BEE2143 – Signals & Networks

Chapter 2 – Elementary Signals

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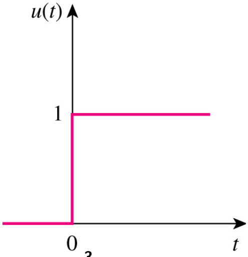
Elementary signals

i Unit step function

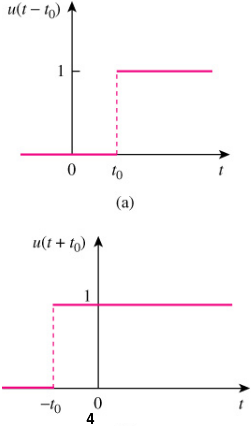
- The unit step function $u(t)$ is 0 for negative values of t and 1 for positive values of t.

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

- The unit step function is undefined at $t = 0$, where it changes abruptly from 0 to 1.



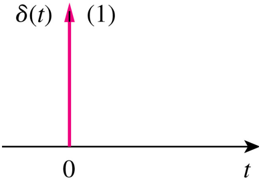
- Unit step function (cont.)
  - If the abrupt change occurs at $t = t_0$ or $t = -t_0$ (where $t_0 > 0$) instead of $t = 0$, the unit step function becomes $u(t - t_0)$ or $u(t + t_0)$ which is the same as saying that $u(t)$ is delayed or advanced by t_0 seconds.



ii Unit impulse function

- The unit impulse function $\delta(t)$ is zero everywhere except at $t = 0$, where it is undefined (infinity).

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



ii Unit impulse function (cont.)

- It may be visualized as a very short duration pulse of unit area.

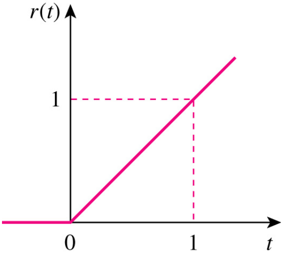
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$$

iii Unit ramp function

- The unit ramp function is zero for negative values of t and has a unit slope for positive values of t.

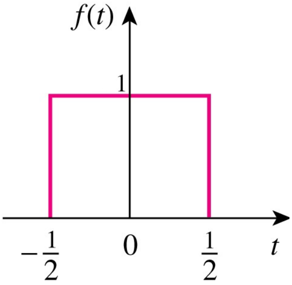
$$\mathrm{ramp}(t) = \begin{cases} 0, & t < 0 \\ t, & t > 0 \end{cases}$$
$$= tu(t)$$



iv Rectangular function

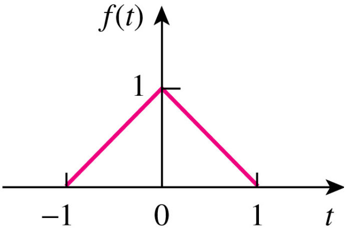
- The rectangular function is defined as

$$\mathrm{rect}(t) = \begin{cases} 1, & -\frac{1}{2} < t < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$
$$= u(t + \tfrac{1}{2}) - u(t - \tfrac{1}{2})$$



- Triangular function
  - The rectangular function is defined as

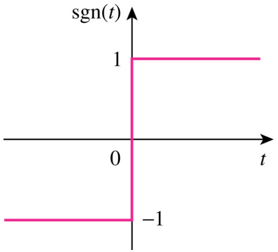
$$\mathrm{tri}(t) = \begin{cases} 1 - t, & -1 < t < 0 \\ 1 + t, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$
$$= (t+1)u(t+1) - 2tu(t) + (t-1)u(t-1)$$



vi Signum function

- The signum function or sign function is an odd mathematical function that extracts the sign of a real number.

$$
\begin{aligned}
\operatorname{sgn}(t) &= \begin{cases} -1, & t < 0 \\ 1, & t > 0 \end{cases} \\
&= 2u(t) - 1
\end{aligned}
$$



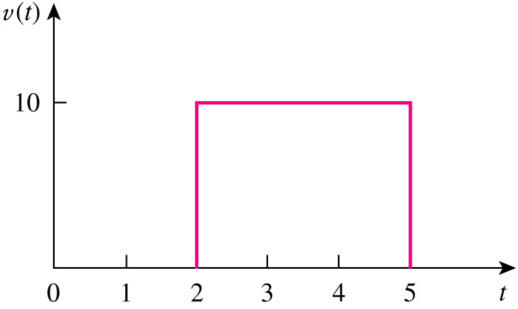
vii Sinc function

- The sinc function, also called the "sampling function," is a function that arises frequently in signal processing and the theory of Fourier transforms. The full name of the function is "sine cardinal," but it is commonly referred to by its abbreviation, "sinc".

$$\mathrm{sinc}(t) = \frac{\sin t}{t}$$

Example 7.6, pg. 269 (Alexander & Sadiku, 2009)

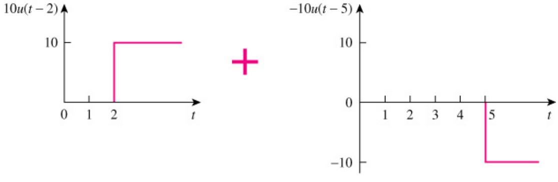
Express the voltage pulse in Fig. 7.31 in terms of the unit step. Calculate its derivative and sketch it.



