Universiti Malaysia PAHANG

Engineering • Technology • Creativity

FACULTY OF ELECTRICAL & ELECTRONICS ENGINEERING

FINAL EXAMINATION

COURSE	:	**SIGNALS AND NETWORKS**
COURSE CODE	:	**BEE2143**
LECTURER	:	**FARADILA NAIM** **NURUL WAHIDAH ARSHAD** **ASSOC. PROF. DR. ZUWAIRIE IBRAHIM**
DATE	:	**5 JANUARY 2015**
DURATION	:	**3 HOURS**
SESSION/SEMESTER	:	**SESSION 2014/2015 SEMESTER I**
PROGRAMME CODE	:	**BEC/BEE/BEP**

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of **FIVE (5)** questions. Answer **ALL** questions in **PART A** and **TWO** questions for **PART B**.
2. All answers to a new question should start on new page.
3. All the calculation answer and assumptions must be clearly stated.
4. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

EXAMINATION REQUIREMENTS

1. **APPENDIX I** – Table of Formula
2. Semilog Graph Paper

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **FOURTEEN (14)** printed pages including front page.

PART A (Answer ALL questions)

QUESTION 1

(a) For each pair of functions in Figure 1, provide the values of the constants A, t_0 and a. The relationship between $h_1(t)$ and $h_2(t)$ is $h_2(t) = A\, h_1((t-t_0)/a)$.

[9 Marks]

[CO1, PO1, C2]

