**Universiti Malaysia PAHANG**

Engineering • Technology • Creativity

**FACULTY OF ELECTRICAL & ELECTRONICS ENGINEERING**

**FINAL EXAMINATION**

| | | |
|---|---|---|
| **COURSE** | : | **SIGNALS AND NETWORKS** |
| **COURSE CODE** | : | **BEE2143** |
| **LECTURER** | : | **NURUL WAHIDAH ARSHAD**<br>**DR. SUNARDI**<br>**ASSOC. PROF. DR. ZUWAIRIE IBRAHIM** |
| **DATE** | : | **16 JUNE 2014** |
| **DURATION** | : | **3 HOURS** |
| **SESSION/SEMESTER** | : | **SESSION 2013/2014 SEMESTER II** |
| **PROGRAMME CODE** | : | **BEC/BEE/BEP** |

**INSTRUCTIONS TO CANDIDATES**

1. This question paper consists of **FIVE (5)** questions. Answer **ALL** questions in **PART A** and **ONE** question for **PART B**.
2. All answers to a new question should start on new page.
3. All the calculation answer and assumptions must be clearly stated.
4. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

**EXAMINATION REQUIREMENTS**

1. **APPENDIX I** – Table of Formula
2. Semilog Graph Paper

**DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO**

This examination paper consists of **TEN (10)** printed pages including front page.

## PART A (Answer ALL questions)

### QUESTION 1

(a) Name two (2) properties of linear system.

**[2 Marks]**
**[CO1, PO1, C2]**

(b) Sketch $x(-3t-6)$ if $x(t)=3u(t+3)-u(t)+3u(t-3)-5u(t-6)$.

**[4 Marks]**
**[CO1, PO1, C2]**

(c) Given $r(t)=5u(t-1)-5u(t-3)$ and $s(t)=t^2[u(t)-u(t-2)]$ as the input signal and impulse response of a linear time-invariant (LTI) system, respectively.

(i) Sketch both $r(t)$ and $s(t)$.

(ii) If $y(t)$ is the output signal of the LTI system, find $y(t)$.

(iii) Using the same input signal, the impulse response is changed to $v(t)=t^2[u(t)-u(t-2)]+\delta(t)$. Without using the calculation of convolution, find the new output signal, $z(t)$, in terms of $y(t)$ and $r(t)$.

**[14 Marks]**
**[CO1, PO1, C2]**

**QUESTION 2**

(a) Explain the Gibbs Phenomenon.

**[2 Marks]**

**[CO2, PO1, C2]**

(b) A periodic signal, $x(t)$, is given in Figure 1.

(i) Calculate the angular frequency $\omega_o$, $c_o$, and $c_n$.

(ii) Express the signal $x(t)$ based on the complex exponential Fourier series until the fifth ($5^{th}$) harmonics.

(iii) Sketch the double-sided magnitude and phase spectrum of signal $x(t)$. The x-axis must be in terms of angular frequency, $\omega$.

**[19 Marks]**

**[CO2, PO1, C3]**

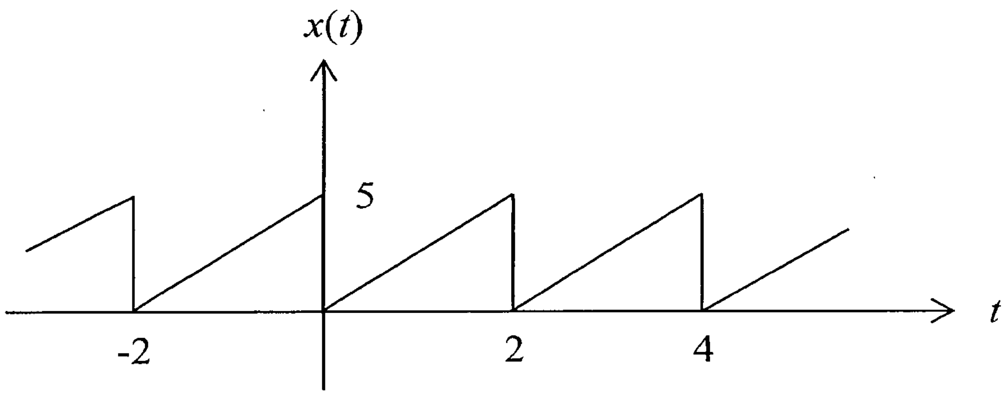



Figure 1

(c) The signal $x(t)$ is subjected to an amplifier with a voltage gain $K = 10$ and then the scaled signal is delayed by $t_0 = 2$ second. Find the new $c_o$ and $c_n$.

