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BEE1133 Circuit Analysis

Chapter 3B
Circuit Theorem(DC Circuits)

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Chapter Description

Aims

This chapter is aimed to:

1. Explain the Thevenin's theorem and Norton's theorem principle in solving problem related to electric circuit

Expected Outcomes

Student should be able to

1. Understand and apply the step for solving the circuit problem using Thevenin's Theorem
2. Understand and apply the step for solving the circuit problem using Norton's Theorem
3. Find the maximum power transfer

References

1. C. Alexander and M. Sadiku, "Fundamentals of Electric Circuits", 4th ed., McGraw-Hill, 2008.
2. J. Nilsson and S. Riedel, "Electric Circuits", 8th ed., Prentice Hall, 2008.

BASIC CONCEPT

7.1 Thevenin's and Norton's Theorem (Independent and Dependent Source)

7.2 Maximum power transfer

THEVENIN’S THEOREM

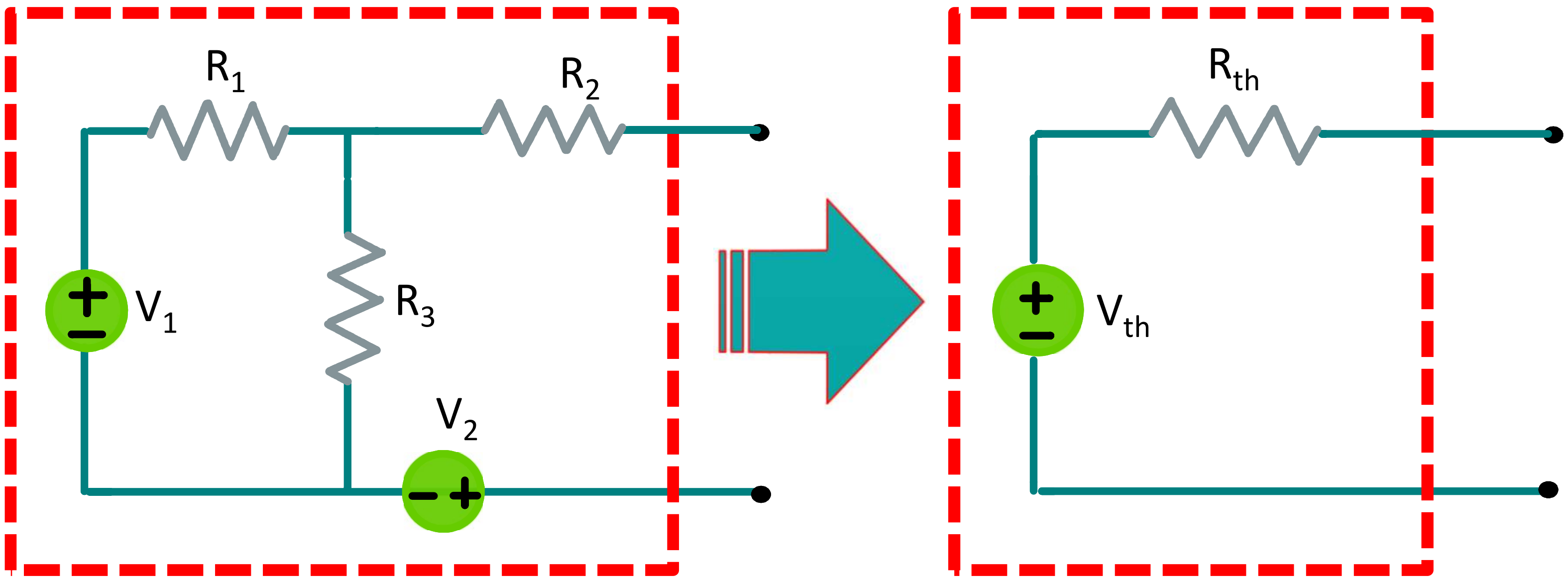
Thevenin's Theorem

- Purpose: Replace the whole circuit between the two terminals with an equivalent simple circuit (R_{th} and V_{th})
- **A voltage source, V_{th} in SERIES with one resistance R_{th}**

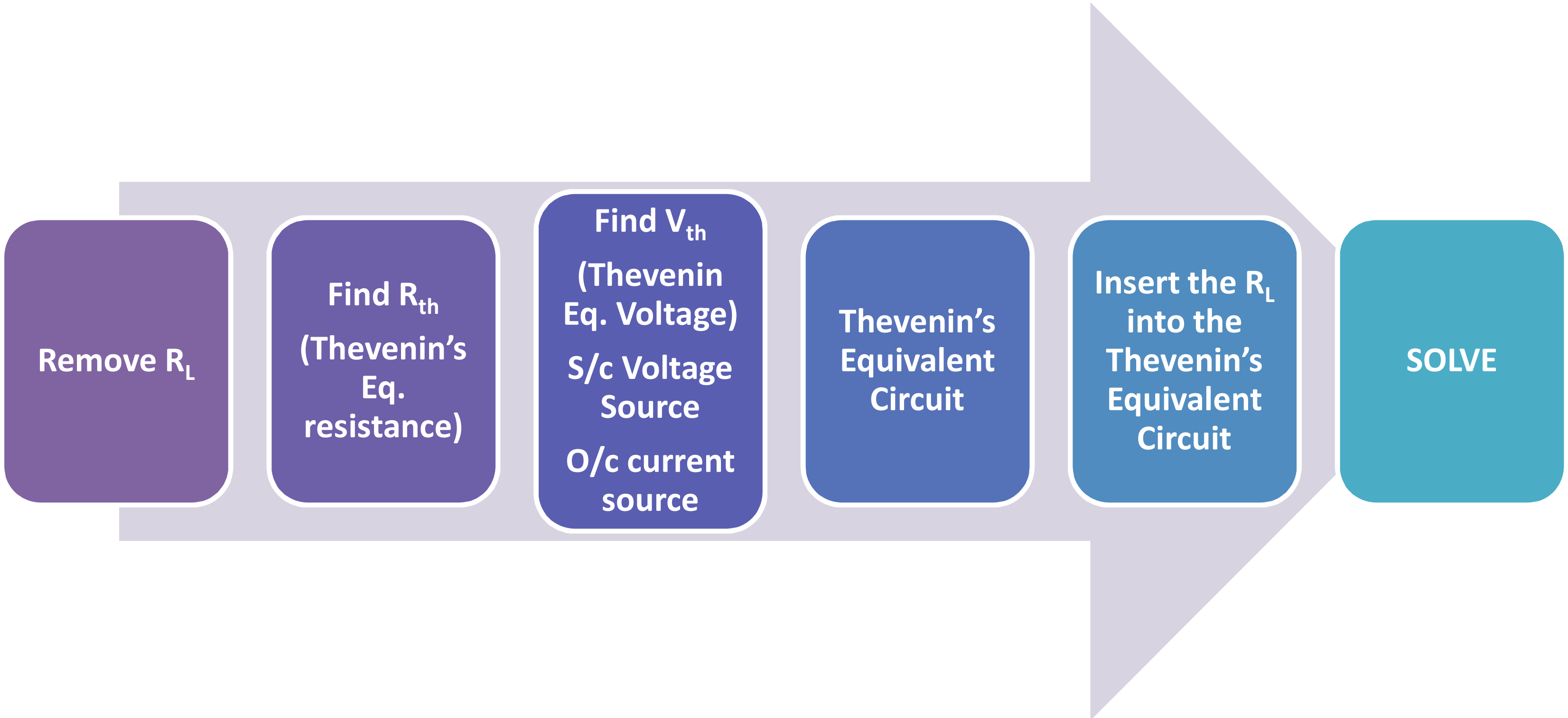
Thevenin's Theorem State that:

- *"Any two-terminal linear circuit can be replaced by an equivalent circuit consisting of voltage source in series with a single equivalent resistance."*