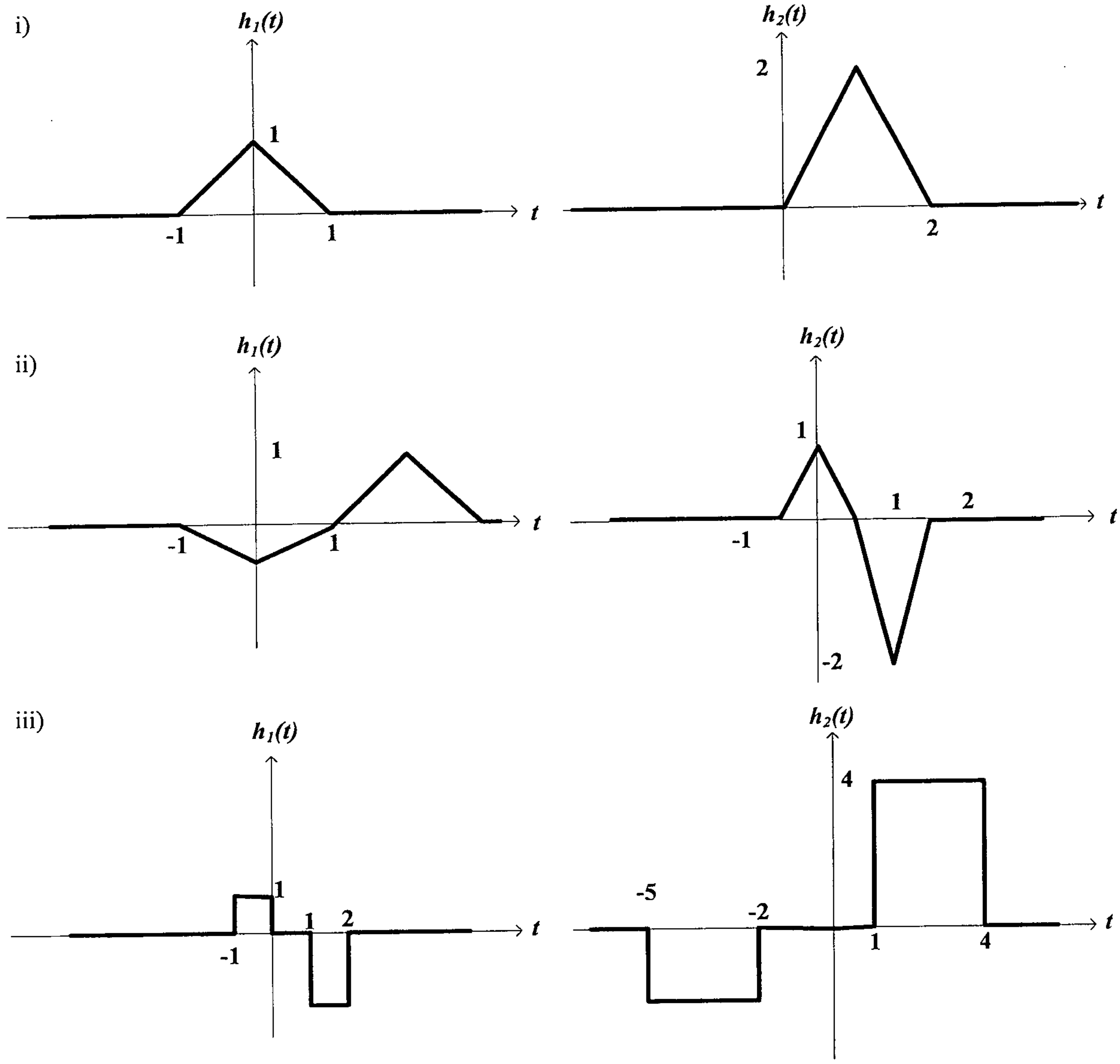


Figure 1

(b) Figure 2 shows the rectangular function. Sketch $g(t)$, if $g(t) = rect\ (t-1) * rect\ (t/2)$.

[11 Marks]

[CO1, PO1, C2]

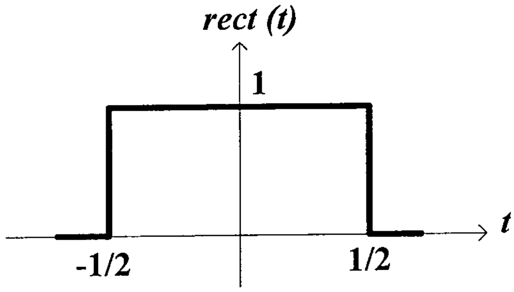


Figure 2

QUESTION 2

(a) Classify each of these frequency responses as lowpass, highpass, bandpass or bandstop.

(i) $H(f) = \dfrac{1}{1 + jf}$

(ii) $H(f) = \dfrac{jf}{1 + jf}$

(iii) $H(j\omega) = \dfrac{j10\omega}{100 - \omega^2 + j10\omega}$

[9 Marks]

[CO3, PO2, C3]

(b) Given circuit in Figure 3, where $V_i(t)$ is the input, and $V_L(t)$ is the output.

(i) Find the transfer function

(ii) Determine the type of filter

(iii) Draw asymptotic magnitude and phase Bode diagrams for the frequency response.

(iv) Obtain the cut-off frequency from your answer in (iii).

[21 Marks]

[CO3, PO2, C4]

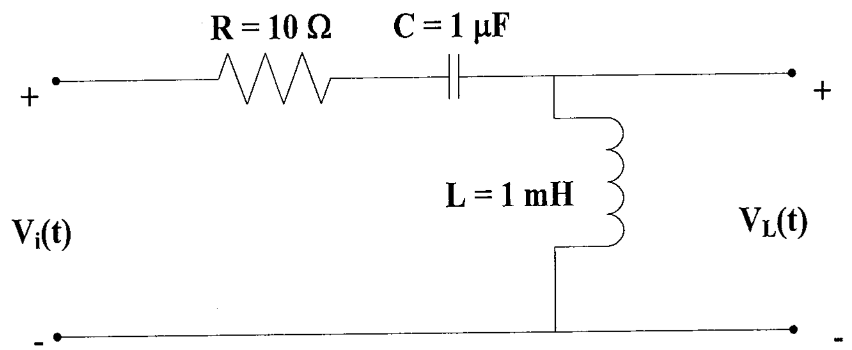


Figure 3

PART B (Answer only TWO question)

QUESTION 3

(a) Given the signal $x(t)$ in Figure 4.

(i) Find the expression for $x(t)$ using Complex Exponential Fourier Series up to the 10^{th} harmonic.

(ii) Plot the double-sided spectrum of $x(t)$ for magnitude and phase.

