Universiti Malaysia PAHANG

Engineering • Technology • Creativity

FACULTY OF ELECTRICAL & ELECTRONICS ENGINEERING

FINAL EXAMINATION

COURSE	:	**SIGNALS & NETWORKS**
COURSE CODE	:	**BEE2143**
LECTURER	:	**FARADILA NAIM** **NURUL WAHIDAH ARSHAD** **PROF. MADYA DR. ZUWAIRIE IBRAHIM**
DATE	:	**20 JUNE 2013**
DURATION	:	**3 HOURS**
SESSION/SEMESTER	:	**SESSION 2012/2013 SEMESTER II**
PROGRAMME CODE	:	**BEE/BEC/BEP**

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of **FIVE (5)** questions. Answer **ALL** questions in **PART A** and **ONE** question for **PART B**.
2. All answers to a new question should start on new page.
3. All the calculation answer and assumptions must be clearly stated.
4. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

EXAMINATION REQUIREMENTS

1. **Appendix I** – Table of Formula
2. Semilog Graph

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **FOURTEEN (14)** printed pages including front page.

PART A (Answer ALL questions)

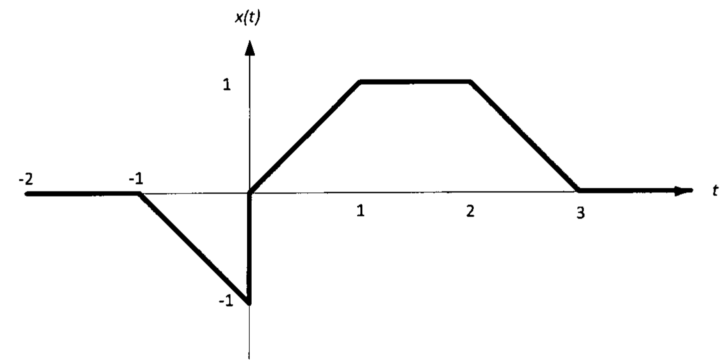
QUESTION 1

(a) Let *x(t)* and *y(t)* be given in Figure 1(a) and (b), respectively. Solve and sketch

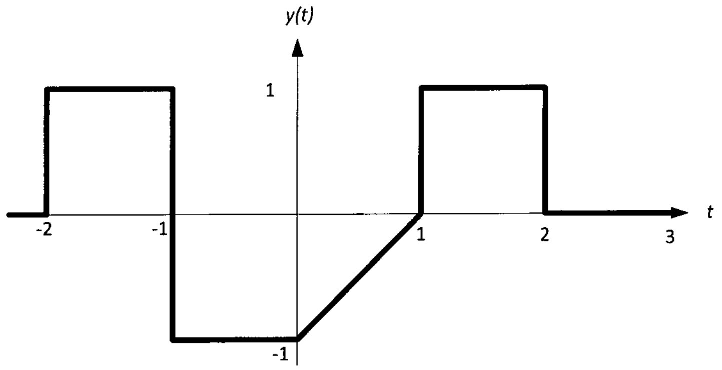
$$z(t) = x(2t).y\left(\frac{1}{2}t+1\right)$$

[8 Marks]

[CO1, PO2, C3]



(a)



(b)
Figure 1

(b) Evaluate the convolution integrals of given signal *f(t)* and *g(t)* in Figure 2.

[12 Marks]

[CO1, PO1, C3]

