L\frac{di}{dt}$$

$$L\frac{di}{dt} + Ri = 0$$

$$\vdots$$

$$i(t) = i(0)e^{-\left(\frac{R}{L}\right)t}$$

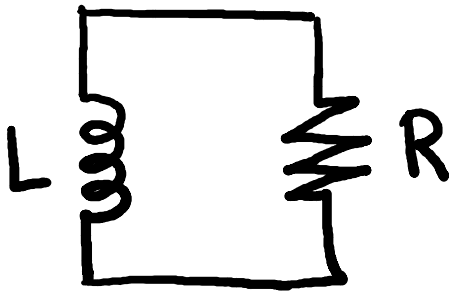
$i(0) = i_o$

$\tau = \frac{L}{R}$ time constant (s)

$$\therefore i(t) = i_o e^{-\frac{t}{\tau}}$$

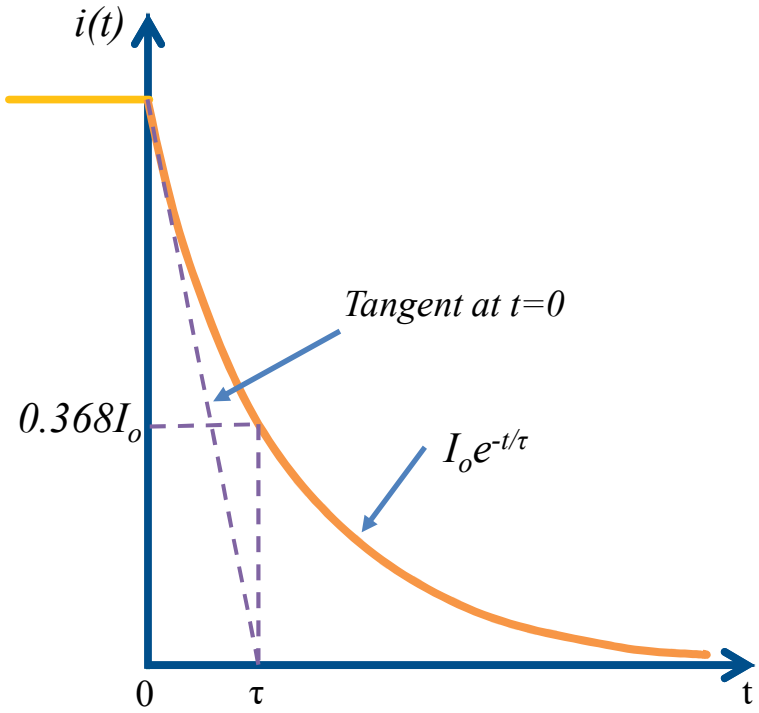
RL Circuit

$$i(t) = i_o e^{-\frac{t}{\tau}}$$

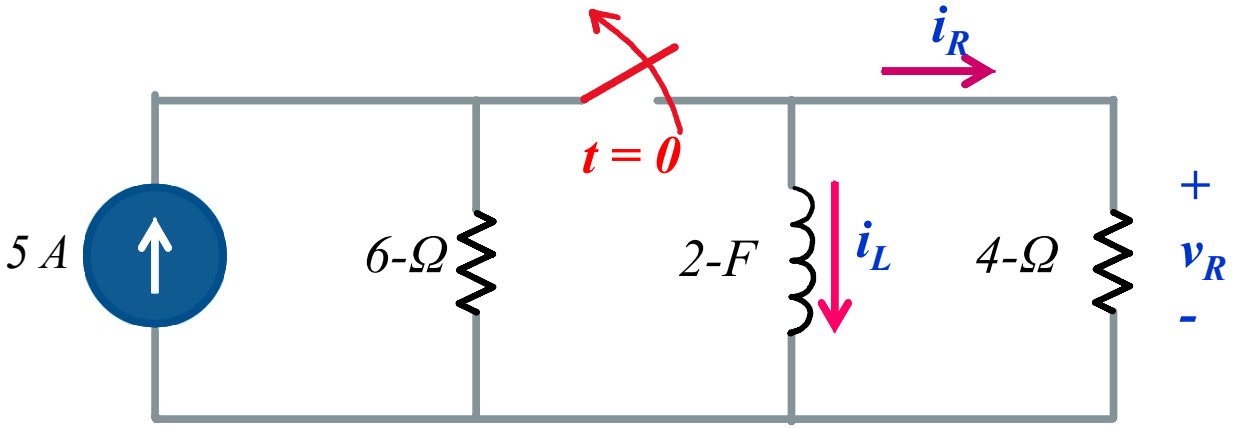


t=0	t>0
