BEE1133 Circuit Analysis

Chapter 2A
Methods of Analysis (DC Circuits)

by

Nor Rul Hasma Abdullah
Faculty of Electrical & Electronics Engineering
hasma@ump.edu.my

Chapter Description

Aims

This chapter is aimed to:

1. Explain the Nodal Analysis technique in solving problem related to electric circuit

Expected Outcomes

Student should be able to

1. Identify the essential node
2. Identify the supernode in the circuit
3. Determine the equation of ohm’s law
4. Determine the group of equation for each node for solving the electric circuit problem.

References

1. C. Alexander and M. Sadiku, “Fundamentals of Electric Circuits”, 4th ed., McGraw-Hill, 2008.
2. J. Nilsson and S. Riedel, “Electric Circuits”, 8th ed., Prentice Hall, 2008.

Basic Concept

4.1 Nodal Analysis

4.2 Nodal Analysis with voltage source: Supernode

Nodal Analysis

- ☐ Finds the node voltages by performing KCL at the essential nodes.
- ☐ KCL: Summation of current in nodes equal to zero
- ☐ KCL is performed with the current going out of the node as positive (i.e. currents going out are added, going in are subtracted)

Remember!

Please understand on how to write the equation of current, I for each branch

How to write the equation for I?

☐ The branch must consist of resistor, **R** (*Ohm’s Law)

☐ If the branch do not consist of R, then do the KCL for others branch.

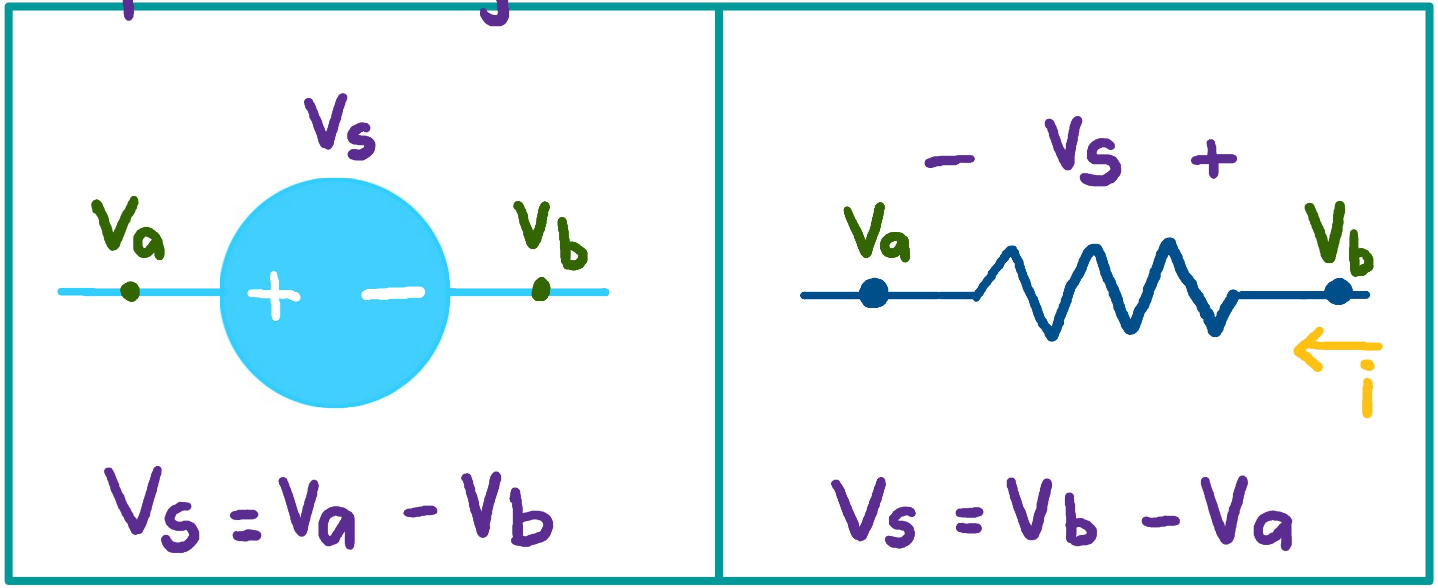
☐ Know the **VOLTAGE** for the resistor, R either value or symbol

Reminder!

Finding the voltage

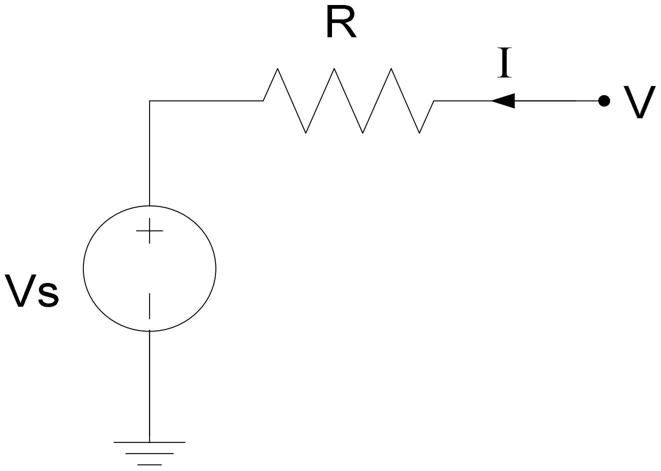
Current entering the resistor, the polarity setting is always positive

Tips : Voltage for element ($V_{at\ +ve} - V_{at\ -ve}$)