[20 Marks]

[CO3, PO1, C4]

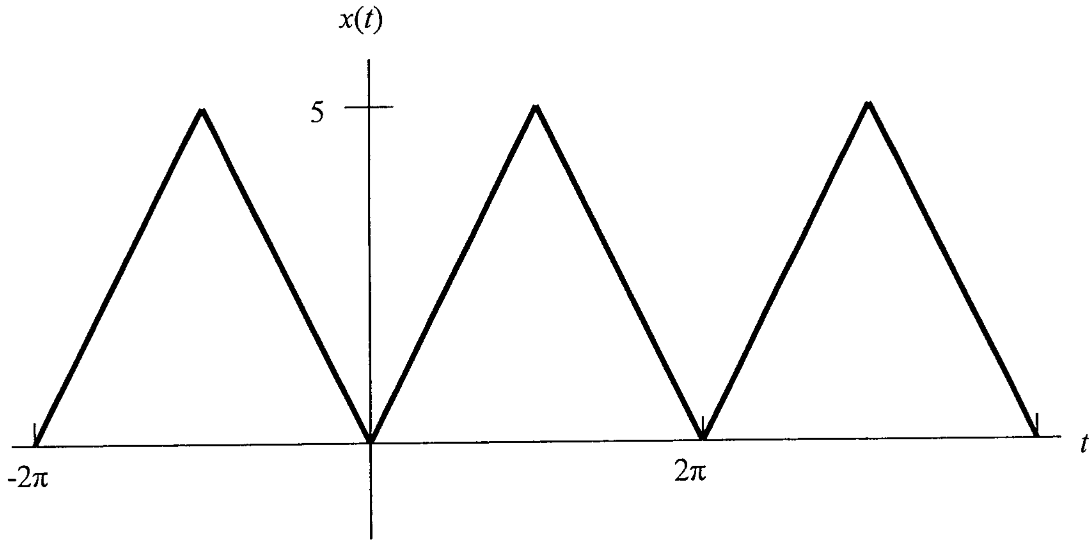


Figure 4

(b) The signal $x(t)$ in Figure 4 has been subjected to several transformation to obtain $y(t)$ as shown in Figure 5. Based on the Complex Exponential Fourier Series of $x(t)$, find the Complex Exponential Fourier Series of $y(t)$.

[5 Marks]

[CO3, PO1, C4]

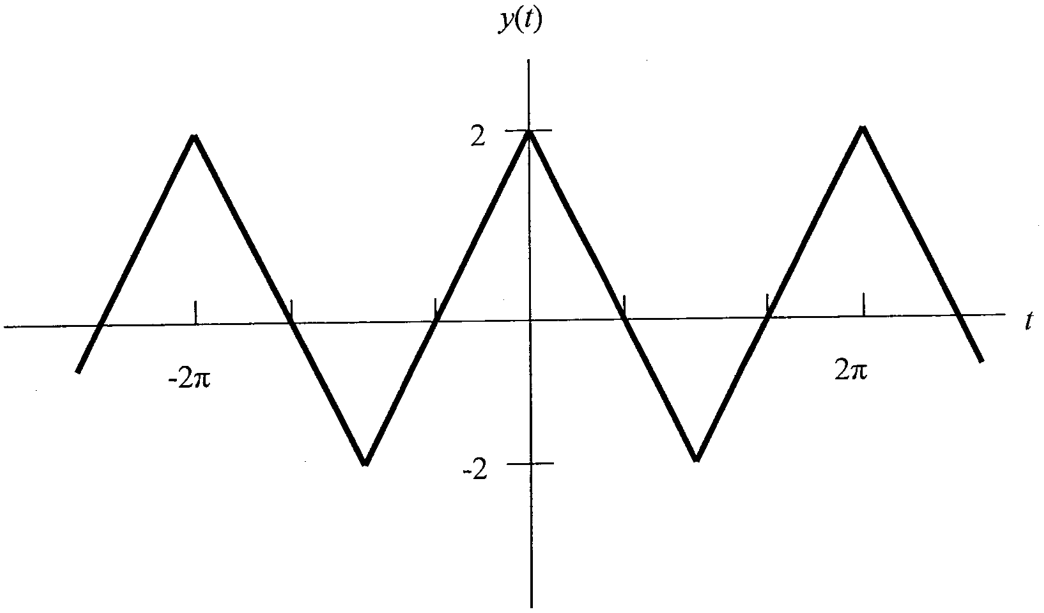


Figure 5

QUESTION 4

(a) Let Find the Fourier transform (FT) of the signals given. State the properties and FT pairs involved.

