BEE2143 – Signals & Networks

Chapter 6 – Applications of the Fourier Transform

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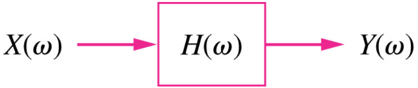
References

Circuit analysis

- The Fourier transform generalizes the phasor technique to nonperiodic functions
- Therefore, we apply Fourier transforms to circuits with nonsinusoidal excitations in exactly the same way we apply phasor techniques to circuits with sinusoidal excitations
- Thus, Ohm's law is still valid
- Once we transform the functions for the circuit elements into the frequency domain and take the Fourier transforms of the excitations, we can use circuit techniques
- Although the Fourier transform method produces a response that exists for $-\infty < t < \infty$, Fourier analysis cannot handle circuits with initial conditions

- The transfer function is defined as the ratio of the output response $Y(\omega)$ to the input excitation $X(\omega)$

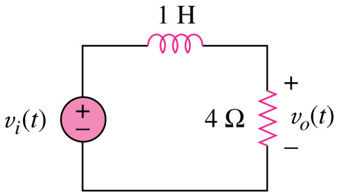
$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$



- This equation shows that if we know the transfer function and the input, we can readily find the output

Practice Problem 18.7, pg. 830 (Alexander & Sadiku, 2009)

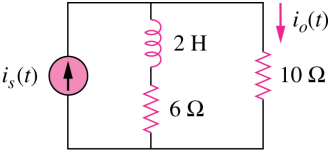
Determine $v_o(t)$ in Fig. 18.19 if
$v_i(t) = 5\,\mathrm{sgn}(t) = (-5 + 10u(t))$ V.



Answer: $-5 + 10(1 - e^{-4t})u(t)$ V

Practice Problem 18.8, pg. 831 (Alexander & Sadiku, 2009)

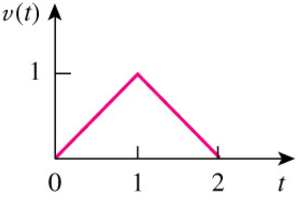
Find the current $i_o(t)$ in the circuit in Fig. 18.21, given that $i_s(t) = 5\cos 4t$ A.



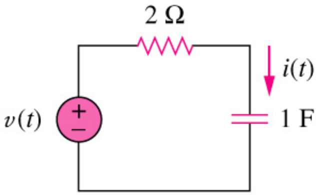
Answer: $2.95\cos(4t + 26.57°)$ A

Problem 18.40, pg. 845 (Alexander & Sadiku, 2009)

Determine the current $i(t)$ in the circuit of Fig. 18.42(b), given the voltage source shown in Fig. 18.42(a).



(a)



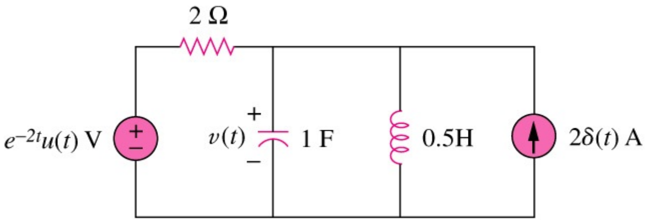
(b)

Answer:

$$i(t) = (1-e^{-0.5t})u(t)-2[1-e^{-0.5(t-1)}]u(t-1)+[1-e^{-0.5(t-2)}]u(t-2)\ \text{A}$$

Problem 18.41, pg. 845 (Alexander & Sadiku, 2009)

Determine the Fourier transform of $v(t)$ in the circuit shown in Fig. 18.43.



Answer:

$$V(\omega) = \frac{j2\omega(4.5 + j2\omega)}{(2 + j\omega)(4 - 2\omega^2 + j\omega)}$$

Parseval's theorem

- Parseval's theorem states that the total energy delivered to a 1-Ω resistor equals the total area under the square of $f(t)$ or $1/2\pi$ times the total area under the square of the magnitude of the Fourier transform of $f(t)$

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

- Since $|F(\omega)|^2$ is an even function, we may integrate from to and double the result; that is,

$$W_{1\Omega} = \frac{1}{\pi}\int_{0}^{\infty} |F(\omega)|^2 d\omega$$

- We may also calculate the energy in any frequency band $\omega_1 < \omega < \omega_2$ as

$$W_{1\Omega} = \frac{1}{2\pi}\int_{\omega_1}^{\omega_2} |F(\omega)|^2 d\omega$$

Practice Problem 18.9, pg. 834 (Alexander & Sadiku, 2009)

(a) Calculate the total energy absorbed by a 1-Ω resistor with $i(t) = 5e^{-2|t|}$ A in the time domain.