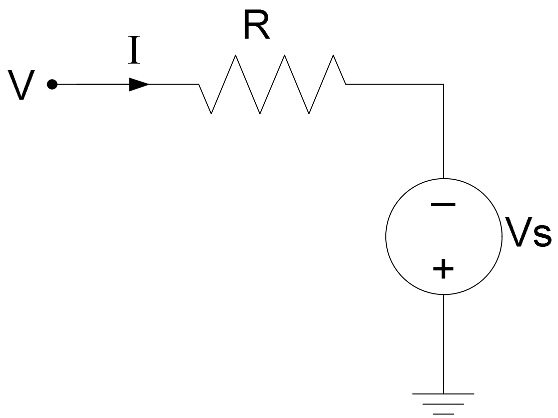


$$V_s = V_a - V_b$$

$$V_s = V_b - V_a$$



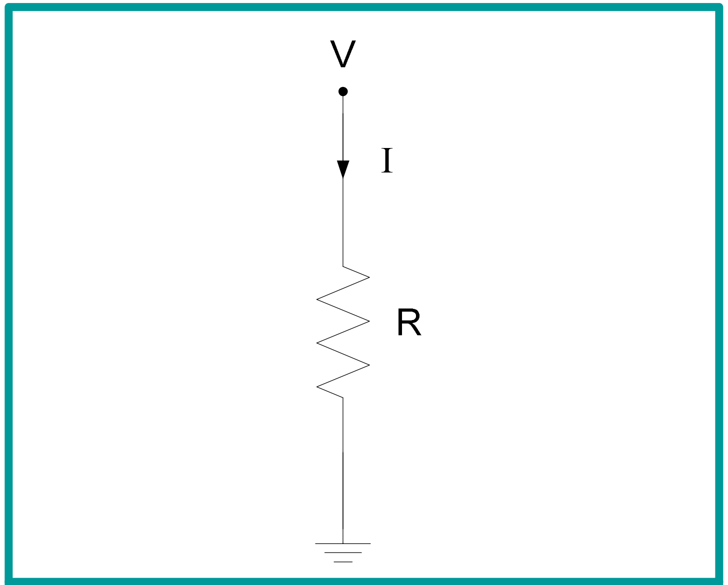
$$I = \frac{V - V_s}{R}$$



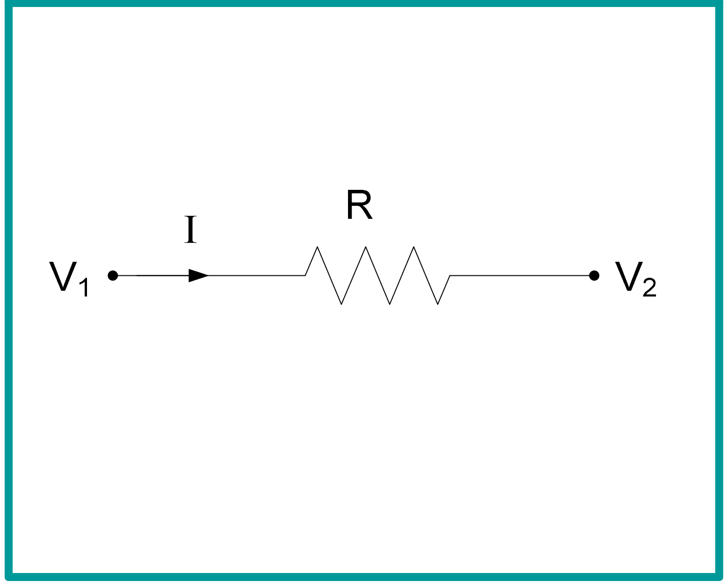
$$I = \frac{V - (-V_s)}{R}$$

$$= \frac{V + V_s}{R}$$

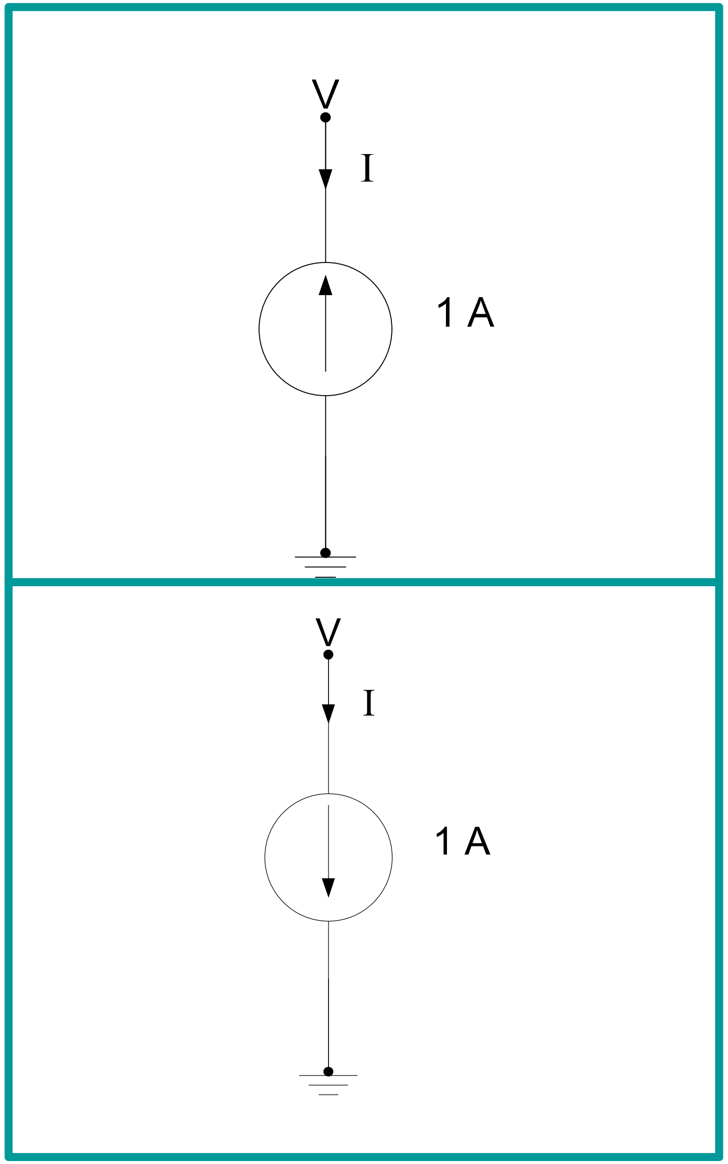
FINDING THE CURRENT FOR EACH BRANCHES



$$I = \frac{V - 0}{R} = \frac{V}{R}$$

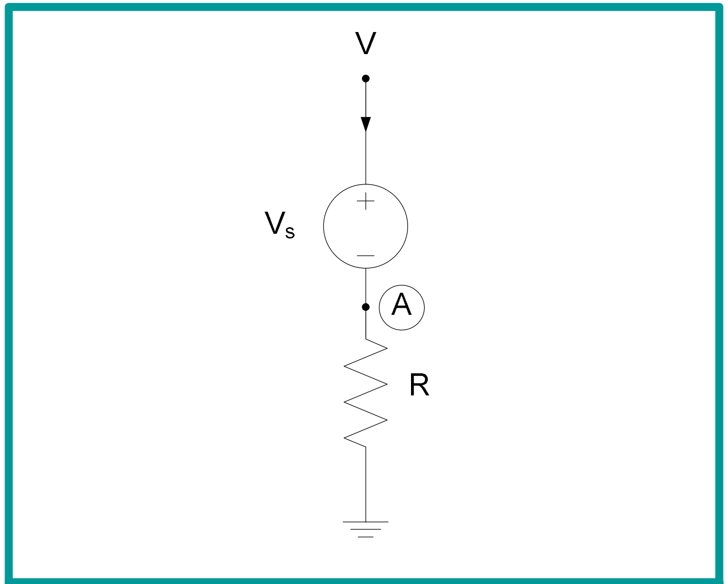


$$I = \frac{V_1 - V_2}{R}$$