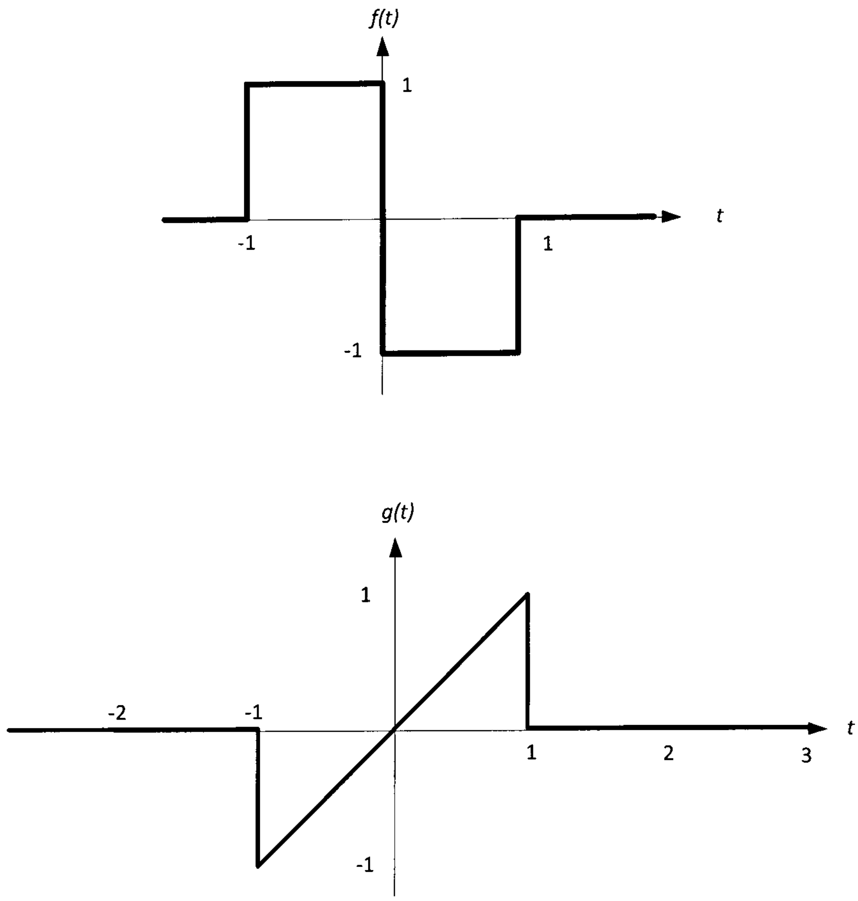


Figure 2

QUESTION 2

(a) The periodic waveform $f(t)$ in Figure 3. Derive the trigonometric Fourier series of $f(t)$.

[13 Marks]

[CO2, PO1, C3]

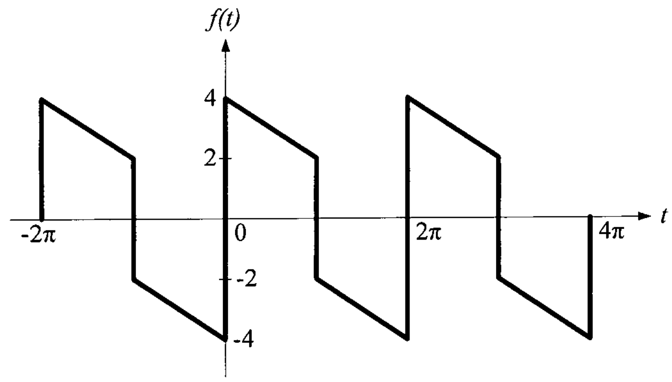


Figure 3

(b) (i) Refer to sawtooth signal in FOURIER SERIES APPENDIX, find the exponential Fourier Series of $y(t)$ up to 3rd Harmonics as shown in Figure 4.

(ii) Sketch the double sided line spectrum (magnitude and phase) of the signal $y(t)$.

[12 Marks]

[CO2, PO1, C3]

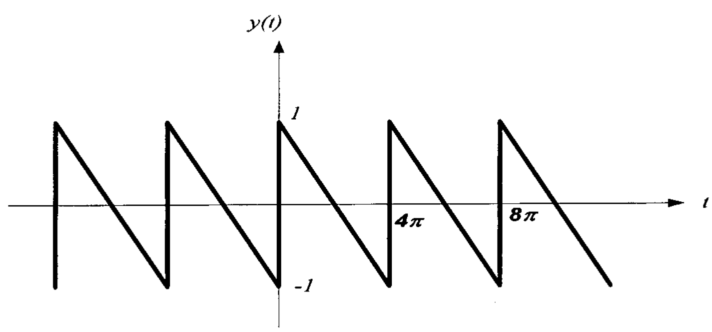


Figure 4

QUESTION 3

(a) Briefly explain the effect of damping ratio, ζ ,to the second order low pass filter.