Find the current flow through L (L is short circuit)	Calculate $\tau=L/R$ (no source exist in the circuit when the switch started to operate)

Current Response



RL Circuit



The switch in the circuit has been closed for a long time before it open at t=0. Find

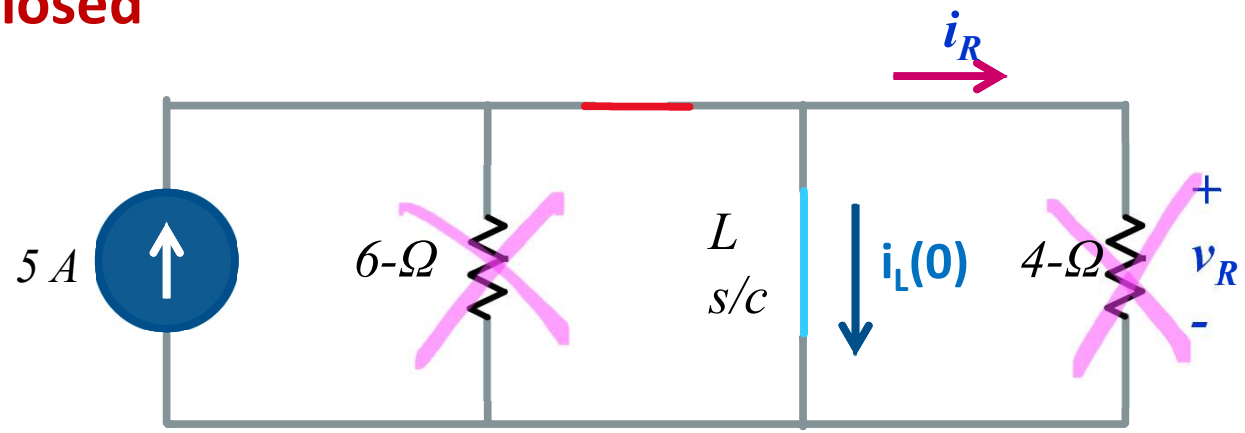
i. $i_L(t)$ for $t \geq 0$.

ii. $i_R(t)$ for $t \geq 0$.

iii. $v_R(t)$ for $t \geq 0$.

STEP 1: Find the initial current, $i_L(0)$ through the inductor, t = 0 (L short circuit)