$$I = -1\ A$$

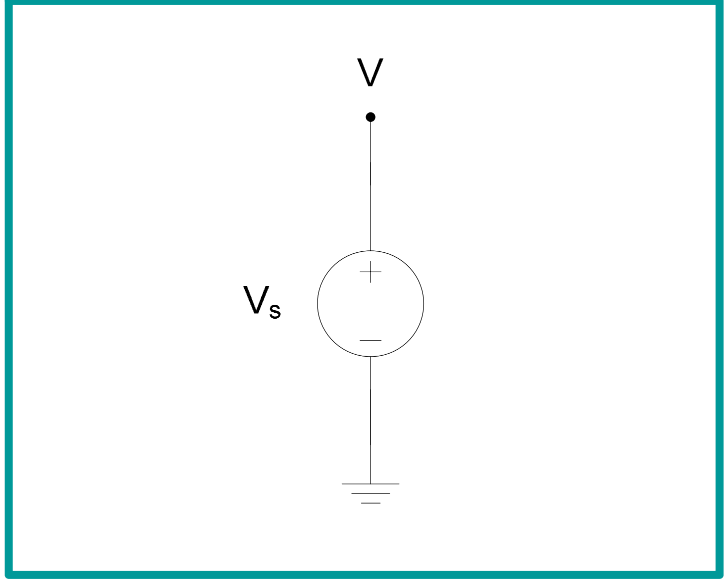
$$I = 1\ A$$



$$V - A = V_s$$

$$A = V - V_s$$

$$I = \frac{A - 0}{R} = \frac{V - V_s}{R}$$

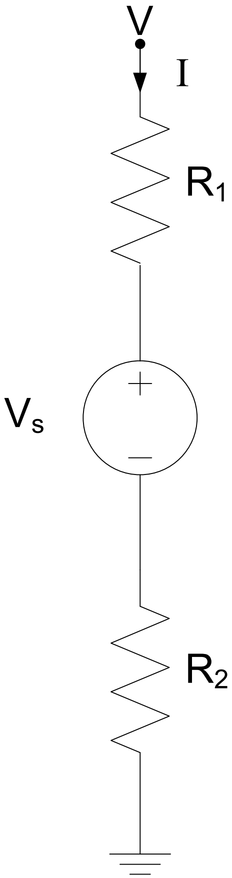


$$V = V_s$$

If the branch consist of this type of combination, DO NOT do KCL at this node. Why?
Because the node voltage is already given.

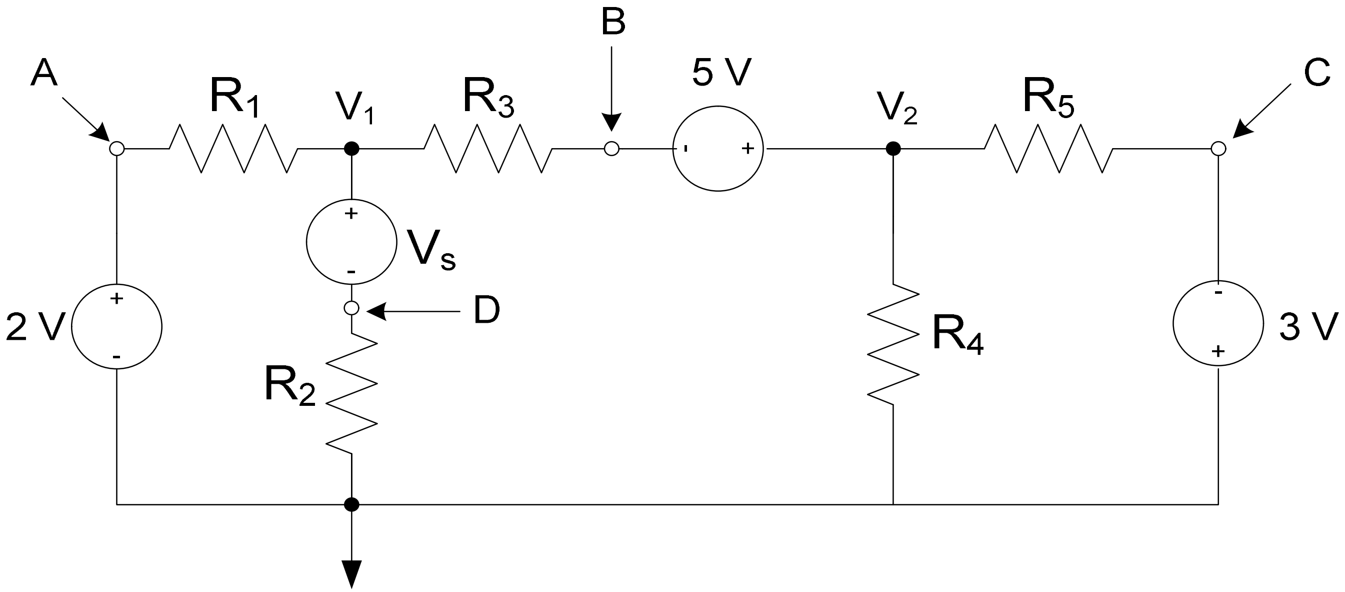
Remember!

The objective is to find the node voltage.