[4 Marks]

[CO3, PO4, C3]

(b) A transfer function of an RL circuit is given as

$$H(s) = \frac{s\tau}{1 + s\tau}$$

Proof that the RL circuit behaves as a high-pass filter.

[4 Marks]

[CO3, PO4, C4]

(c) Figure 5 shows a filter circuit.

(i) Determine the transfer function $H_1(s) = y(t)/x(t)$ in terms ofR_1, R_2 dan C.

(ii) Plot the magnitud and phase response of $H_1(s)$. AssumeR_1 = $10\,k\Omega$, R_2 = $1\,k\Omega$, and C = $7.5nF$.

(iii) Determine the type of filter based on the magnitude response.

(iv) Using Bode plot (or any method), find the expression of $H_1(s)$ in term of $H_2(s)$.

[22 Marks]

[CO3, PO4, C4]

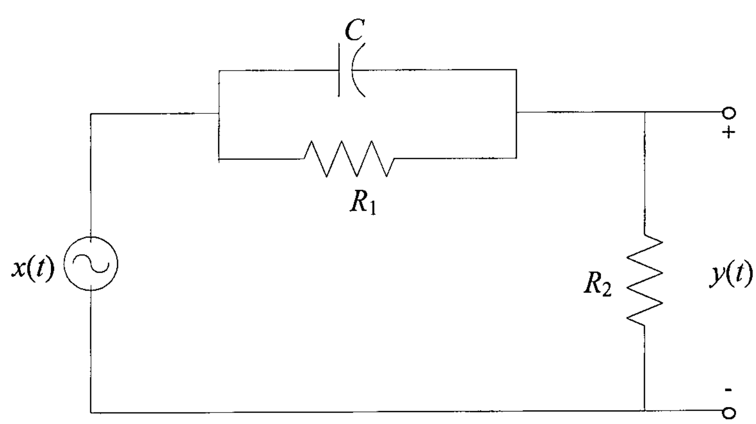


Figure 5

PART B (Answer only ONE question)

QUESTION 4

(a) Briefly discuss **TWO** differences between Fourier series and Fourier transform in the application of signal analysis and processing.

[4 Marks]
[CO, PO1, C2]

(b) *x(t)* is the input to an LTI system with unit impulse response *h(t)*. The mathematical expression of signal *x(t)* is given as $x(t) = 4 + e^{jt} + 2e^{j3t}$. Signal *h(t)* is given in Figure 6.

(i) Find the Fourier Transform of signal *x(t)* and sketch spectrum *X(ω)*.

(ii) By using properties of Fourier Transform, find the Fourier Transform of signal *h(t)* and then sketch spectrum *H(ω)*.

(iii) Sketch the output spectrum of the LTI system, *Y(ω)* and then write the expression of the output signal *y(t)*.

[12 Marks]
[CO, PO1, C3]