(i) $f(t) = e^{-j4t}$

(ii) $g(t) = u'(\frac{1}{2}t)$

[7 Marks]

[CO2, PO1, C3]

(b) Find the Inverse Fourier transform for the signals given. State other properties and FT pair involved.

(i) $F(\omega) = \frac{j\omega+10}{(-\omega^2+7j\omega+12)}$

(ii) $G(\omega) = \frac{4e^{-j2\omega}}{j\omega}$

[8 Marks]

[CO2, PO1, C3]

c) In the system in Figure 6, $m(t) = cos2t$, $\omega_c = 10$. Signal $m(t)$ will be modulated to get transmitted signal $y_t(t)$. The transmitted signal is then demodulated to acquire $y_d(t)$. Finally, the signal will be filtered with lowpass filter to estimate the original $m(t)$.

(i) Plot the frequency response of $M(\omega), Y_t(\omega)$, and $Y_d(\omega)$.

(ii) Sketch the LPF filter according to $Y_d(\omega)$ to get back the original signal.

[10Marks]

[CO2, PO1, C4]

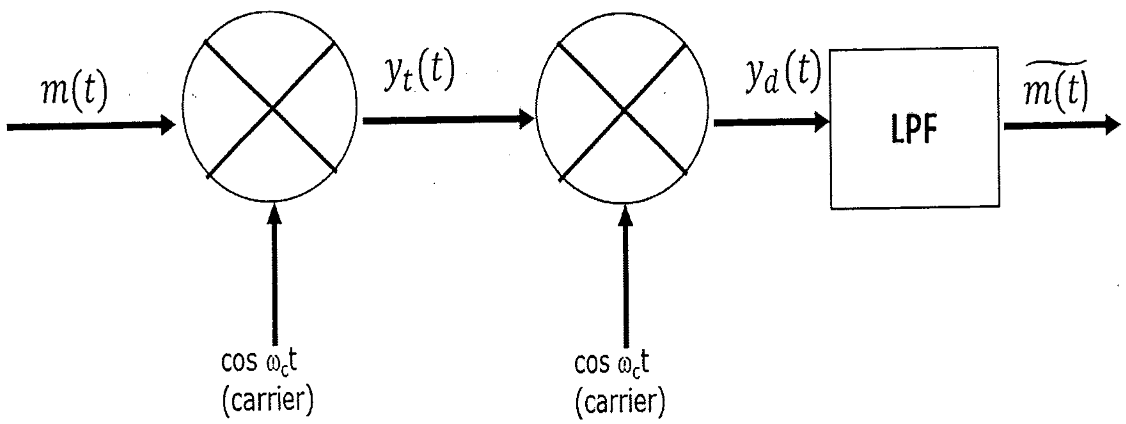


Figure 6

QUESTION 5

(a) The Laplace transform is a widely used integral transform in mathematics and is commonly employed in electrical engineering. Explain how the Laplace transform can be used to simplify the analysis and problem solving in time domain.

[2 Marks]

[CO2, PO1, C3]

(b) Find the Laplace transform of these signals:

(i) $f(t) = (t-1)[u(t-1) - u(t-2)]$

(ii) $f(t) = e^{-t}u(t-\tau)$

[6 Marks]

[CO2, PO1, C3]

(c) A circuit network receives an input signal $f(t) = 2e^{-0.5t}$. The output of the network is the signal $f(t)$ which has been delayed by 2 seconds.

(i) Write the equation of the output signal.

(ii) Find the Laplace transform of the output signal.