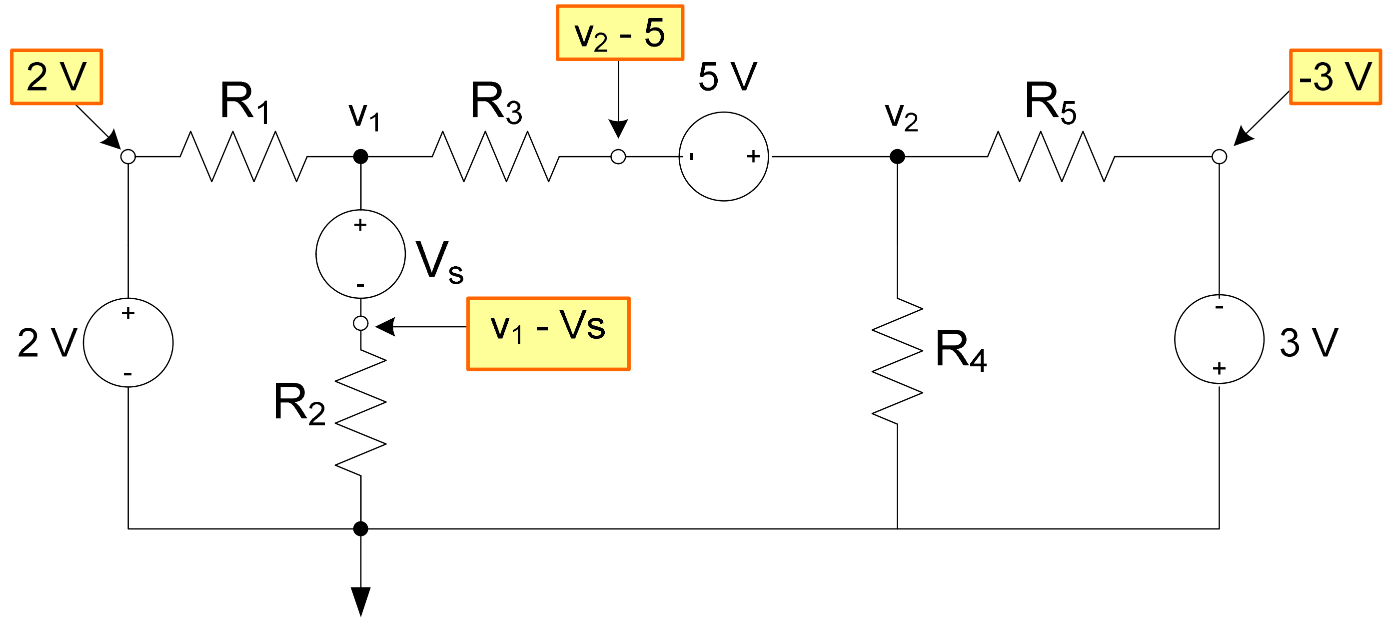


$$I = \frac{V - V_s}{R_1 + R_2}$$

Test your understanding



Answer



General Step For Using Nodal Analysis

Step 1

Mark the essential node

Step 2

Reference Node

Choose the essential node that joins the most number of elements

Step 3

Assign unknown node voltage

Step 4

Decide on number of equations required

*1 Node=1 KCL

* 1 Supernode= 1 KCL (combine 2 loop)+Supernode Equation

*1 Dependent=1 Constraint Eq.

Note: If the branch do not consist R, no need to do KCL.

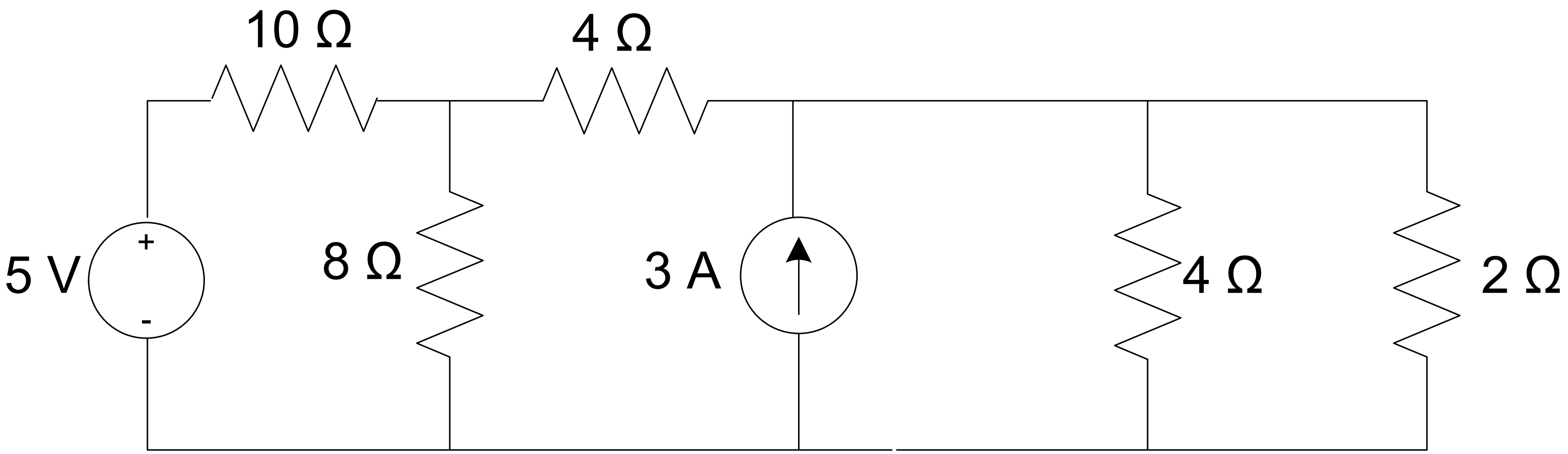
Step 5

Perform KCL at the selected nodes

Step 5

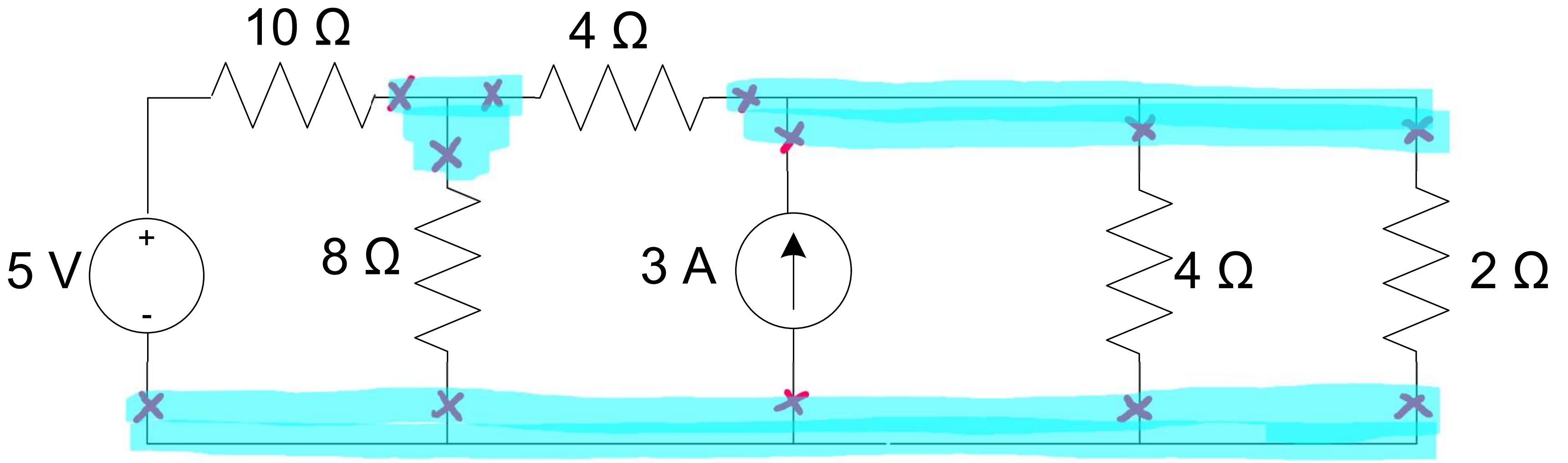
Calculate the node voltage using Cramer/Calculator