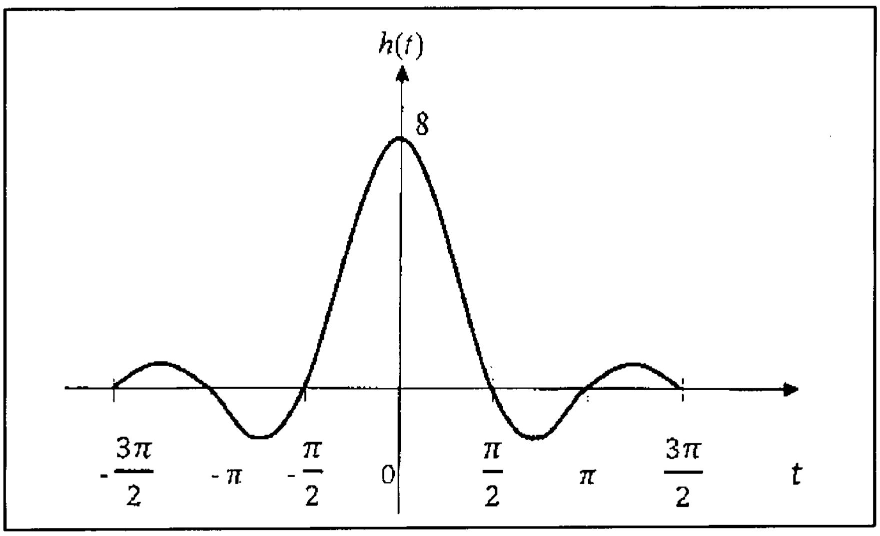


Figure 6

(c) Consider a system as shown in Figure 7. An input signal, *x(t)* is multiplied by a carrier signal cos*(At)*. The resulting signal is then passed through an ideal filter, *h(t)*. The signal is then multiplied by carrier signal cos*(Bt)*. The spectrum of $X(\omega)$ and $H(\omega)$ are shown in Figure 7(b).

(i) Determine the value of ω_1 in order to get the following spectrum of $B(\omega)$ as shown in Figure 7(c).

(ii) If ω_1 is now set at 500 rad/s, determine the value of ω_2 and sketch the spectrum of filter $H_2(\omega)$ in order for the output signal $Y(\omega)$to be equal to the input $X(\omega)$. Show your calculation by drawing spectrum $A(\omega)$,$B(\omega)$ and $C(\omega)$.

[9 Marks]

[CO, PO1, C4]