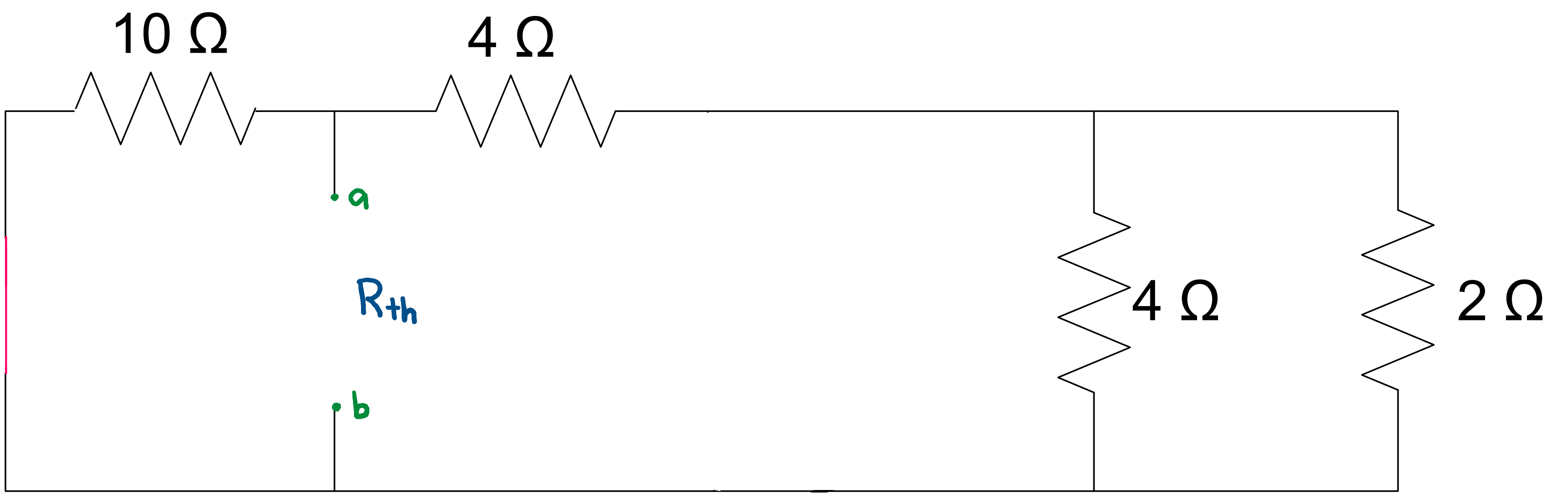
Equivalent Circuit



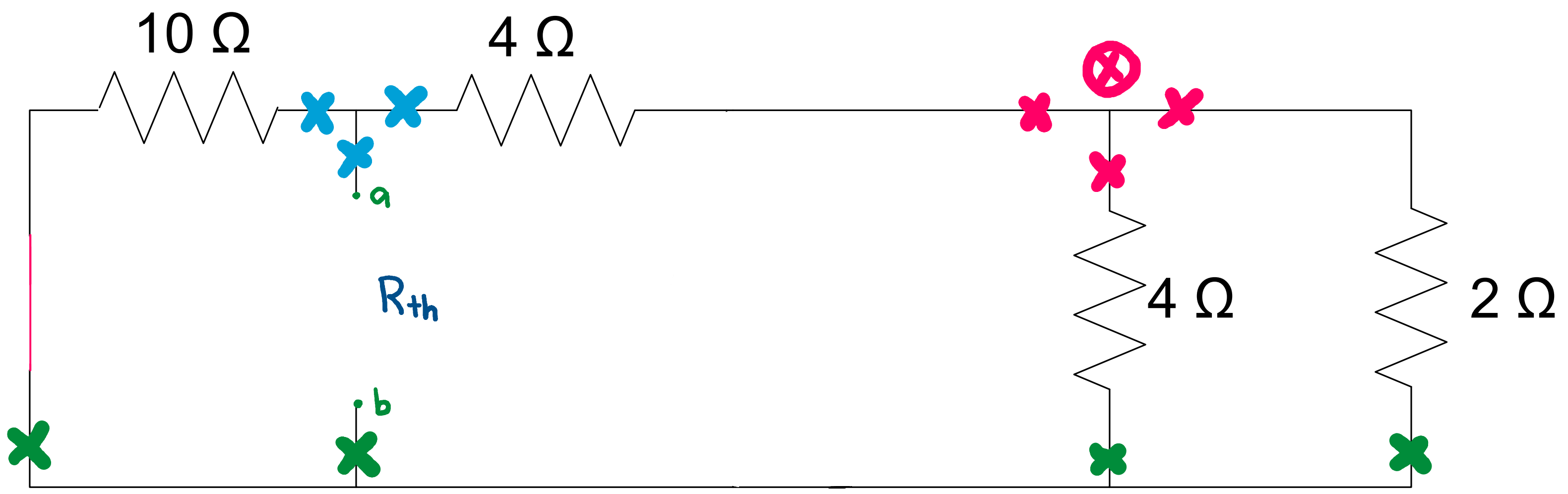
Process Flow



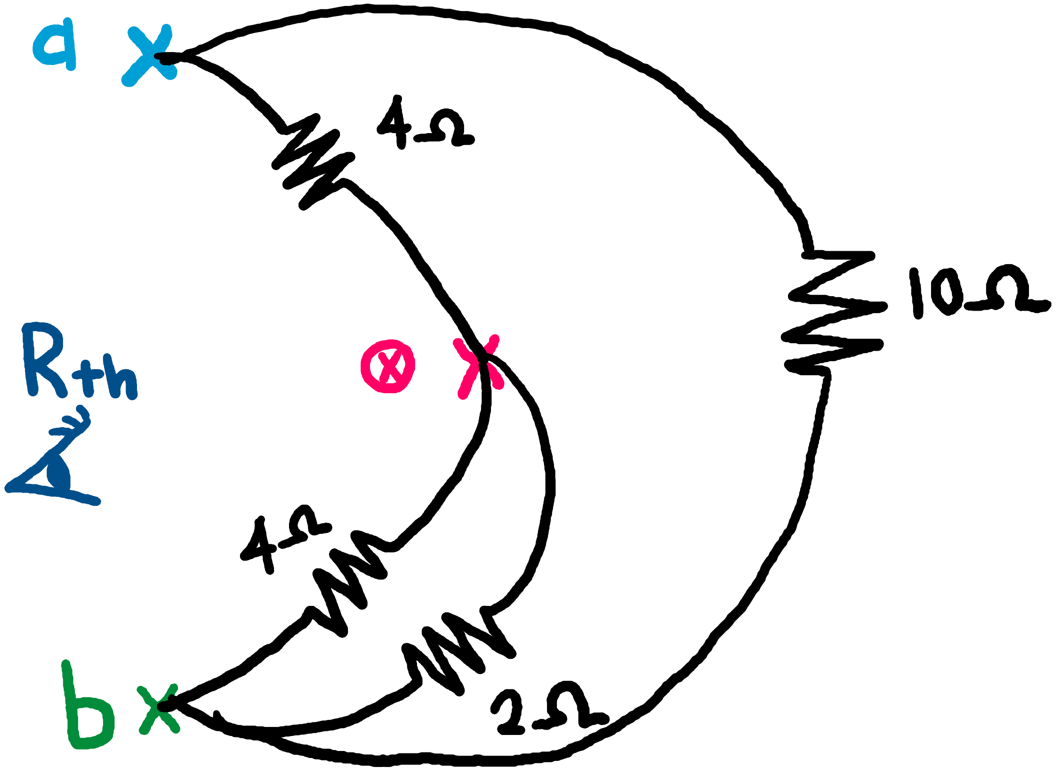
Step 3: Find R_{th}



Step 3: Find R_{th}



Step 3: Find R_{th}: Simplified the circuit



$$R_{xb} = 4 \parallel 2 = \frac{4}{3}\,\Omega$$

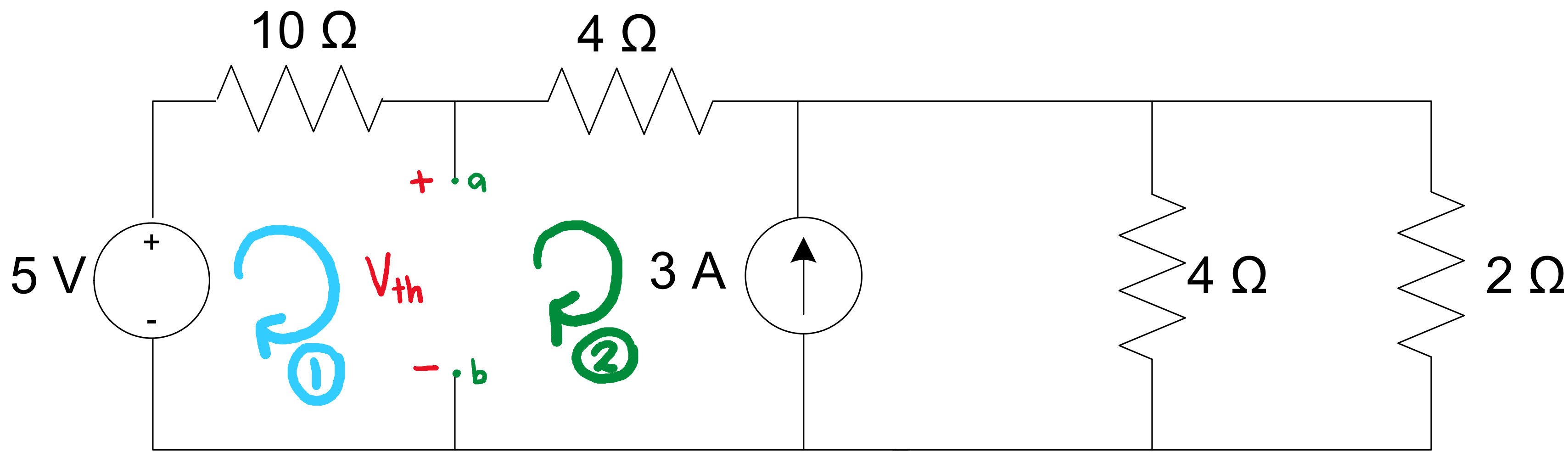
$$R_{ab} = R_{\frac{4}{3},4} \parallel R_{10}$$

$$= \frac{16}{3} \parallel 10\,\Omega$$

$$= 3.48\,\Omega$$

$$= R_{th}$$

Step 3:Find V_{th}

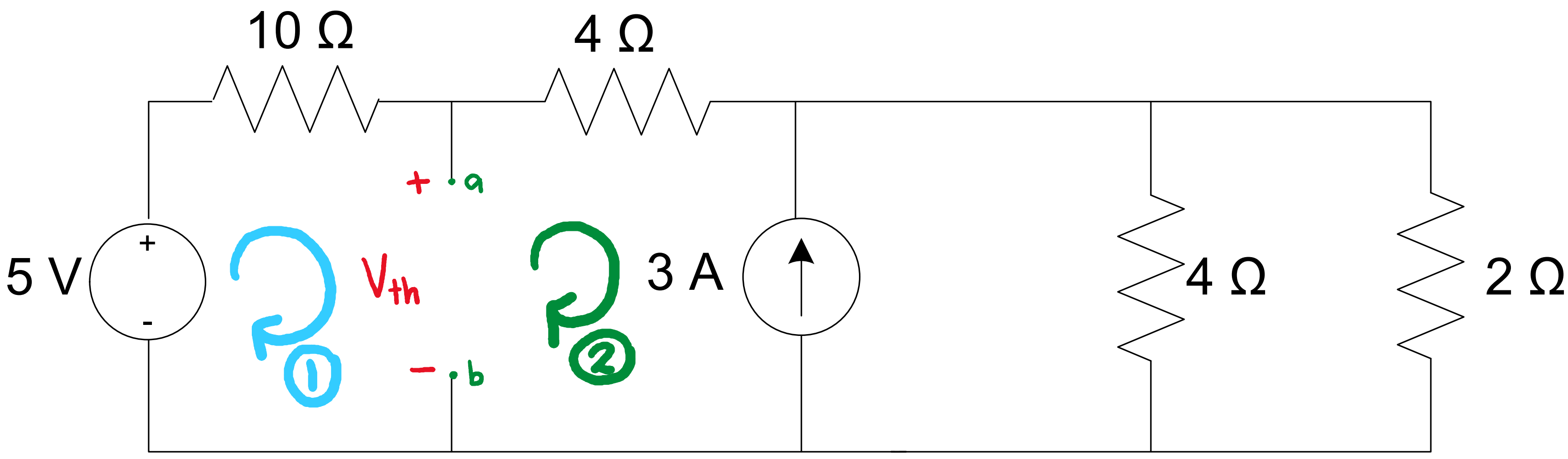


There are 2 choices(KVL)