BASIC STEP



Assume that we are trying to find the voltage across and the current through all the elements

Step 1: Mark essential node

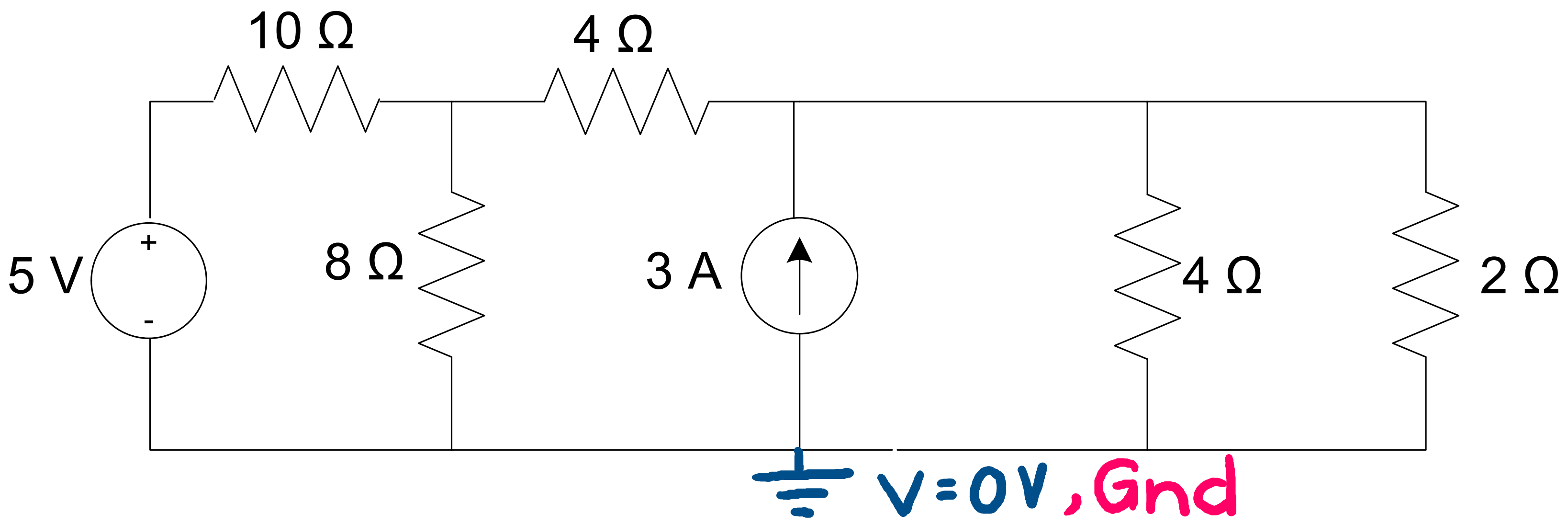


3 Essential Nodes

1 (Gnd)

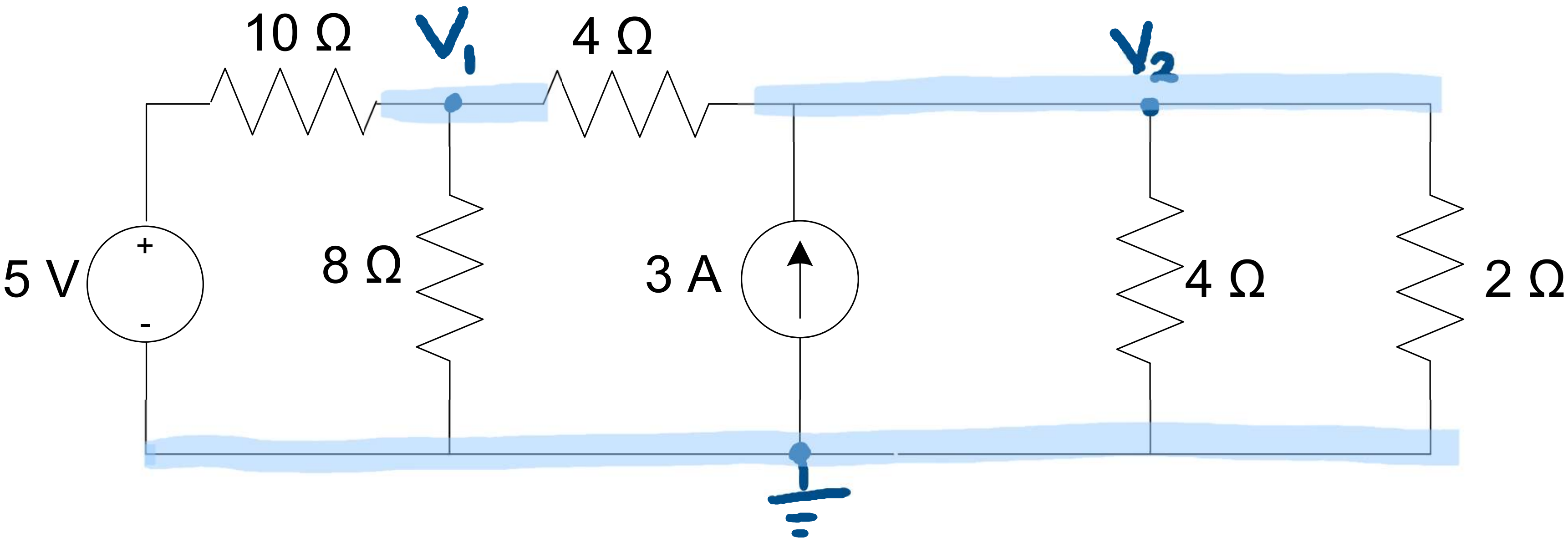
2(Node Voltage, V_1 & V_2)

Step 2: Reference Node



- ☐ Mark the reference node with the earth sign.
- ☐ A reference node is the node from where all the other node voltages the node that is considered to be at 0 V.