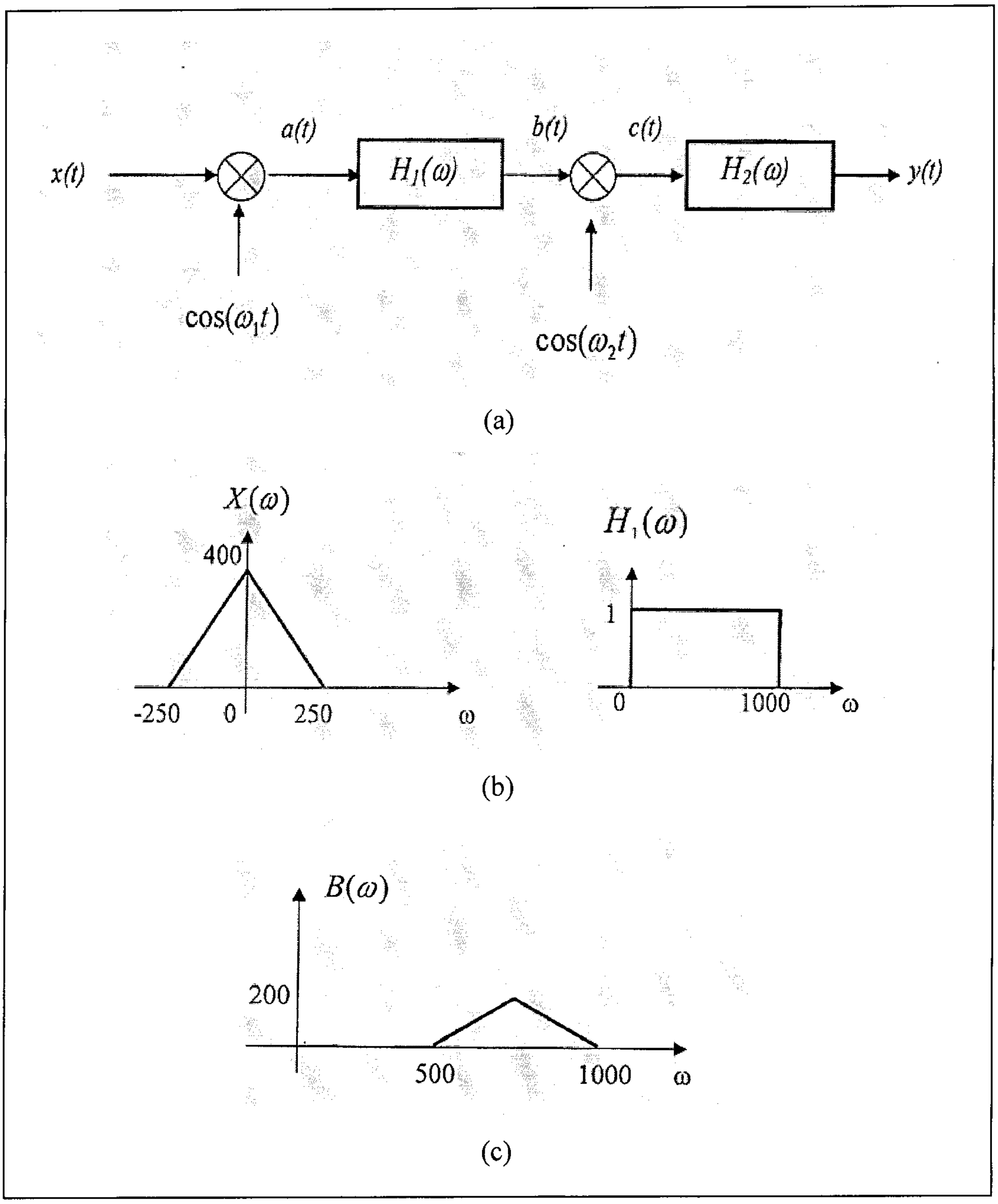

x(t)
a(t)
H1(ω)
b(t)
c(t)
H2(ω)
y(t)
cos(ω1t)
cos(ω2t)
(a)
X(ω)
400
-250
0
250
ω
H1(ω)
1
0
1000
ω
(b)
B(ω)
200
500
1000
ω
(c)

Figure 7

QUESTION 5

(a) The Laplace transform of a time function $f(t)$ is denoted and defined as

$$L\{F(t)\} = F(s) = \int_0^{\infty} e^{-\infty} f(t)dt$$

(i) By using the Laplace transform pairs as given in the table, find the Laplace transform for a function $g(t) = -9\sin(4t)$.

(ii) By using the first principle (equation above), verify that the Laplace transform for $g(t)$ is the same as the answer in Q5(a)(i).

[8 Marks]

[CO2, PO1, C4]

(b) The signal $x(t)$ shown in Figure 8 is a superposition of sawtooth, square wave, and pulses.

(i) Write a mathematical expression for $x(t)$ in the general form of the unit step function.

(iii) Find the Laplace transform, $X(s)$.

[7 Marks]

[CO2, PO1, C4]

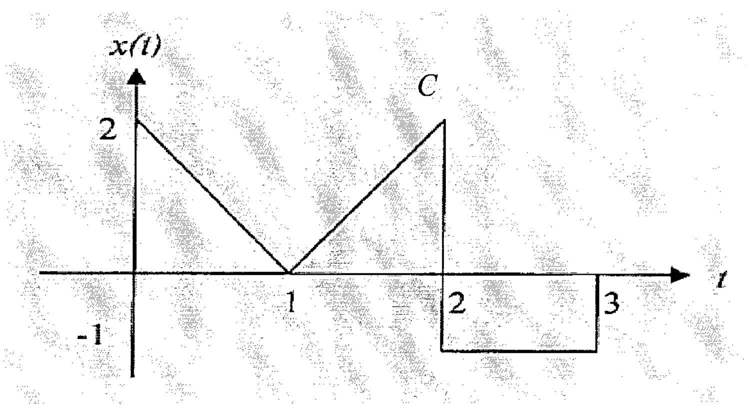


Figure 8

(c) The coils of a large horseshoe type of electromagnet used in a generator are represented by a circuit model shown in Figure 9. The switch is closed at $t = 0$.

(i) Sketch the Laplace equivalent circuit at $t = 0$.

(ii) Find the current $i_1(t)$ provided by the power supply after the switch is closed. Note that $i_1(0) = i_2(0) = 0$.

(iii) After sometimes, a disconnection occurs in the circuit at point 'B'. Draw the Laplace equivalent circuit. Make a brief comment on the circuit.