(1) Left loop

(2) Right loop

Step 3:Find V_{th}

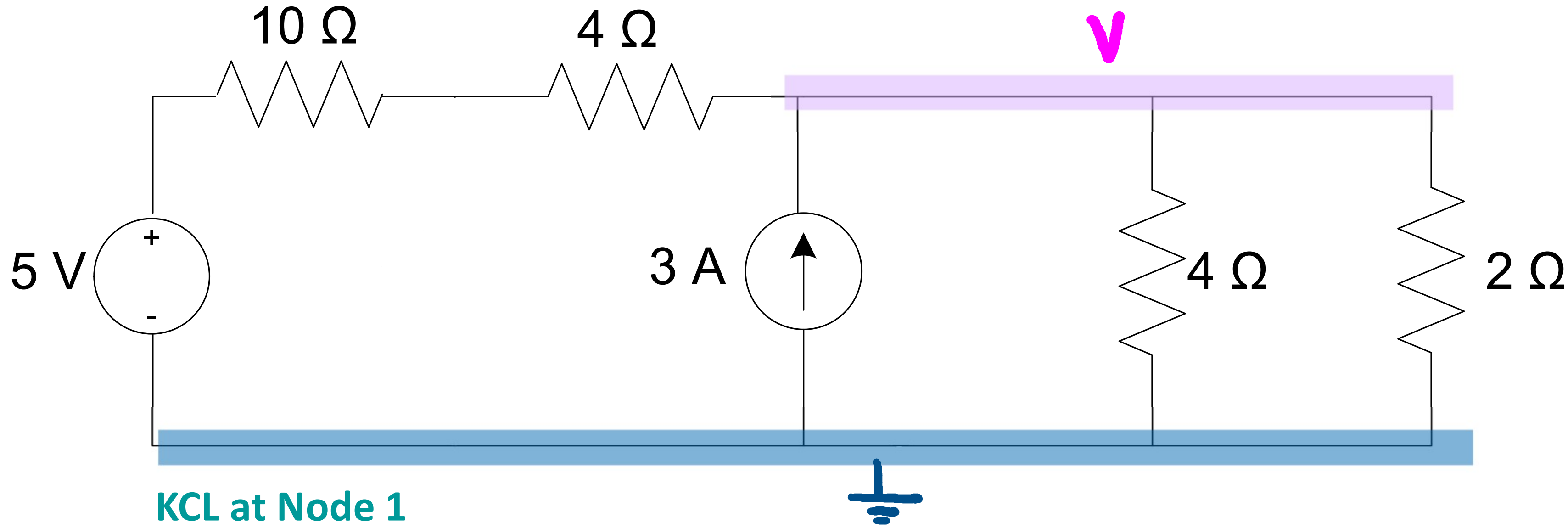


2 Choices either Nodal or Mesh analysis

1. If nodal analysis (2 Node)=2-1(Gnd)=1 node=1 KCL equation
2. If mesh analysis (2 loop)=2-1(Supermesh)+ 1 Supermesh equation
   =2 (1 KVL,1 Supermesh equation)

The different in between those 2, is time. (1 eq. vs 2 eq.)

Step 3:Find V_{th} (Nodal Analysis)



KCL at Node 1

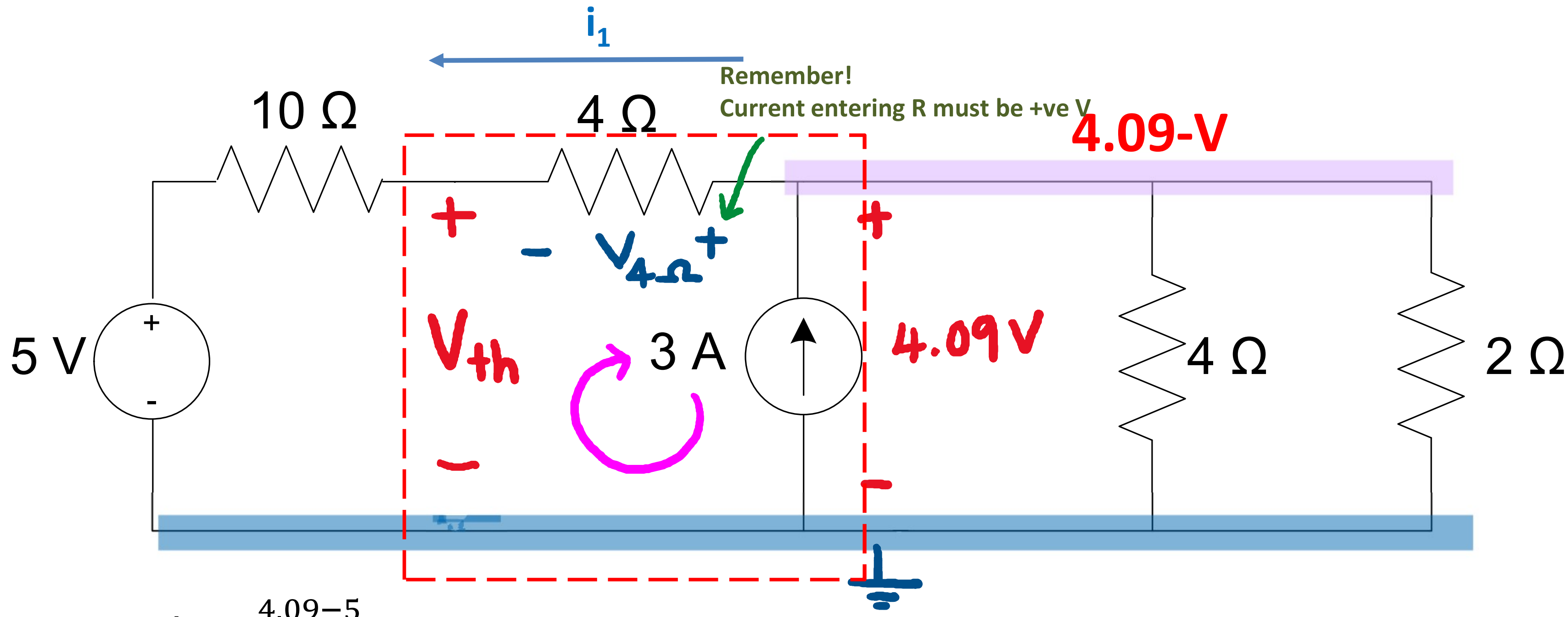
$$\frac{V-5}{14} - 3 + \frac{V}{4} + \frac{V}{2} = 0$$

$$V\left(\frac{1}{14} + \frac{1}{4} + \frac{1}{2}\right) = \frac{5}{14} + 3$$

$$V\left(\frac{23}{28}\right) = \frac{47}{14}$$

$$V = \frac{\left(\frac{47}{14}\right)}{\left(\frac{23}{28}\right)} = \frac{94}{23} = 4.09\ V$$