Step 3: Assign unknown node voltages



Methods of Analysis by N.R.H. Abdullah
http://ocw.ump.edu.my/course/view.php?id=251

Step 4: Decide on number of equations required

☐ Decide on the number of equations required to solve the circuit.

☐ Referring to the example, there are 2 unknowns (i.e. V_1 and V_2).

Step 5: Perform KCL at the selected nodes

KCL is performed with the current going out of the node as positive. **Assume ALL the current exit the node.**

KCL: Node 1:

$$\frac{v_1 - 5}{10} + \frac{v_1 - v_2}{4} + \frac{v_1}{8} = 0$$ ——①

KCL: node 2:

$$\frac{v_2 - v_1}{4} - 3 + \frac{v_2}{4} + \frac{v_2}{2} = 0$$ ——②

Step 6: Solve the equations

Solving the simultaneous equation by applying Cramer's Rule or using calculator

From 1

$$\frac{v_1-5}{10}+\frac{v_1-v_2}{4}+\frac{v_1}{8}=0$$

$$v_1\left(\frac{1}{10}+\frac{1}{4}+\frac{1}{8}\right)+v_2\left(-\frac{1}{4}\right)=\frac{5}{10}$$

$$v_1\left(\frac{19}{40}\right)+v_2\left(-\frac{1}{4}\right)=\frac{1}{2}$$

From 2

$$\frac{v_2-v_1}{4}-3+\frac{v_2}{4}+\frac{v_2}{2}=0$$

$$v_2\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{2}\right)+v_1\left(-\frac{1}{4}\right)=3$$

$$v_2(1)+v_1\left(-\frac{1}{4}\right)=3$$

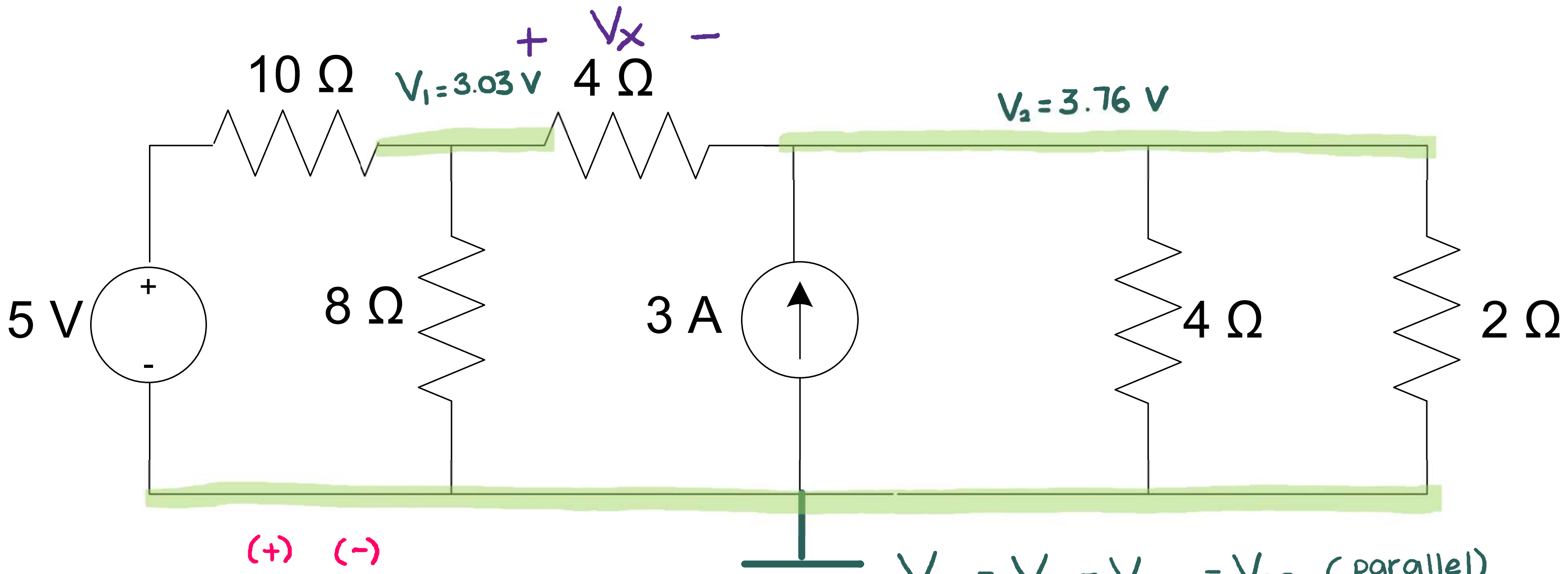
Cramers

$$\begin{bmatrix}\frac{19}{40} & -\frac{1}{4}\\ -\frac{1}{4} & 1\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\ 3\end{bmatrix}$$

$$\left.\begin{aligned}v_1&=\frac{100}{33}=3.03V\\ v_2&=\frac{124}{33}=3.76V\end{aligned}\right\}$$

Use calculator

Answer



$$V_x = \overset{(+)}{V_1} - \overset{(-)}{V_2}$$

$V_2 = V_{3A} = V_{4\Omega} = V_{2\Omega}$ (parallel)

$V_1 = V_{8\Omega}$

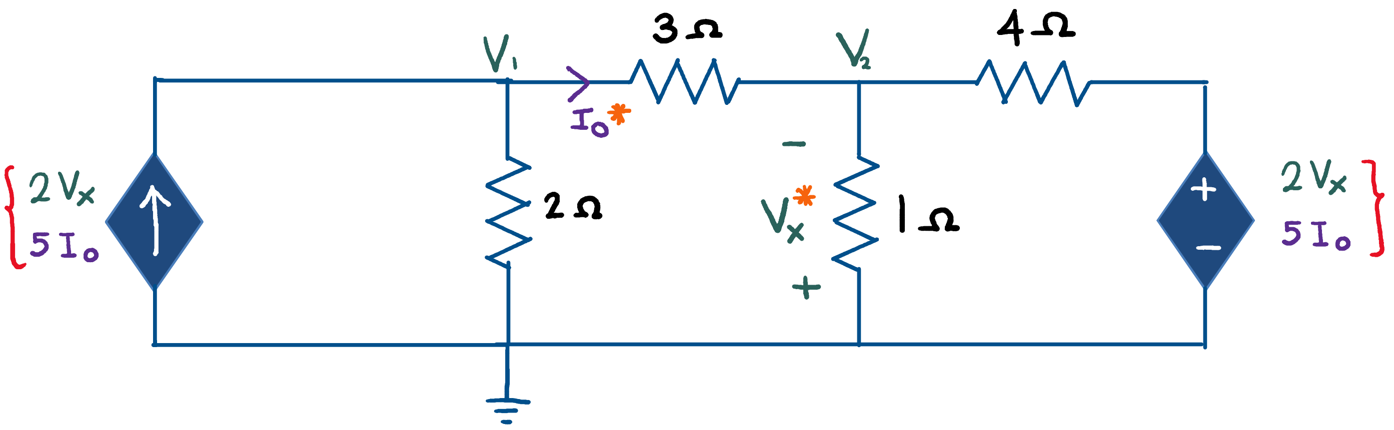
In order to know the value of the current in a branch, use the voltage at R and divide it with R. If the branch do not consist R, use KCL by using other branches.