[6 Marks]

[CO2, PO1, C3]

(d) Figure 7 shows an electrical circuit that drives a motor. The inductor component represents the rotor and it rotates when the switch is closed. A UMP student accidently touched the switch which made it open. Assume this accident is happened at $t = 0$. You are asked to perform an analysis of the current $i(t)$:

(i) Before the switch is opened.

(ii) After the switch is opened.

Given

$$L^{-1}\left(\frac{As+B}{s^2+2as+c}\right) = re^{-at}\cos(bt+\theta)u(t)$$

where

$$r = \sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}}, \quad \theta = \tan^{-1}\frac{Aa-B}{A\sqrt{c-a^2}}$$

$$b = \sqrt{c-a^2}$$

[11 Marks]

[CO2, PO1, C3]

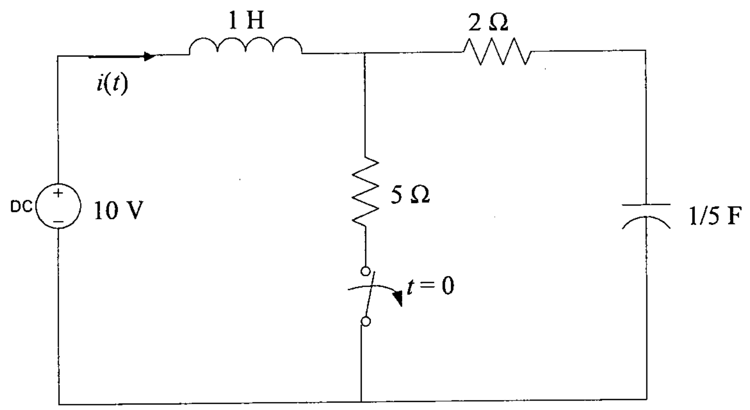


Figure 7

END OF QUESTION PAPER

APPENDIX I – Table of Formulas

MATHEMATICAL FORMULAS

Trigonometric identities

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\pm\cos x = \sin(x \pm 90°)$$
$$\mp\sin x = \cos(x \pm 90°)$$
$$-\cos x = \cos(x \pm 180°)$$

$$e^{\pm jx} = \cos x \pm j\sin x$$
$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$
$$\sin x = \frac{e^{jx} - e^{-jx}}{j2}$$

$$\cos n\pi = (-1)^n$$
$$\sin n\pi = 0$$
$$\cos 2n\pi = 1$$
$$\sin 2n\pi = 0$$

Complex numbers

$$\frac{1}{j} = -j \quad , \quad j^2 = -1$$
$$z = x + jy = r\angle\phi = re^{j\phi}$$
where $r\angle\phi = \sqrt{x^2 + y^2}\angle\tan^{-1}\frac{y}{x}$

Integrals

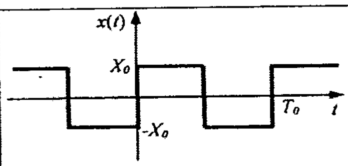
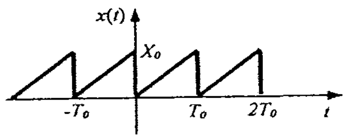
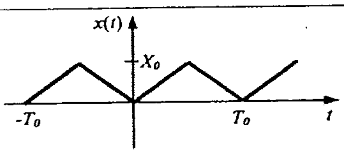
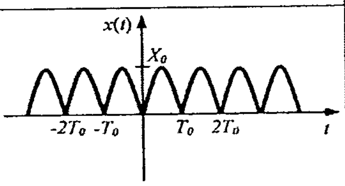
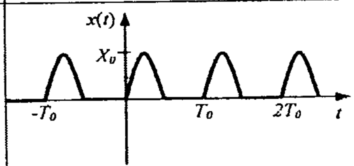
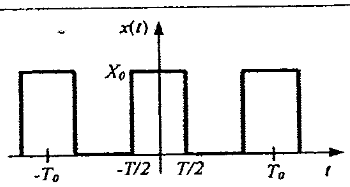
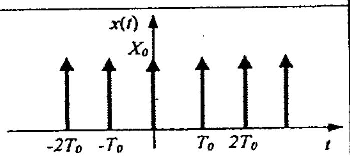
$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$
$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$
$$\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$
$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$
$$\int e^{ax}dx = \frac{e^{ax}}{a}$$
$$\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax - 1)$$
$$\int x^2e^{ax}dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$
$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$$
$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx)$$
$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$
$$\int \frac{dx}{(a^2 + x^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2 + a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right)$$