[10 Marks]

[CO2, PO2, C3]

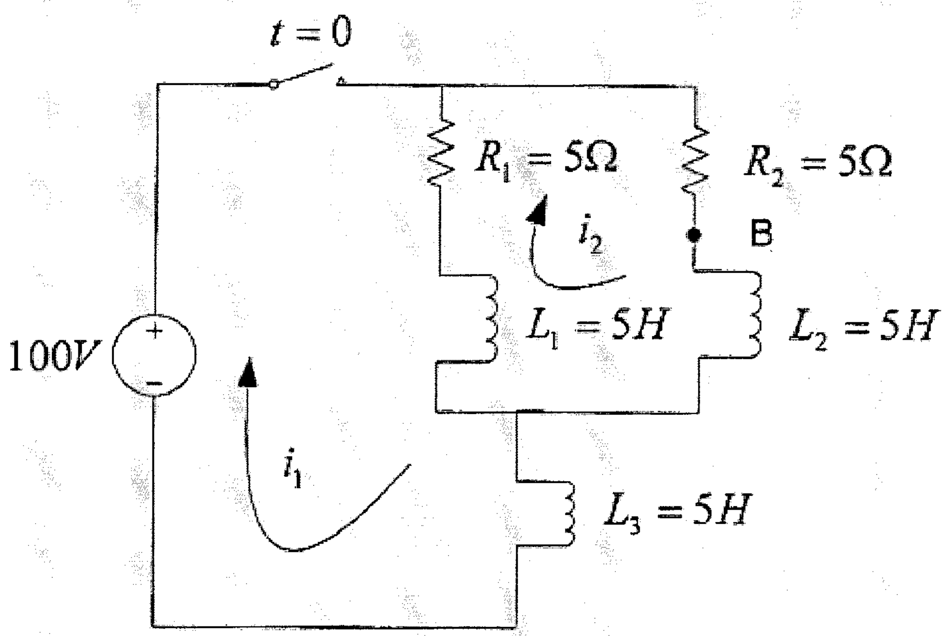


Figure 9

END OF QUESTION PAPER

Appendix I – Table of Formulas

MATHEMATICAL FORMULAS

Trigonometric identities

$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
$\pm\cos x = \sin(x \pm 90°)$
$\mp\sin x = \cos(x \pm 90°)$
$-\cos x = \cos(x \pm 180°)$

$e^{\pm jx} = \cos x \pm j\sin x$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{j2}$$

$\cos n\pi = (-1)^n$
$\sin n\pi = 0$
$\cos 2n\pi = 1$
$\sin 2n\pi = 0$

Complex numbers

$\frac{1}{j} = -j \quad , \quad j^2 = -1$

$z = x + jy = r\angle\phi = re^{j\phi}$

where $r\angle\phi = \sqrt{x^2 + y^2}\angle\tan^{-1}\frac{y}{x}$

Integrals

$$\int \sin ax dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax dx = \frac{1}{a}\sin ax$$

$$\int x\sin ax dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$

$$\int x\cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$$\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2 e^{ax} dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

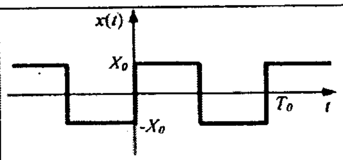
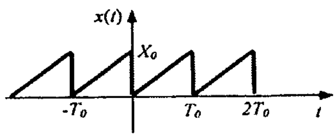
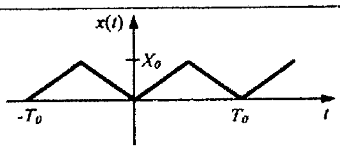
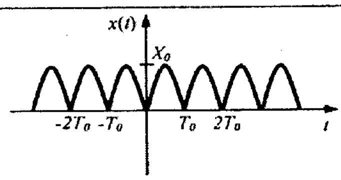
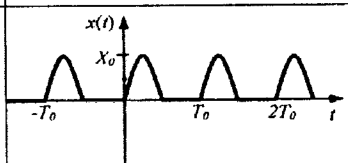
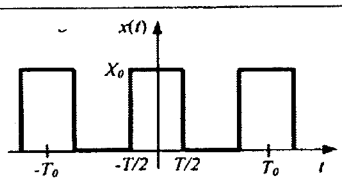
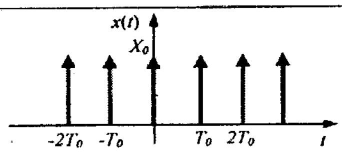
$$\int \frac{dx}{(a^2 + x^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2 + a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right)$$

FOURIER SERIES

Exponential form

$$f(t) = c_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_n e^{jn\omega_0 t} \text{ where } c_0 = a_0, \qquad c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}dt,\ c_n = \frac{a_n - jb_n}{2}$$