Circuits With Dependent Sources

☐ If a dependent source is present in the circuit, a constraint equation imposed by the presence of the dependent source.

☐ The constraint equation is an equation describing the dependent term (of the dependent source) in terms of node voltages or values.

Circuits With Dependent Sources



Dependent current source (A)

Dependent voltage source (V)

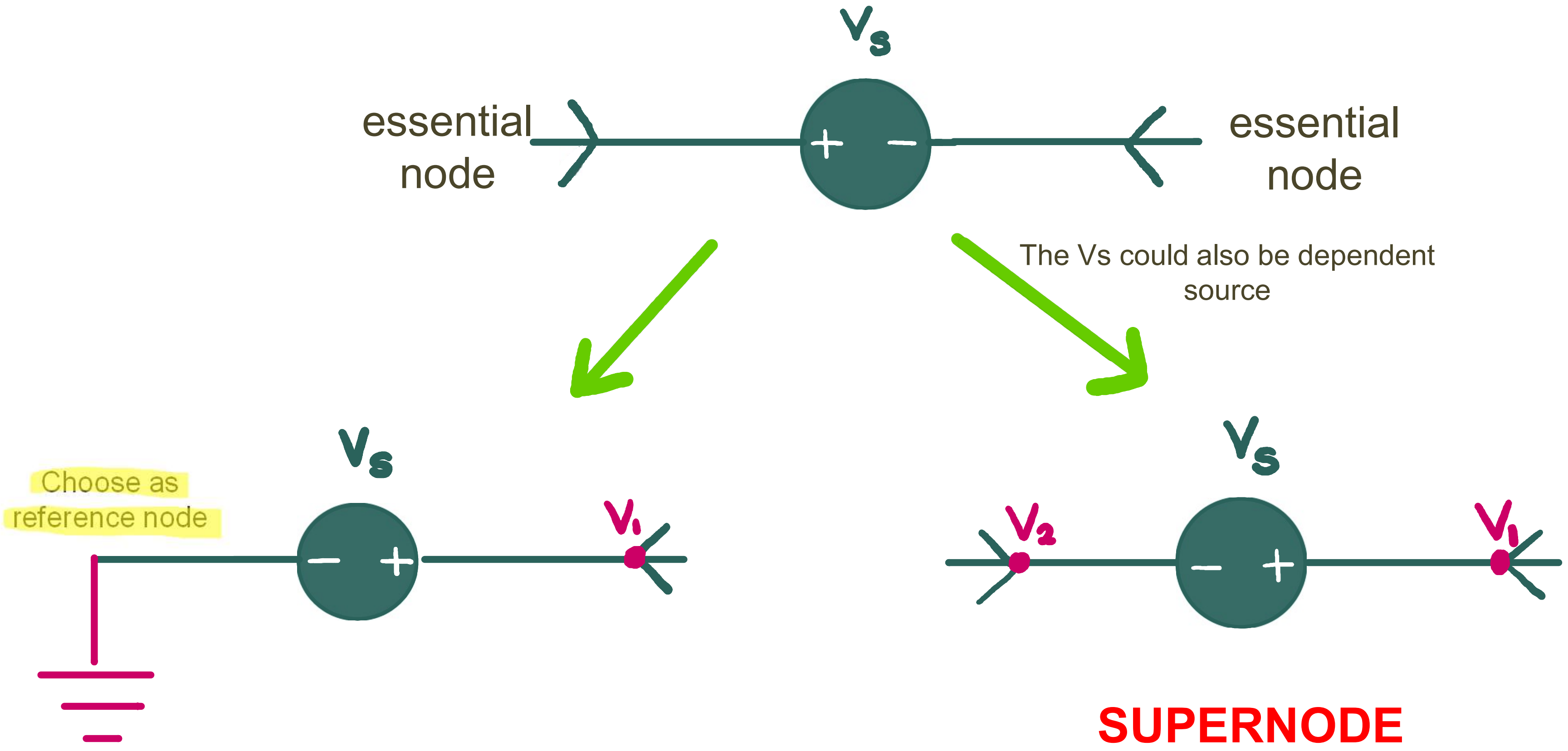
Constraint Equation,*

$$\boxed{\begin{array}{l} I_o = \dfrac{V_1 - V_2}{5} \\ V_x = -V_2 \end{array}}$$

This equation is required together with the KCL equations

Methods of Analysis by N.R.H. Abdullah
http://ocw.ump.edu.my/course/view.php?id=251