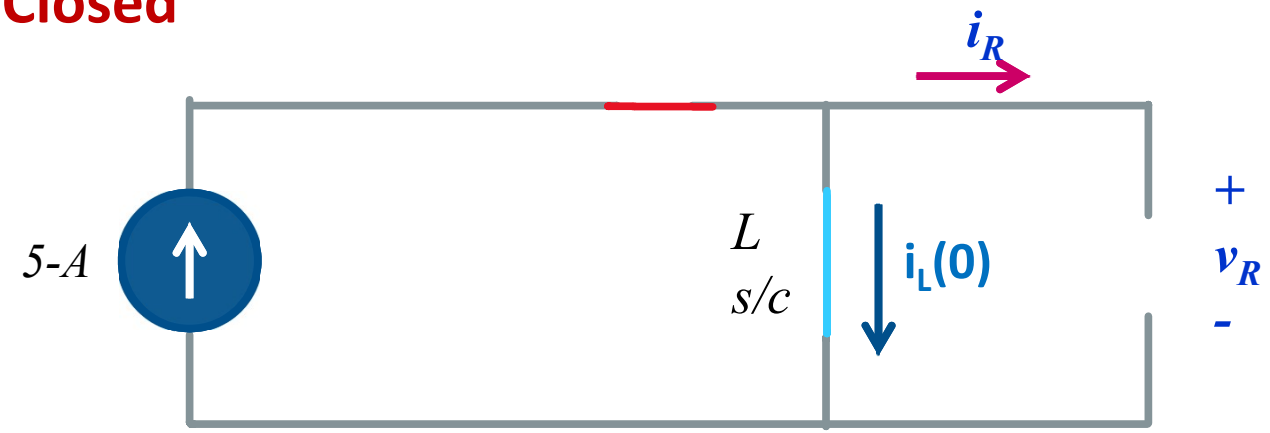
Switch Closed



- Assume,5 A =constant current/dc & switch closed for a long time
- L appears S/C prior to release of the stored energy
- Current, 5-A is not flowing across R

Cont.

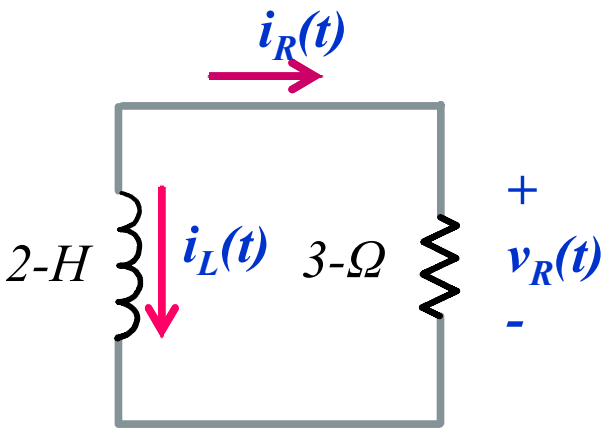
Switch Closed



$$i_L(0) = 5\ A$$

STEP 2: Find the time constant of the circuit, τ (t>0)

Switch Open



$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5$$

$i_L(t)$

i. $i_L(t)$ for $t \geq 0$.

t = 0 { $i_L(0) = 5$ A

t > 0 { $\tau = 0.5$

Substitute

$$i_L(t) = i_L(0)e^{-\frac{t}{\tau}}$$

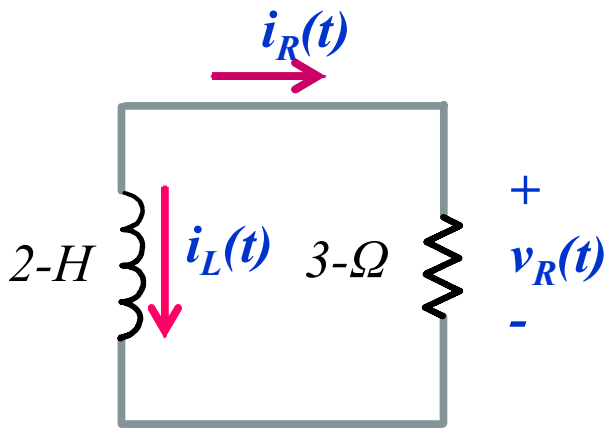
$$= 5\, e^{-\frac{t}{0.5}} \text{ A}$$

$$= 5\, e^{-2t} \text{ A}$$

$i_R(t)$ and $v_R(t)$

ii. $i_R(t)$ for t ≥ 0.

iii. $v_R(t)$ for t ≥ 0.



$$i_R(t) = -i_L(t) = -5\,e^{-2t}\ \text{A}$$

$$v_R(t) = i_R(t)\ x\ R_{3\text{-}\Omega} = (-5\,e^{-2t})(3) = -15\,e^{-2t}\ V$$

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