FOURIER SERIES

Exponential form

$$f(t) = c_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_n e^{jn\omega_0 t} \quad \text{where } c_0 = a_0, c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}\,dt\,,\; c_n = \frac{a_n - jb_n}{2}$$

Fourier Coefficients Table

Name	Waveform	C_0	$C_n, n \neq 0$	Comments
1. Square Wave		0	$-j\dfrac{2X_0}{\pi n}$	$C_n = 0$, n even
2. Sawtooth		$\dfrac{X_0}{2}$	$j\dfrac{X_0}{2\pi n}$	
3. Triangular Wave		$\dfrac{X_0}{2}$	$\dfrac{-2X_0}{(\pi n)^2}$	$C_n = 0$, n even
4. Full-wave rectified		$\dfrac{2X_0}{\pi}$	$\dfrac{-2X_0}{\pi(4n^2-1)}$	
5. Half-wave rectified		$\dfrac{X_0}{\pi}$	$\dfrac{-X_0}{\pi(n^2-1)}$	$C_n = 0$, n odd, except for $C_1 = -j\dfrac{X_0}{4}$, and $C_{-1} = j\dfrac{X_0}{4}$
6. Square Wave		$\dfrac{TX_0}{T_0}$	$\dfrac{TX_0}{T_0}\operatorname{sinc}\dfrac{Tn\omega_0}{2}$	$\dfrac{Tn\omega_0}{2} = \dfrac{\pi Tn}{T_0}$
7. Impulse train		$\dfrac{X_0}{T_0}$	$\dfrac{X_0}{T_0}$	

FOURIER TRANSFORM

Properties of the Fourier Transform

Property	F(t)	F(ω)
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency shift	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Modulation	$\cos(\omega_0 t) f(t)$	$\frac{1}{2}[F(\omega + \omega_0) + F(\omega - \omega_0)]$
Time differentiation	$\frac{df}{dt}$ $\frac{d^n f}{dt^n}$	$j\omega F(\omega)$ $(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)dt$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$

Fourier Transform Pairs

f(t)	F(ω)	f(t)	F(ω)
$\delta(t)$	1	$e^{-a\lvert t \rvert}$	$\frac{2a}{a^2 + \omega^2}$
1	$2\pi\delta(\omega)$	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega$
$u(t+\tau) - u(t-\tau)$	$2\frac{\sin \omega\tau}{\omega}$	$\cos \omega_0 t$	$\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0$
$\lvert t \rvert$	$-\frac{2}{\omega^2}$	$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
$e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$rect(t/\tau)$	$\tau \,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$
$e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$tri(t/\tau)$	$\tau \,\mathrm{sinc}^2\left(\frac{\omega\tau}{2}\right)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$e^{-at} \sin \omega_0 t \, u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
		$e^{-at} \cos \omega_0 t \, u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$

LAPLACE TRANSFORM

Definition of Laplace Transform

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st}\,dt$$

Properties of the Laplace transform

Property	*f(t)*	*F(s)*
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$ $\frac{d^2 f}{dt^2}$ $\frac{d^n f}{dt^n}$	$sF(s) - f(0^-)$ $s^2 F(s) - sf(0^-) - f'(0^-)$ $s^n F(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \ldots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(t)dt$	$\frac{1}{s}F(s)$
Frequency differentiation	$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{d\omega^n}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^{\infty} F(s)ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}} = \frac{1}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$

Laplace transform pairs

f(t)	*F(s)*
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t^n	$\frac{n!}{s^{n+1}}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)$	$\frac{s\sin\theta + \omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

s-domain equivalents

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$$

or

$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$V_C(s) = \frac{1}{sC}I_C(s) + \frac{v_C(0^-)}{s}$$

or

$$I_C(s) = sCV_C(s) - Cv_C(0^-)$$