Fourier Coefficients Table

Name	Waveform	C_0	$C_n, n \neq 0$	Comments
1.Square Wave		0	$-j\frac{2X_0}{\pi n}$	$C_n = 0$, n even
2. Sawtooth		$\frac{X_0}{2}$	$j\frac{X_0}{2\pi n}$	
3. Triangular Wave		$\frac{X_0}{2}$	$\frac{-2X_0}{(\pi n)^2}$	$C_n = 0$, n even
4. Full-wave rectified		$\frac{2X_0}{\pi}$	$\frac{-2X_0}{\pi(4n^2-1)}$	
5. Half-wave rectified		$\frac{X_0}{\pi}$	$\frac{-X_0}{\pi(n^2-1)}$	$C_n = 0$, n odd, except for $C_1 = -j\frac{X_0}{4}$, and $C_{-1} = j\frac{X_0}{4}$
6. Square Wave		$\frac{TX_0}{T_0}$	$\frac{TX_0}{T_0}\text{sinc}\frac{Tn\omega_0}{2}$	$\frac{Tn\omega_0}{2} = \frac{\pi Tn}{T_0}$
7. Impulse train		$\frac{X_0}{T_0}$	$\frac{X_0}{T_0}$	

FOURIER TRANSFORM

Properties of the Fourier Transform

Property	F(t)	F(ω)
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency shift	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Modulation	$\cos(\omega_0 t) f(t)$	$\frac{1}{2}[F(\omega+\omega_0) + F(\omega-\omega_0)]$
Time differentiation	$\frac{df}{dt}$ $\frac{d^n f}{dt^n}$	$j\omega F(\omega)$ $(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)dt$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$

Fourier Transform Pairs

f(t)	F(ω)	f(t)	F(ω)
$\delta(t)$	1	$e^{-a\lvert t \rvert}$, $\text{Re}(a) > 0$	$\frac{2a}{a^2+\omega^2}$
1	$2\pi\delta(\omega)$	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\sin \omega_0 t$	$j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega$
$u(t+\tau) - u(t-\tau)$	$2\frac{\sin \omega\tau}{\omega}$	$\cos \omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0$
$\lvert t \rvert$	$-\frac{2}{\omega^2}$	$\text{sgn}(t)$	$\frac{2}{j\omega}$
$e^{at}u(-t)$, $\text{Re}(a) > 0$	$\frac{1}{a-j\omega}$	$rect(t/\tau)$	$\tau \,\text{sinc}\left(\frac{\omega\tau}{2}\right)$
$e^{-at}u(t)$, $\text{Re}(a) > 0$	$\frac{1}{a+j\omega}$	$tri(t/\tau)$	$\tau \,\text{sinc}^2\left(\frac{\omega\tau}{2}\right)$
$t^n e^{-at}u(t)$, $\text{Re}(a) > 0$	$\frac{n!}{(a+j\omega)^{n+1}}$	$e^{-at}\sin \omega_0 t\, u(t)$, $\text{Re}(a) > 0$	$\frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$
		$e^{-at}\cos \omega_0 t\, u(t)$, $\text{Re}(a) > 0$	$\frac{a+j\omega}{(a+j\omega)^2 + \omega_0^2}$

LAPLACE TRANSFORM

Definition of Laplace Transform

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$$

Properties of the Laplace transform

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$ $\frac{d^2 f}{dt^2}$ $\frac{d^n f}{dt^n}$	$sF(s) - f(0^-)$ $s^2F(s) - sf(0^-) - f'(0^-)$ $s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \ldots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(t)dt$	$\frac{1}{s}F(s)$
Frequency differentiation	$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{d\omega^n}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}} = \frac{1}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$

Laplace transform pairs

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t^n	$\frac{n!}{s^{n+1}}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t+\theta)$	$\frac{s\sin\theta + \omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t+\theta)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

s-domain equivalents

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$$

or

$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$V_C(s) = \frac{1}{sC}I_C(s) + \frac{v_C(0^-)}{s}$$

or

$$I_C(s) = sCV_C(s) - Cv_C(0^-)$$