Circuits With Voltage Sources



essential node

essential node

The Vs could also be dependent source

Choose as reference node

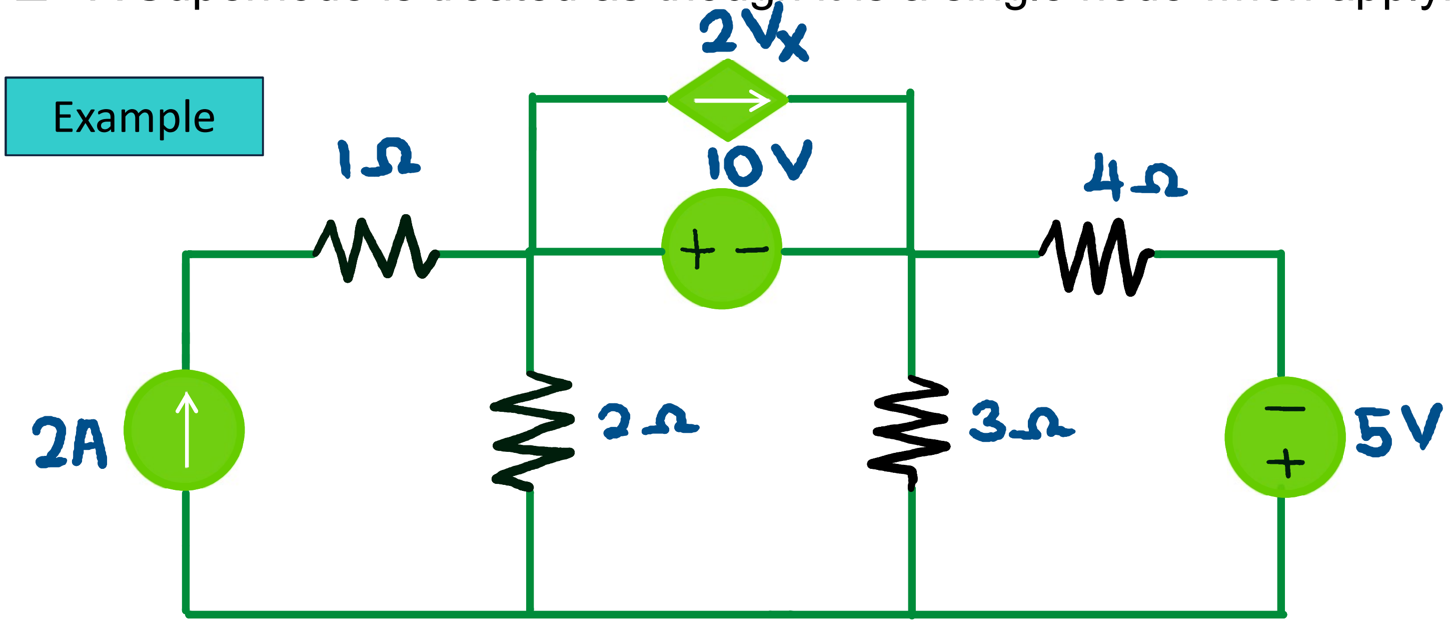
SUPERNODE

Methods of Analysis by N.R.H. Abdullah
http://ocw.ump.edu.my/course/view.php?id=251

SUPERNODE

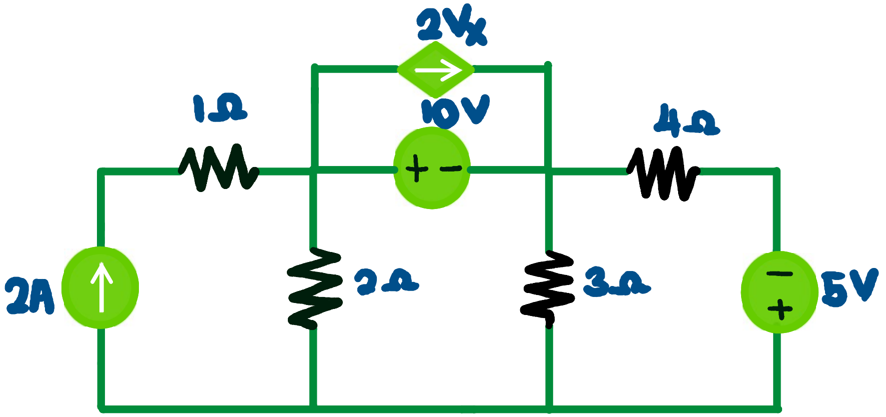
- ☐ Group the 2 nodes that enclose the voltage source to form a SUPERNODE.
- ☐ A Supernode is treated as though it is a single node when applying KCL.

Example



Where is supernode?
Why?

SUPERNODE



1. How many essential node?
2. Choose 1 node as ground.
3. How many node left?
4. Assign Node Voltage by V_1, V_2,.., V_n.
5. Is any Supernode exist? If YES, group the nodes.
6. Do the KCL equations.

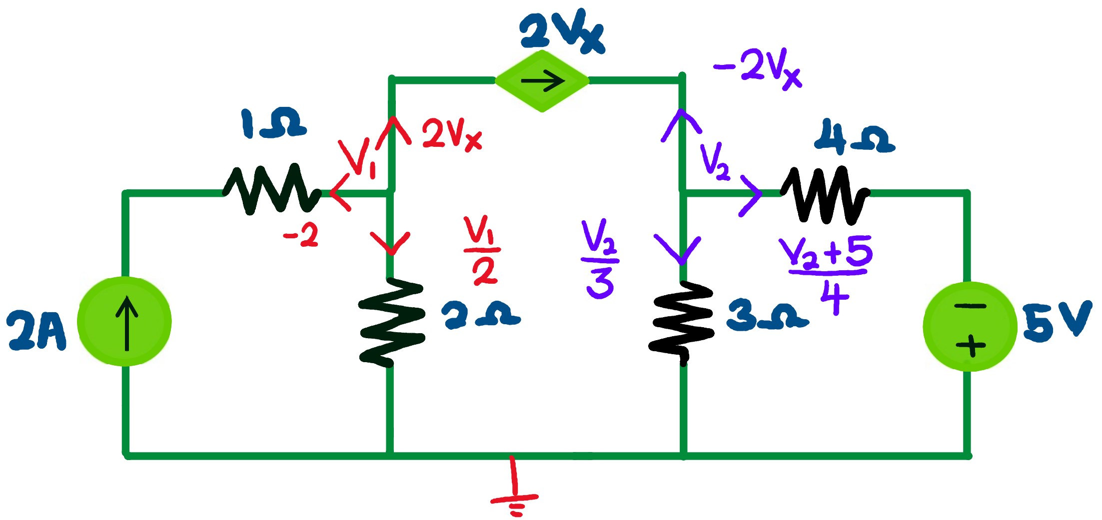
SUPERNODE → KCL at supernode
SUPERNODE → Supernode Equation

combine 2 node

Example

Total Node = 3
Gnd = 3 − 1 (Gnd) = 2
Supernode = 2 − 1 (Supernode) + 1 (supernode eq)
= 2

→ KCL at Supernode
→ supernode equation



① Assign ALL node voltage : Gnd, V_1, V_2

② Refer previous slide, ① KCL at Supernode

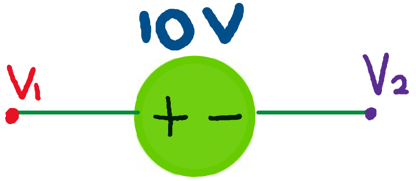
ⓐ Group Node V_1 & V_2 as 1 KCL

ⓑ Assume ALL current exit the node

ⓒ Do the KCL

$$-2+2V_x+\frac{V_1}{2}+\frac{V_2}{3}+\frac{V_2+5}{4}-2V_x=0$$

③ Supernode Eq.



What is 10V?

answer :

$$V_1-V_2=10$$

Author Information

Nor Rul Hasma Abdullah (Ph. D)
Senior Lecturer
Email:
hasma@ump.edu.my
Google Scholar:
Nor Rul Hasma
Scopus ID :
35791718100