**[4 Marks]**

**[CO2, PO1, C3]**

## QUESTION 3

(a) Given Bode plot in Figure 2, obtain the transfer function $H(\omega)$.

**[6 Marks]**

**[CO3, PO1, C4]**

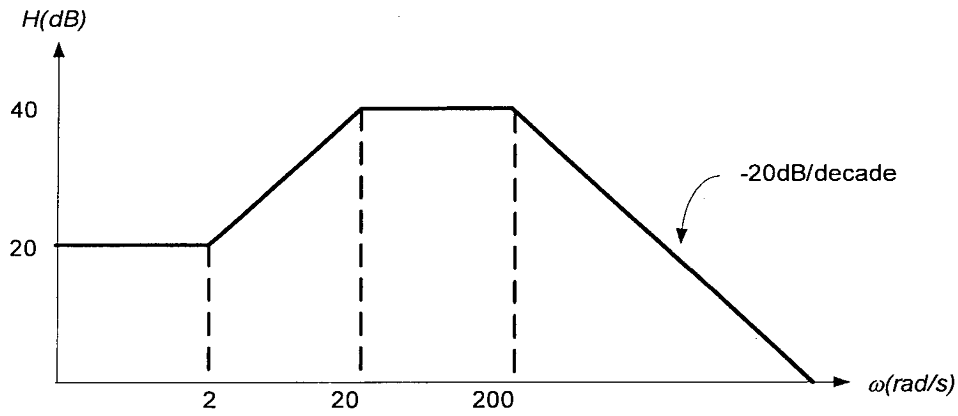



Figure 2

(b) Given circuit in Figure 3. Assume value of C1 and C2 are 1F, while R1 and R2 are 1Ω.

(i) Find the transfer function

(ii) Determine the type of filter

(iii) Obtain the cut-off frequency

(iv) Prove the type of filter in Q3 (b) (ii) by using Bode plot.

**[24 Marks]**

**[CO3, PO1, C4]**

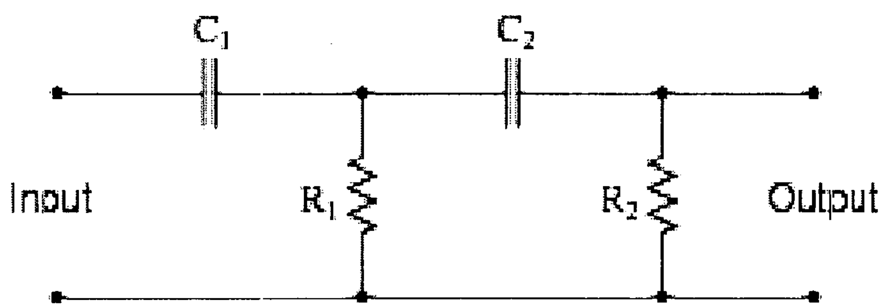



Figure 3

## PART B (Answer only ONE question)

## QUESTION 4

a) Let $f(t) = 8(1+6\cos 400\pi t)\cos 2{,}000\pi t$ as an AM signal.

(i) Find the Fourier Transform for $f(t)$ using modulation property

(ii) Sketch the spectrum of modulating signal, carrier signal, and AM signal.