(b) Repeat (a) in the frequency domain.

Answer: (a) 12.5 J, (b) 12.5 J

Practice Problem 18.10, pg. 835 (Alexander & Sadiku, 2009)

A 2-Ω resistor has $i(t) = 2e^{-t}u(t)$ A. What percentage of the total energy is in the frequency band $-4 < \omega < 4$ rad/s?

Answer: 84.4%

Amplitude modulation

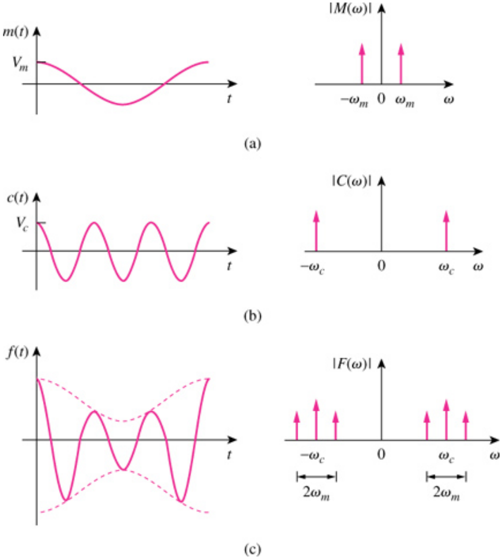
- Amplitude modulation (AM) is a process whereby the amplitude of the carrier is controlled by the modulating signal
- Suppose the audio information, such as voice or music (or the modulating signal in general) to be transmitted is $m(t) = V_m \cos \omega_m t$, while the high-frequency carrier is $c(t) = V_c \cos \omega_c t$, where $\omega_c >> \omega_m$
- Then an AM signal $f(t)$ is given by

$$f(t) = [1 + m(t)]c(t) = V_c[1 + m(t)] \cos \omega_c t$$

- We can use the amplitude modulation property together with the Fourier transform of the cosine function to determine the spectrum of the AM signal:

$$
\begin{aligned}
F(\omega) &= \mathcal{F}[V_c \cos \omega_c t] + \mathcal{F}[V_c m(t) \cos \omega_c t] \\
&= V_c \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{V_c}{2}[M(\omega - \omega_c) + M(\omega + \omega_c)]
\end{aligned}
$$

- The AM signal consists of the carrier and two other sinusoids
- The sinusoid with frequency is known as the lower sideband, while the one with frequency is known as the upper sideband


$m(t)$
V_m
t
$|M(\omega)|$
$-\omega_m$ 0 ω_m ω
(a)
$c(t)$
V_c
t
$|C(\omega)|$
$-\omega_c$ 0 ω_c ω
(b)
$f(t)$
t
$|F(\omega)|$
$-\omega_c$ 0 ω_c ω
$2\omega_m$
$2\omega_m$
(c)

Example 18.11, pg. 838 (Alexander & Sadiku, 2009)

A music signal has frequency components from 15 Hz to 30 kHz. If this signal could be used to amplitude modulate a 1.2-MHz carrier, find the range of frequencies for the lower and upper sidebands.

Answer: 1,170,000 to 1,199,985 Hz   and   1,200,015 to 1,230,000 Hz

Practice Problem 18.11, pg. 838 (Alexander & Sadiku, 2009)

If a 2-MHz carrier is modulated by a 4-kHz intelligent signal, determine the frequencies of the three components of the AM signal that results.

Answer: 2,004,000 Hz, 2,000,000 Hz, 1,996,000 Hz

List of References

1. C.K. Alexander and M.N.O. Sadiku (2009), Fundamentals of Electric Circuits 4th ed., New York: McGraw-Hill.