Step 3:Find V_{th}



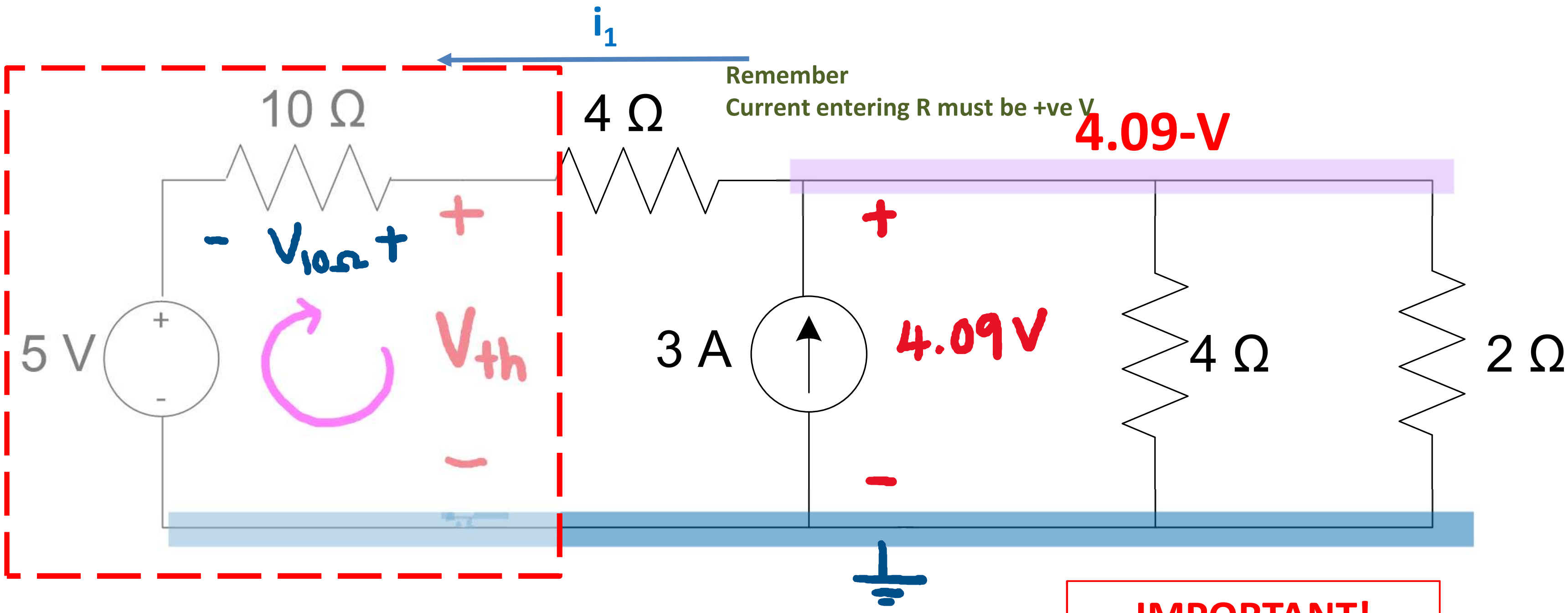
$i_1 = \frac{4.09-5}{10+4}$ =-0.065-V

$V_{4\Omega} = i_1(4) = -0.065(4) = -0.26\ V$

Right Loop: $-V_{th} - V_{4\Omega} + 4.09 = 0$

$$V_{th} = -V_{4\Omega} + 4.09$$
$$= 0.26 + 4.09$$
$$= 4.35\ V$$

Step 3:Find V_{th}



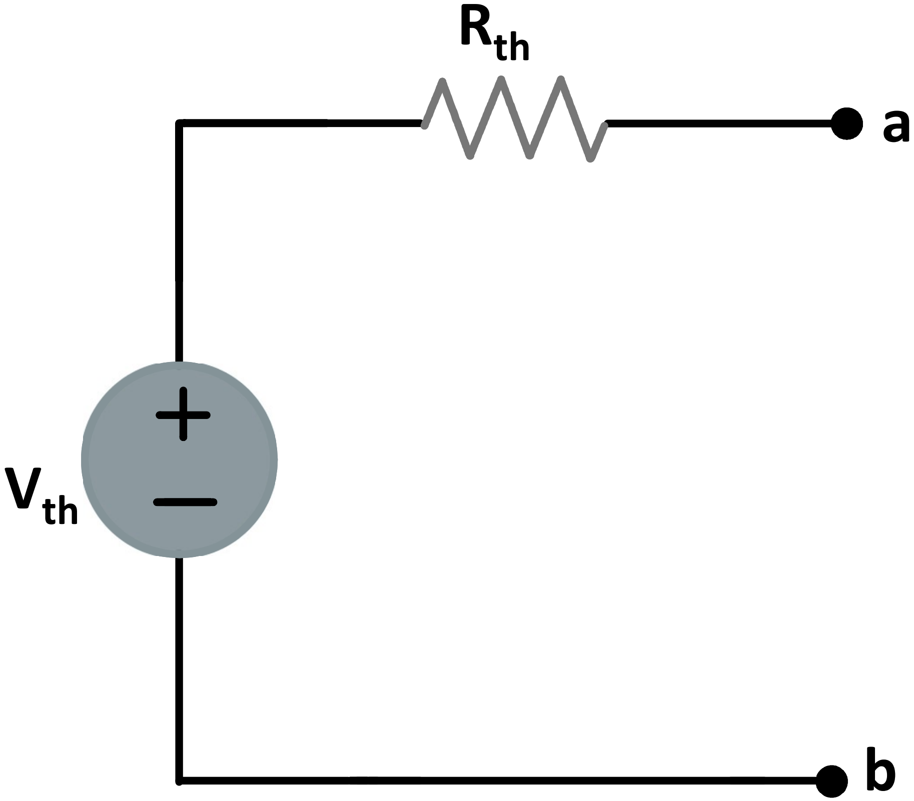
$$V_{10\Omega} = i_1(10) = -0.065(10) = -0.65V$$

Left Loop: $V_{th} - V_{10} - 5 = 0$

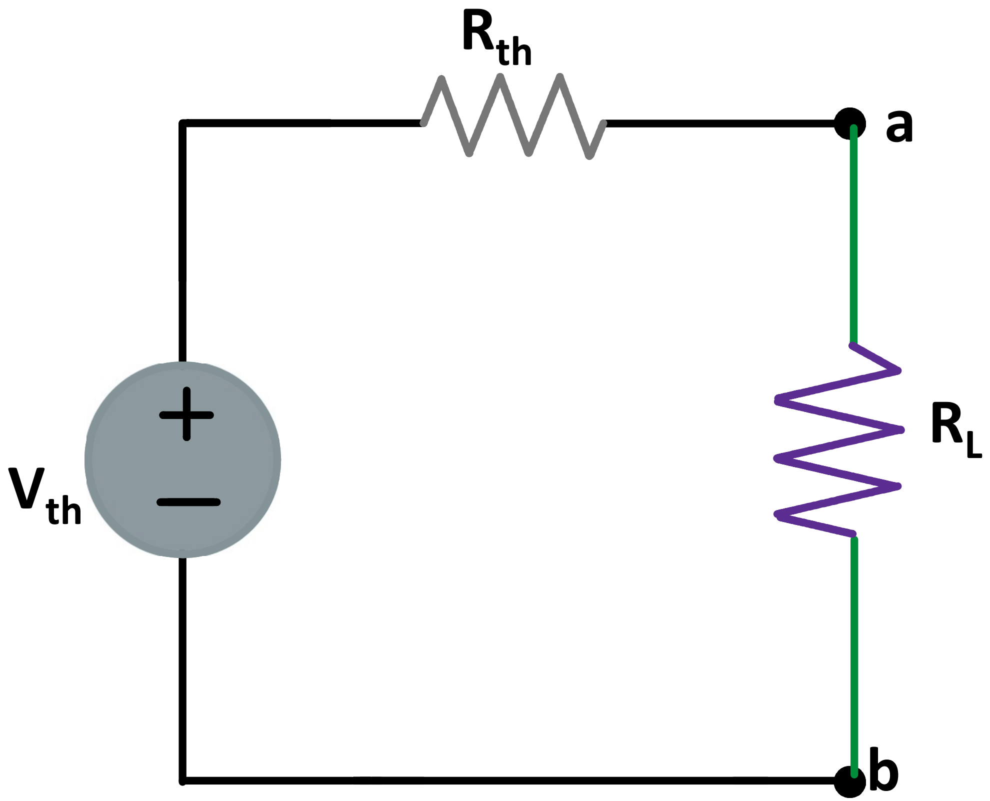
$$V_{th} = V_{10\Omega} + 5$$
$$= -0.65 + 5$$
$$= 4.35\ V$$

IMPORTANT!
You can choose either left or right loop. The answer will be the same.