(iii) Find upper and lower side band frequencies range

(iv) Sketch the spectrum of $y(t)$ based on Figure 1 if $H(\omega) = 2$ for $\omega > 1{,}000\pi$

**[17 Marks]**

**[CO2,PO1,C4]**

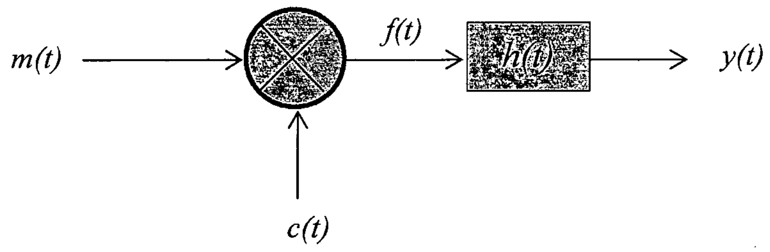



Figure 4

b) Find Fourier transform for $f(t)$ using derivative technique (differentiation property).

$$f(t) = \begin{cases} t+1, & -1 < t < 0 \\ t-1, & 0 < t < 1 \\ 0, & \text{other } t \end{cases}$$

**[8 Marks]**

**[CO2,PO1,C4]**

## QUESTION 5

(a) For the circuit in Figure 5, by assuming zero initial conditions, find

(i) The transfer function $H(s)$

(ii) The impulse response

(iii) The step response

(iv) The response for $v_i(t) = e^{-t}\,u(t)\ V$

**[25 Marks]**

**[CO2, PO1, C3]**

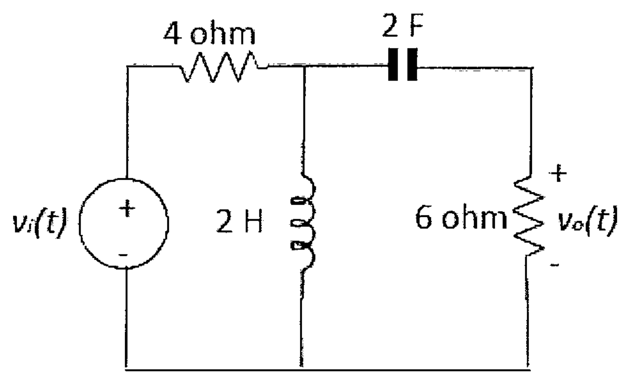



Figure 5

**END OF QUESTION PAPER**

## APPENDIX I – Table of Formulas

## MATHEMATICAL FORMULAS

### Trigonometric identities

$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
$\pm\cos x = \sin(x \pm 90°)$
$\mp\sin x = \cos(x \pm 90°)$
$-\cos x = \cos(x \pm 180°)$

$e^{\pm jx} = \cos x \pm j\sin x$

$\cos x = \dfrac{e^{jx} + e^{-jx}}{2}$

$\sin x = \dfrac{e^{jx} - e^{-jx}}{j2}$

$\cos n\pi = (-1)^n$
$\sin n\pi = 0$
$\cos 2n\pi = 1$
$\sin 2n\pi = 0$

### Complex numbers

$\dfrac{1}{j} = -j \quad , \quad j^2 = -1$

$z = x + jy = r\angle\phi = re^{j\phi}$

where $r\angle\phi = \sqrt{x^2 + y^2}\angle\tan^{-1}\dfrac{y}{x}$

### Integrals

$$\int \sin ax dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax dx = \frac{1}{a}\sin ax$$

$$\int x\sin ax dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$

$$\int x\cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$$\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2 e^{ax} dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