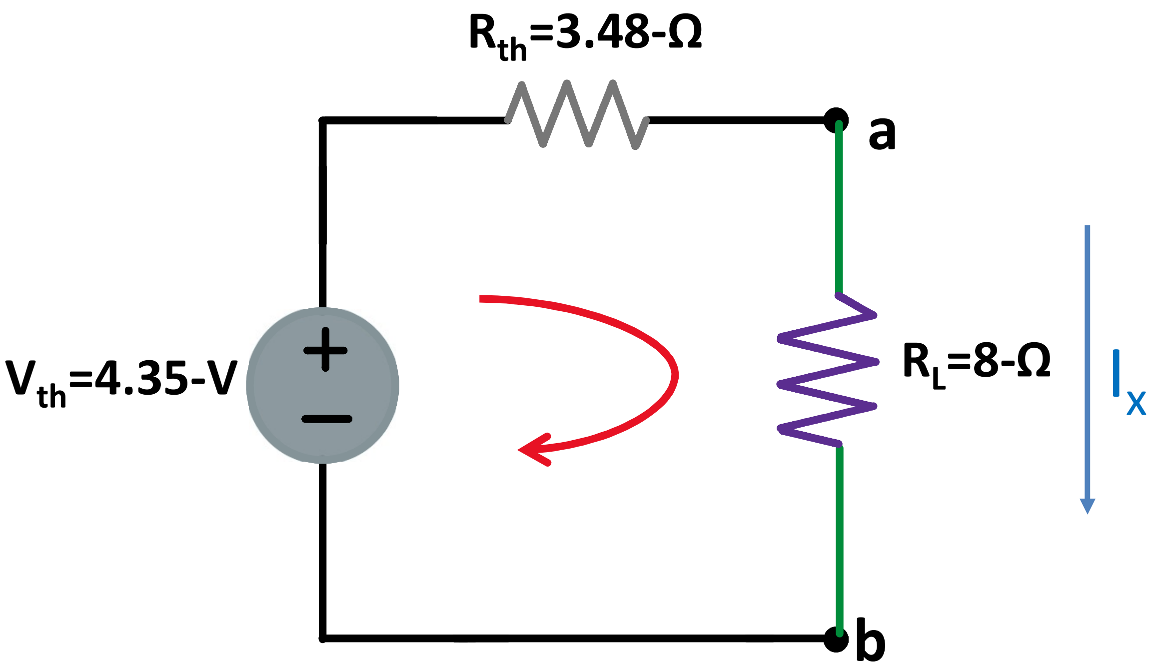
Step 4: Thevenin's Equivalent Circuit



Step 4: Insert the R_L into the Thevenin's Equivalent Circuit



Step 5: Solve



KVL

$$-V_{th} + I_x(R_{th} + R_L) = 0$$

$$I_x = \frac{V_{th}}{R_{th}+R_L}$$

$$= \frac{4.35}{3.48+8}$$

$$= 0.3789\ A$$

NORTON'S THEOREM

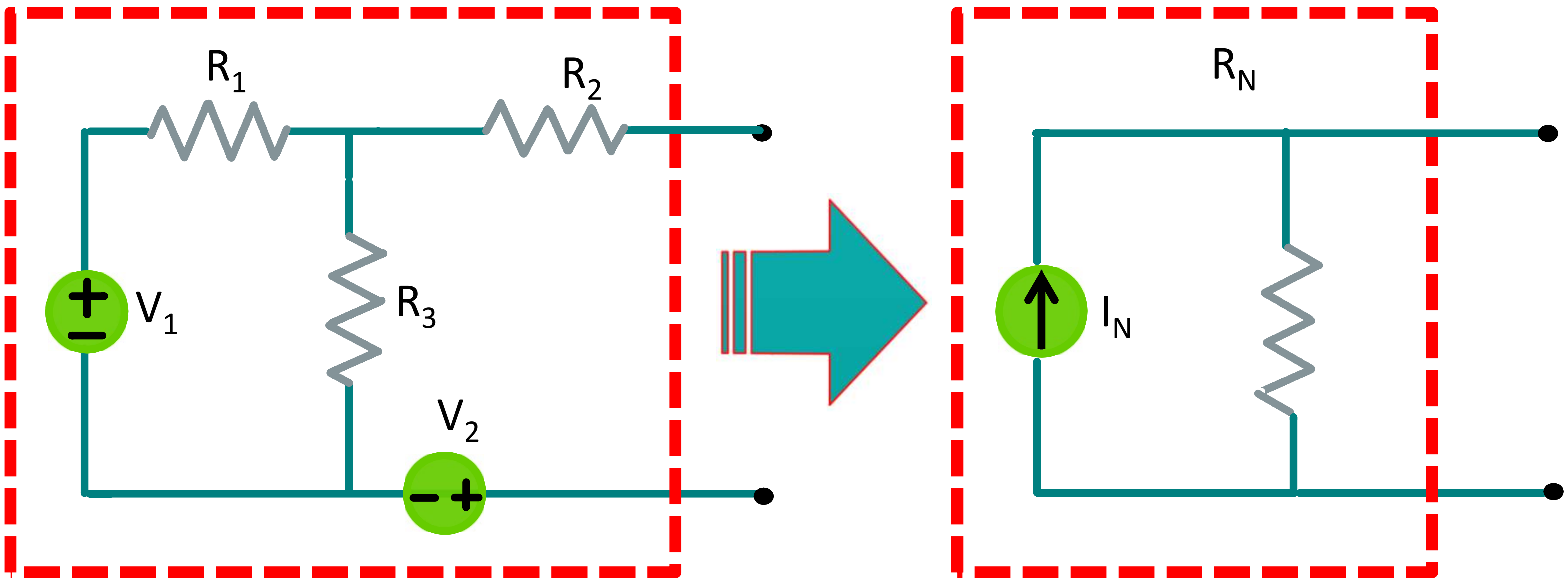
Norton's Theorem

- Purpose: Replace the whole circuit between the two terminals with an equivalent simple circuit (R_N and I_N)
- **A current source, I_N in PARALLEL with one resistance R_N**

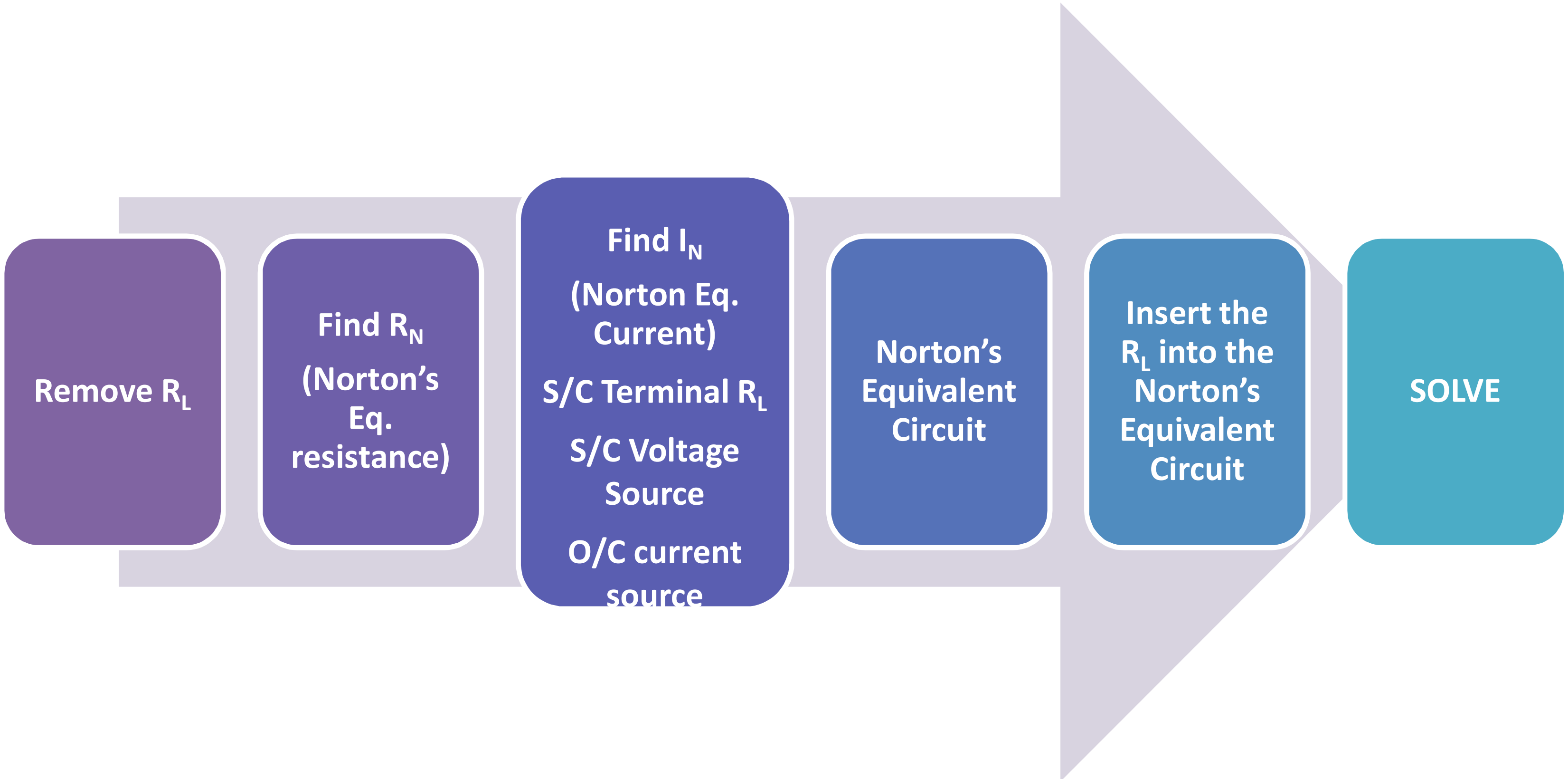
Norton's Theorem State that:

- *"Any two-terminal linear circuit can be replaced by an equivalent circuit consisting of current source in parallel with a single equivalent resistance."*

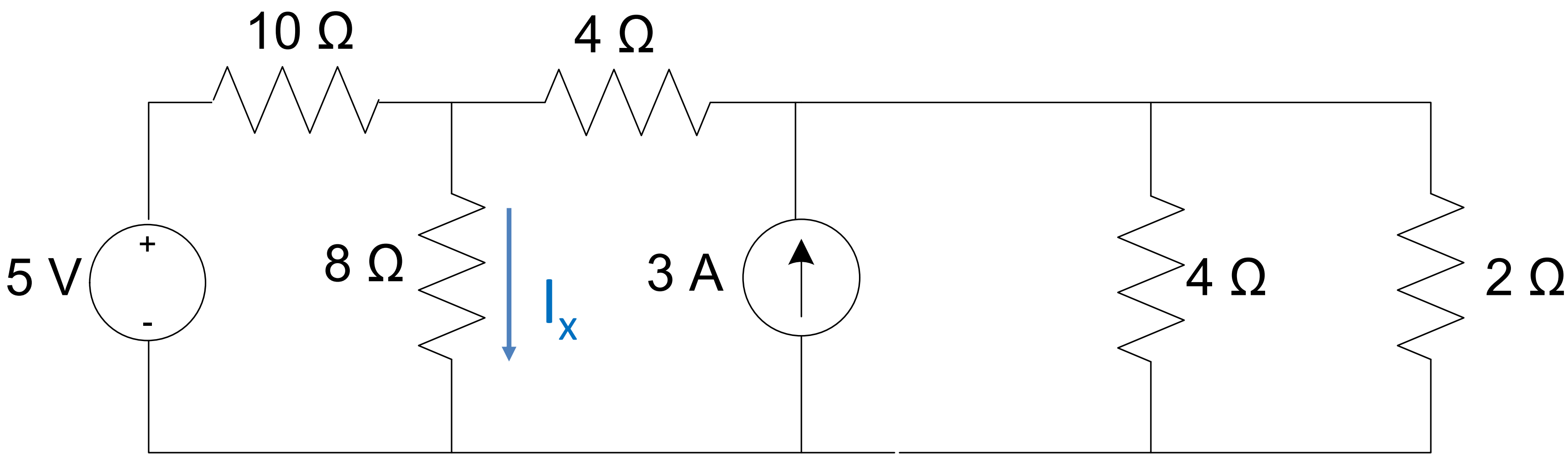
Equivalent Circuit



Process Flow

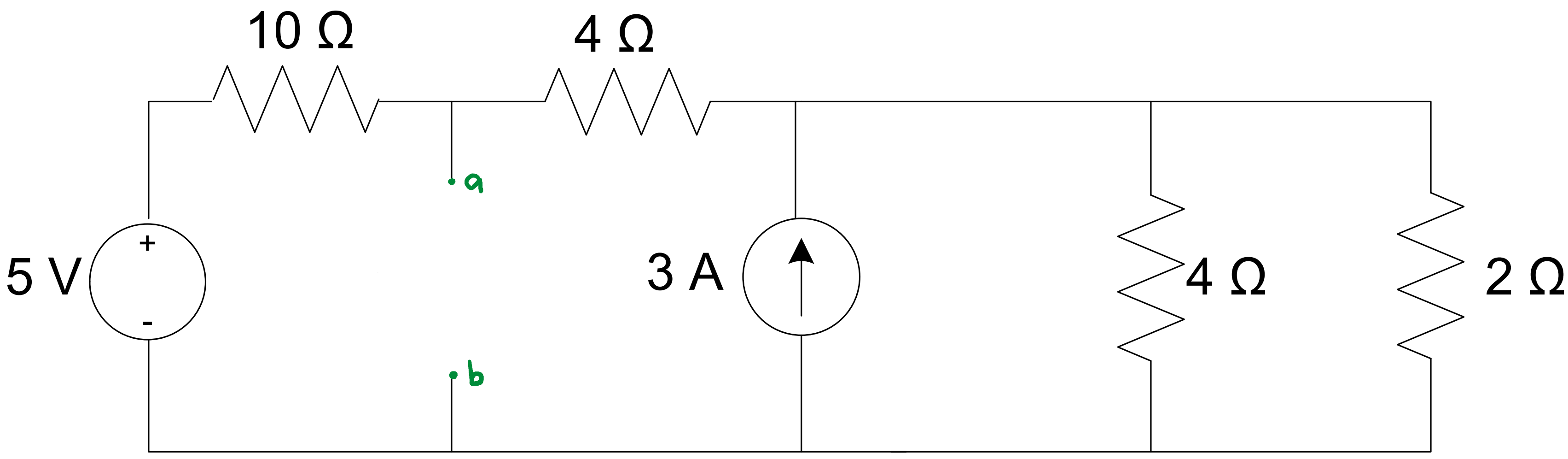


BASIC STEP

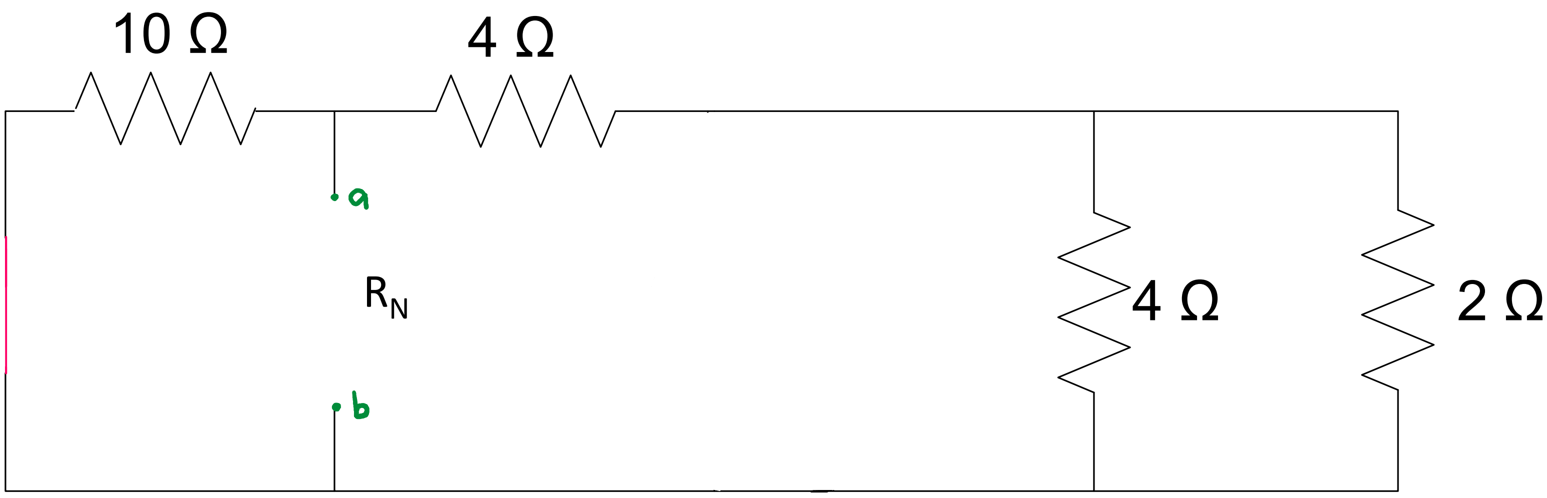


Assume that we are trying to find the current, I_x flow through 8-Ω.

Step 2: Remove R_L

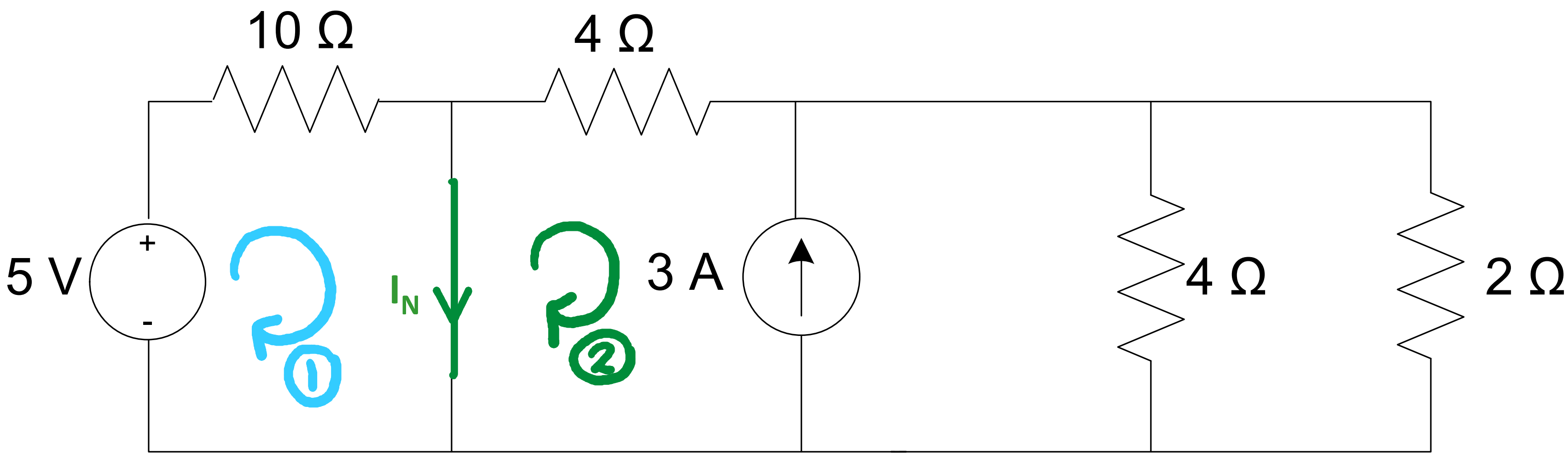


Step 3: Find R_N



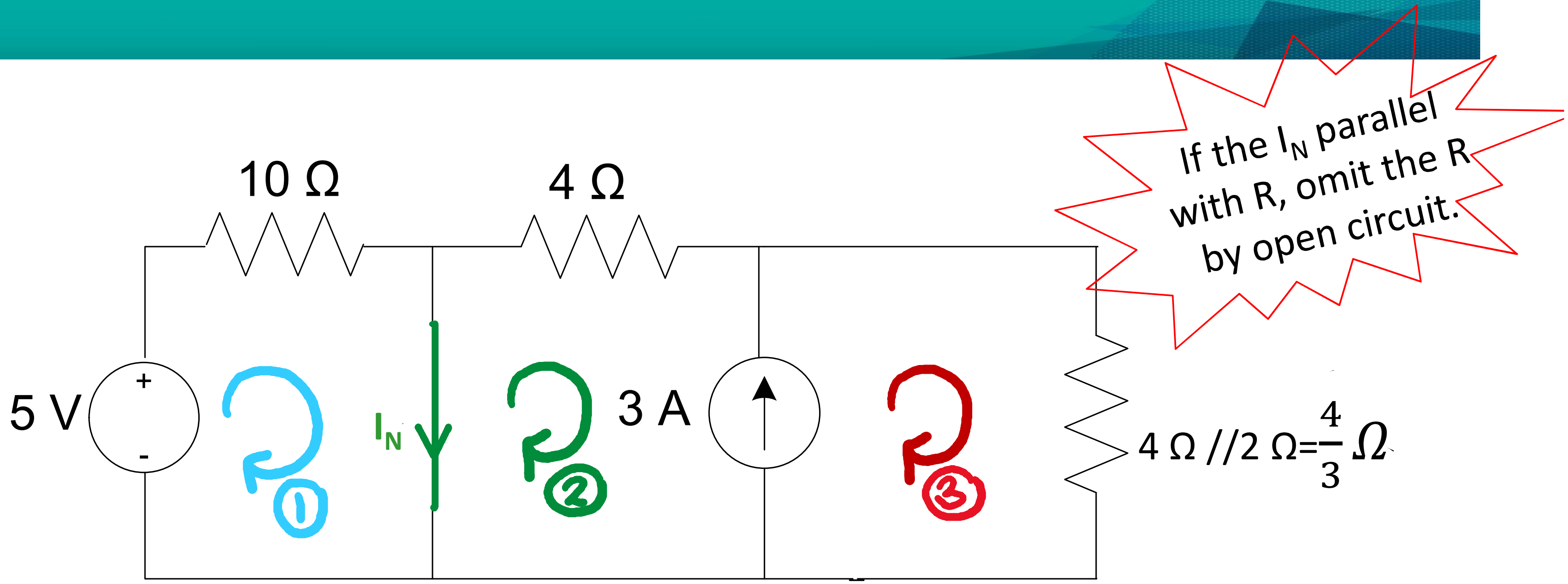
$R_N = R_{th}$ (refer previous note)

Step 3:Find I_N (Flow from terminal a to b)



There is ONLY 1 method to find I_N. → **Mesh Analysis**

Step 3:Find I_N (Flow from terminal a to b)



KVL at Loop 1

$-5 + I_1(10) = 0$

$I_1(10) = 5$

KVL at Loop 1 (Supermesh)

$I_2(4) + I_3(\frac{4}{3}) = 0$

Supermesh Equation

$I_3 - I_2 = 3$

Cramers

$$\begin{bmatrix} 10 & 0 & 0 \\ 0 & 4 & \frac{4}{3} \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$$