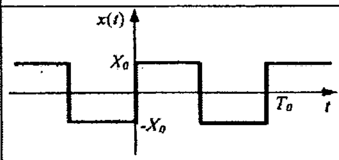
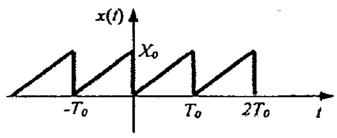
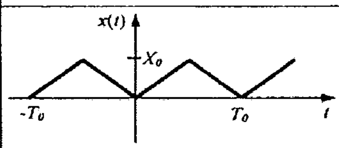
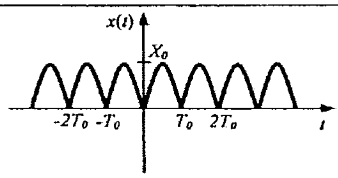
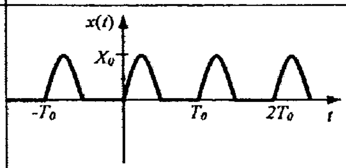
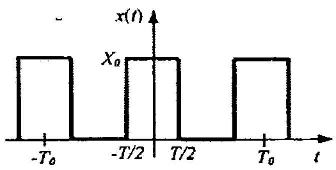
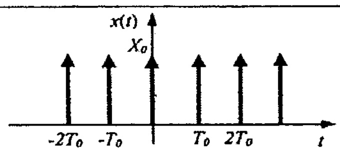
$$\int \frac{dx}{(a^2 + x^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2 + a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right)$$

## FOURIER SERIES

### Exponential form

$$f(t) = c_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_n e^{jn\omega_0 t} \quad \text{where } c_0 = a_0, c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}dt,\ c_n = \frac{a_n - jb_n}{2}$$

**Fourier Coefficients Table**

| Name | Waveform | $C_0$ | $C_n, n \neq 0$ | Comments |
|---|---|---|---|---|
| 1.Square Wave |  | 0 | $-j\dfrac{2X_0}{\pi n}$ | $C_n = 0$, $n$ even |
| 2. Sawtooth |  | $\dfrac{X_0}{2}$ | $j\dfrac{X_0}{2\pi n}$ | |
| 3. Triangular Wave |  | $\dfrac{X_0}{2}$ | $\dfrac{-2X_0}{(\pi n)^2}$ | $C_n = 0$, $n$ even |
| 4. Full-wave rectified |  | $\dfrac{2X_0}{\pi}$ | $\dfrac{-2X_0}{\pi(4n^2-1)}$ | |
| 5. Half-wave rectified |  | $\dfrac{X_0}{\pi}$ | $\dfrac{-X_0}{\pi(n^2-1)}$ | $C_n = 0$, $n$ odd, except for $C_1 = -j\dfrac{X_0}{4}$, and $C_{-1} = j\dfrac{X_0}{4}$ |
| 6. Square Wave |  | $\dfrac{TX_0}{T_0}$ | $\dfrac{TX_0}{T_0}\operatorname{sinc}\dfrac{Tn\omega_0}{2}$ | $\dfrac{Tn\omega_0}{2} = \dfrac{\pi Tn}{T_0}$ |
| 7. Impulse train |  | $\dfrac{X_0}{T_0}$ | $\dfrac{X_0}{T_0}$ | |

# FOURIER TRANSFORM

**Properties of the Fourier Transform**

| Property | F(t) | F(ω) |
|---|---|---|
| Linearity | $a_1 f_1(t) + a_2 f_2(t)$ | $a_1 F_1(\omega) + a_2 F_2(\omega)$ |
| Scaling | $f(at)$ | $\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$ |
| Time shift | $f(t-a)$ | $e^{-j\omega a} F(\omega)$ |
| Frequency shift | $e^{j\omega_0 t} f(t)$ | $F(\omega - \omega_0)$ |
| Modulation | $\cos(\omega_0 t) f(t)$ | $\frac{1}{2}[F(\omega+\omega_0) + F(\omega-\omega_0)]$ |
| Time differentiation | $\frac{df}{dt}$<br>$\frac{d^n f}{dt^n}$ | $j\omega F(\omega)$<br>$(j\omega)^n F(\omega)$ |
| Time integration | $\int_{-\infty}^{t} f(t)dt$ | $\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$ |
| Duality | $F(t)$ | $2\pi f(-\omega)$ |