$I_1 = 0.5$ A,

$I_2 = -0.75$ A,

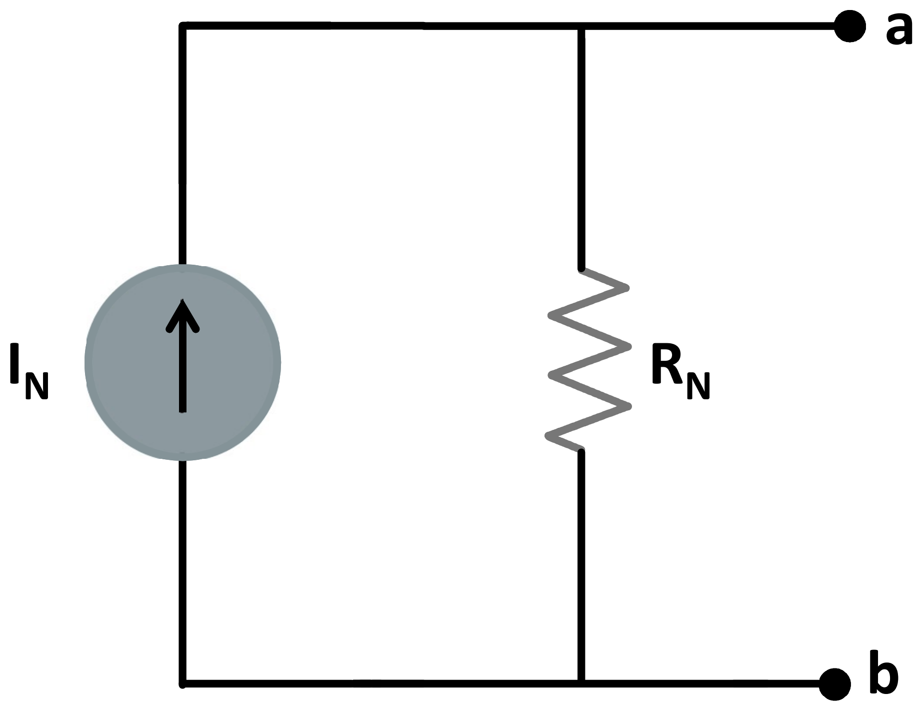
$I_3 = 2.25$ A

$I_N = I_1 - I_2$

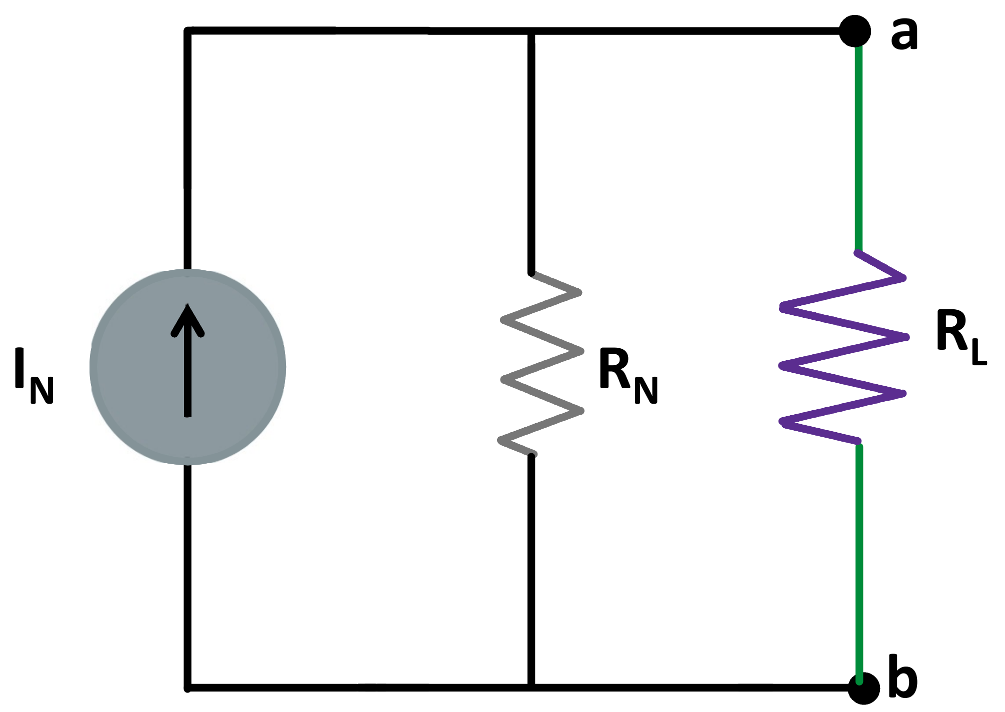
$= 0.5 - (-0.75)$

$= 1.25$ A

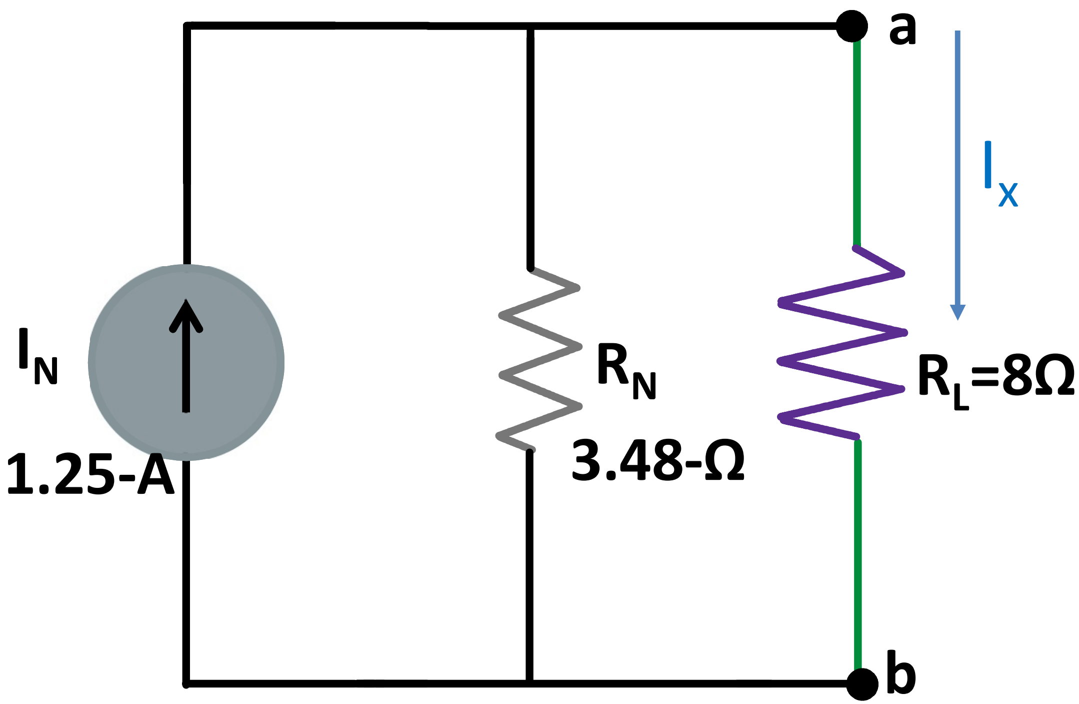
Step 4: Norton's Equivalent Circuit



Step 4: Insert the R_L into the Norton's Equivalent Circuit



Step 5: Solve

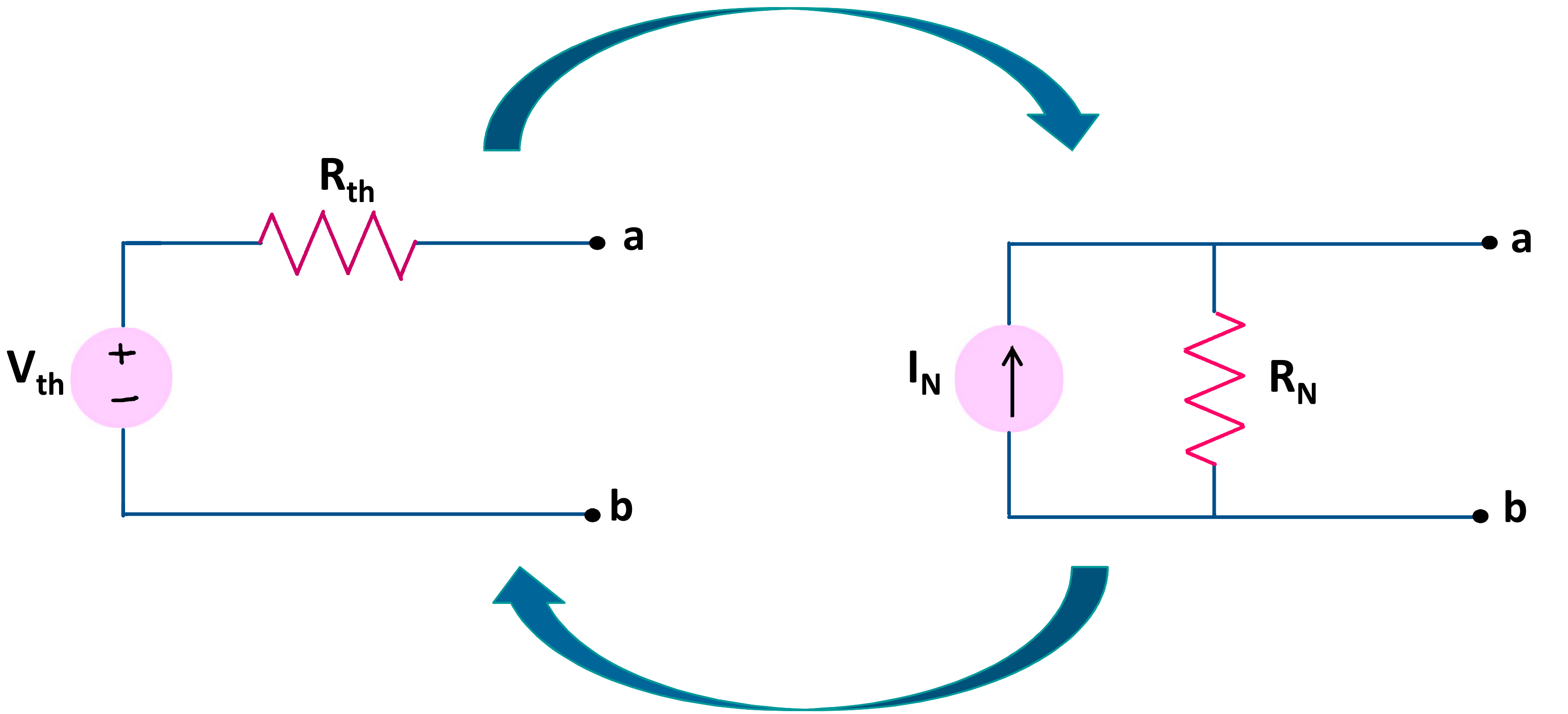


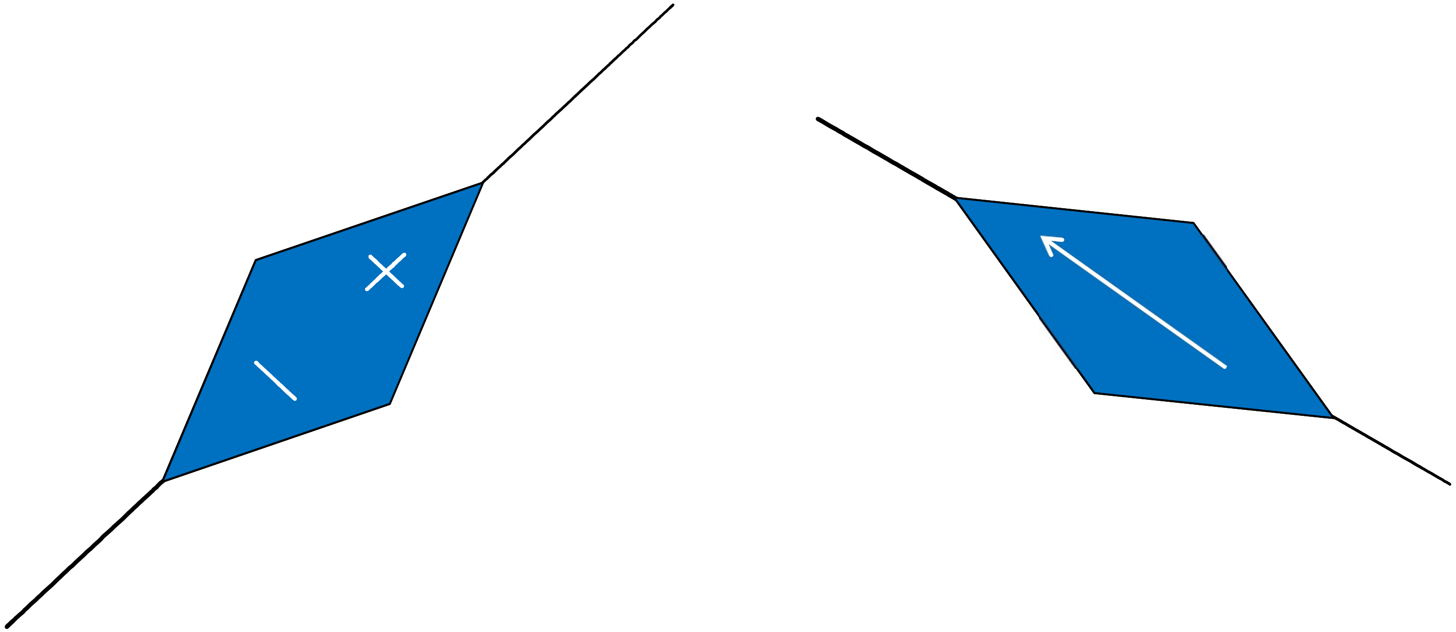
Using Current Divider

$$I_x = \frac{R_N}{R_N + R_L}(I_N)$$
$$= \frac{3.48}{3.48+8}(1.25)$$
$$= 0.3789\ A$$

The answer same as Thevenin's Theorem

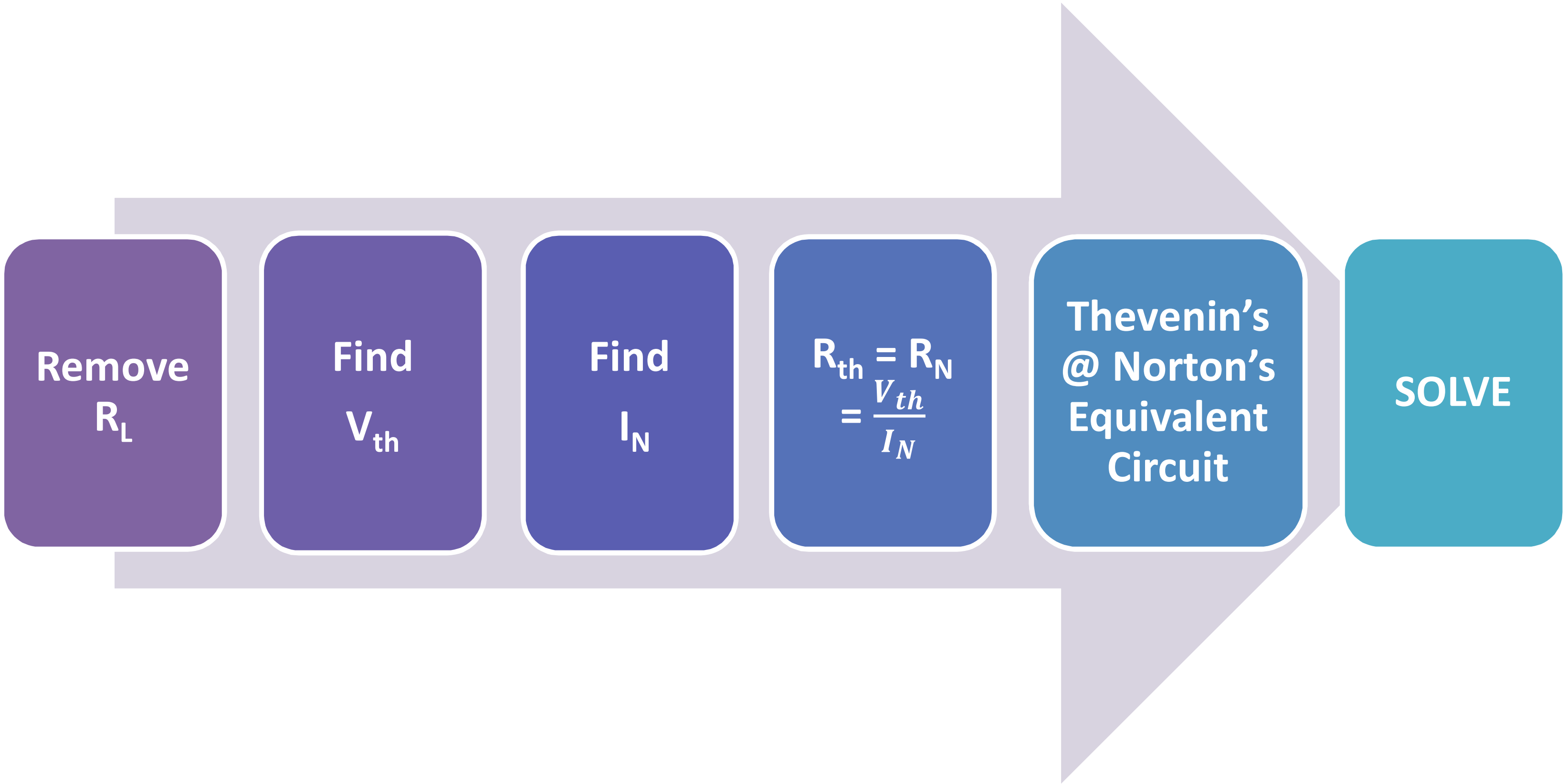
Relationship in between Thevenin and Norton





WHAT IF THE CIRCUIT CONSIST OF DEPENDENT SOURCE?

Process Flow



Maximum Power Transfer

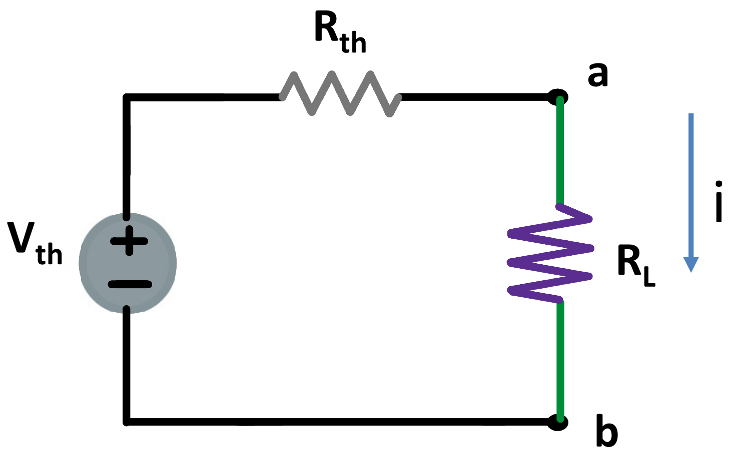
WHY?

- the efficiency of the power transfer
- the amount of power transfer.

Maximum power transfer states:

"A load will receive maximum power from a linear bilateral dc network, when Its total resistive value is equal to the Thevenin resistance of the network seen by the load."

Maximum Power Transfer



Power for R_L

$$P = i^2 R = \left(\frac{V_{th}}{R_{th} + R_L} \right)^2 .R_L$$

V_{th} and R_{th} will be fixed. Therefore the power dissipated will be the R_L

$$R_L = R_{th}$$

$$P_{\max} = \frac{{V_{th}}^2 R_L}{(2R_L)^2} = \frac{{V_{th}}^2}{4R_{th}}$$

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