**Fourier Transform Pairs**

| f(t) | F(ω) | f(t) | F(ω) |
|---|---|---|---|
| $\delta(t)$ | $1$ | $e^{-a\lvert t \rvert}$ , $\mathrm{Re}(a) > 0$ | $\frac{2a}{a^2+\omega^2}$ |
| $1$ | $2\pi\delta(\omega)$ | $e^{j\omega_0 t}$ | $2\pi\delta(\omega-\omega_0)$ |
| $u(t)$ | $\pi\delta(\omega) + \frac{1}{j\omega}$ | $\sin\omega_0 t$ | $j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega$ |
| $u(t+\tau) - u(t-\tau)$ | $2\frac{\sin\omega\tau}{\omega}$ | $\cos\omega_0 t$ | $\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0$ |
| $\lvert t \rvert$ | $-\frac{2}{\omega^2}$ | $\mathrm{sgn}(t)$ | $\frac{2}{j\omega}$ |
| $e^{at}u(-t)$ , $\mathrm{Re}(a) > 0$ | $\frac{1}{a-j\omega}$ | *rect(t/τ)* | $\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$ |
| $e^{-at}u(t)$ , $\mathrm{Re}(a) > 0$ | $\frac{1}{a+j\omega}$ | *tri(t/τ)* | $\tau\,\mathrm{sinc}^2\left(\frac{\omega\tau}{2}\right)$ |
| $t^n e^{-at}u(t)$ , $\mathrm{Re}(a) > 0$ | $\frac{n!}{(a+j\omega)^{n+1}}$ | $e^{-at}\sin\omega_0 t\, u(t)$ , $\mathrm{Re}(a) > 0$ | $\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$ |
| | | $e^{-at}\cos\omega_0 t\, u(t)$ , $\mathrm{Re}(a) > 0$ | $\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$ |

# LAPLACE TRANSFORM

## Definition of Laplace Transform

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$$

## Properties of the Laplace transform

| Property | $f(t)$ | $F(s)$ |
|---|---|---|
| Linearity | $a_1 f_1(t) + a_2 f_2(t)$ | $a_1 F_1(s) + a_2 F_2(s)$ |
| Scaling | $f(at)$ | $\frac{1}{a}F\left(\frac{s}{a}\right)$ |
| Time shift | $f(t-a)u(t-a)$ | $e^{-as}F(s)$ |
| Frequency shift | $e^{-at}f(t)$ | $F(s+a)$ |
| Time differentiation | $\frac{df}{dt}$<br>$\frac{d^2 f}{dt^2}$<br>$\frac{d^n f}{dt^n}$ | $sF(s) - f(0^-)$<br>$s^2F(s) - sf(0^-) - f'(0^-)$<br>$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \dots - f^{(n-1)}(0^-)$ |
| Time integration | $\int_0^t f(t)dt$ | $\frac{1}{s}F(s)$ |
| Frequency differentiation | $t^n f(t)$ | $(-1)^n \frac{d^n F(s)}{d\omega^n}$ |
| Frequency integration | $\frac{f(t)}{t}$ | $\int_s^\infty F(s)ds$ |
| Time periodicity | $f(t) = f(t+nT)$ | $\frac{F_1(s)}{1-e^{-sT}} = \frac{1}{1-e^{-sT}}$ |
| Initial value | $f(0)$ | $\lim_{s\to\infty} sF(s)$ |
| Final value | $f(\infty)$ | $\lim_{s\to 0} sF(s)$ |

## Laplace transform pairs

| $f(t)$ | $F(s)$ |
|---|---|
| $\delta(t)$ | $1$ |
| $u(t)$ | $\frac{1}{s}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $t^n e^{-at}$ | $\frac{n!}{(s+a)^{n+1}}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ |
| $\sin(\omega t + \theta)$ | $\frac{s\sin\theta + \omega\cos\theta}{s^2+\omega^2}$ |
| $\cos(\omega t + \theta)$ | $\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$ |
| $e^{-at}\sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ |
| $e^{-at}\cos \omega t$ | $\frac{s+a}{(s+a)^2+\omega^2}$ |

## s-domain equivalents

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$$

or

$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$V_C(s) = \frac{1}{sC}I_C(s) + \frac{v_C(0^-)}{s}$$

or

$$I_C(s) = sCV_C(s) - Cv_C(0^-)$$