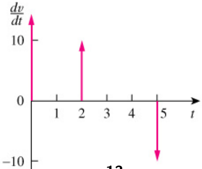
Example 7.6, pg. 269 (cont.)

Answer:

$$v(t) = 10[u(t-2) - u(t-5)], \quad \frac{dv(t)}{dt} = 10[\delta(t-2) - \delta(t-5)]$$

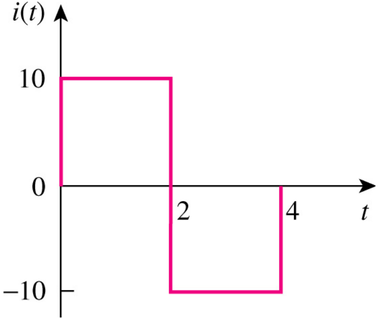


(a)



Practice Problem 7.6, pg. 270 (Alexander & Sadiku, 2009)

Express the current pulse in Fig. 7.33 in terms of the unit step. Find its integral and sketch it.

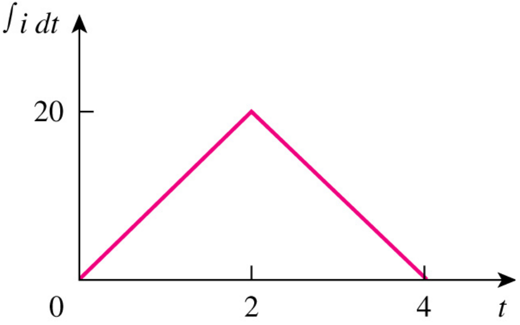


Practice Problem 7.6, pg. 270 (cont.)

Answer:

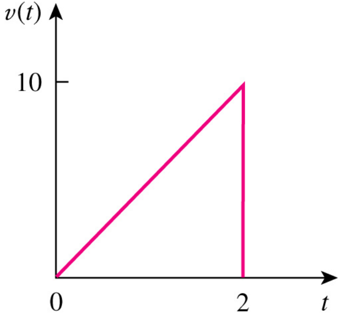
$$i(t) = 10[u(t) - 2u(t-2) + u(t-4)]$$

$$\int i(t)dt = 10[tu(t) - 2(t-2)u(t-2) + (t-4)u(t-4)]$$



Example 7.7, pg. 270 (Alexander & Sadiku, 2009)

Express the sawtooth function shown in Fig. 7.35 in terms of singularity functions.

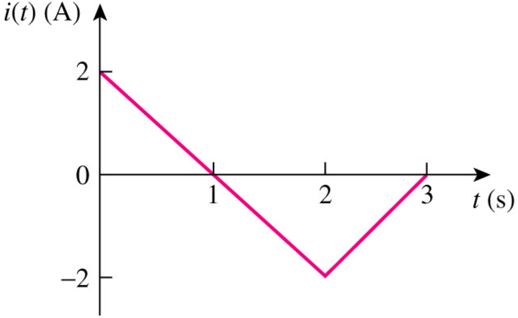


Answer:

$$v(t) = 5t[u(t) - u(t-2)]$$

Practice Problem 7.7, pg. 272 (Alexander & Sadiku, 2009)

Refer to Fig. 7.39. Express $i(t)$ in terms of singularity functions.



Answer:

$$i(t) = (2 - 2t)u(t) + (4t - 8)u(t - 2) - (2t - 6)u(t - 3)$$

Signals Operations

Types of signals operations:

- Time reversal: $f(-t)$
- Time scaling: $f(kt)$
- Time shifting: $f(t - t_0)$
- Amplitude scaling and shifting: $af(t) + b$

Convolution

- The term convolution means "folding"
- Convolution is an invaluable tool to the engineer because it provides a means of viewing and characterizing physical systems
- For example, it is used in finding the response $y(t)$ of a system to an excitation $x(t)$, knowing the system impulse response $h(t)$

- This is achieved through the convolution integral, defined as

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$$

or simply

$$y(t) = x(t) * h(t)$$

where λ is a dummy variable and the asterisk denotes convolution

- The convolution process is commutative:

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

or

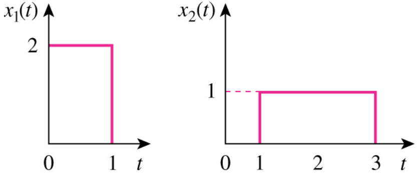
$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

- This implies that the order in which the two functions are convolved is immaterial
- The process of convolving two signals in the time domain is better appreciated from a graphical point of view
- The graphical procedure for evaluating the convolution integral in Eq. (15.70) usually involves four steps.
- Steps to evaluate the convolution integral:
  1. Folding: Take the mirror image of $h(\lambda)$ about the ordinate axis to obtain $h(-\lambda)$.
  2. Displacement: Shift or delay $h(-\lambda)$ by t to obtain $h(t-\lambda)$.
  3. Multiplication: Find the product of $h(t-\lambda)$ and $x(\lambda)$.
  4. Integration: For a given time t, calculate the area under the product $h(t-\lambda)x(\lambda)$ for $0 < \lambda < t$ to get $y(t)$ at t.

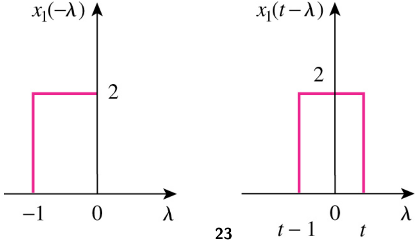
- To apply the four steps, it is necessary to be able to sketch $x(\lambda)$ and $h(t-\lambda)$
- To get $x(\lambda)$ from the original function $x(t)$ involves merely replacing t with λ
- Sketching $h(t-\lambda)$ is the key to the convolution process
- It involves reflecting $h(\lambda)$ about the vertical axis and shifting it by t
- Analytically, we obtain $h(t-\lambda)$ by replacing every t in $h(t)$ by $t-\lambda$
- Since convolution is commutative, it may be more convenient to apply steps 1 and 2 to $x(t)$ instead of $h(t)$

Example 15.12, pg. 700 (Alexander & Sadiku, 2009)

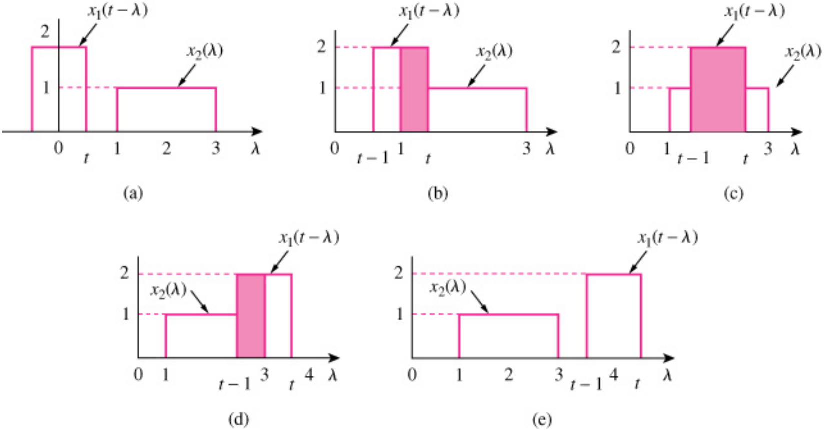
Find the convolution of the two signals in Fig. 15.10.



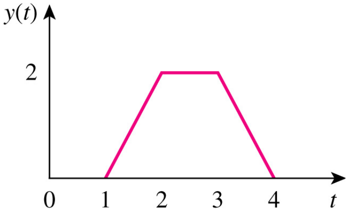
Answer:



Example 15.12, pg. 700 (cont.)



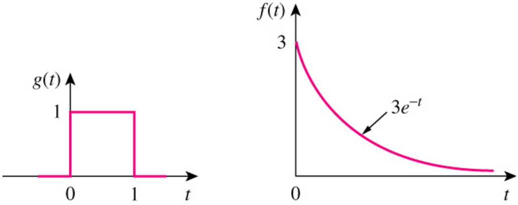
Example 15.12, pg. 700 (cont.)



$$y(t) = \begin{cases} 0, & 0 \leq t \leq 1 \\ 2t - 2, & 1 \leq t \leq 2 \\ 2, & 2 \leq t \leq 3 \\ 8 - 2t, & 3 \leq t \leq 4 \\ 0, & t \geq 4 \end{cases}$$

Practice Problem 15.13, pg. 703 (Alexander & Sadiku, 2009)

Given $g(t)$ and $f(t)$ in Fig. 15.20, graphically find $y(t) = g(t) * f(t)$.

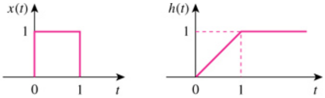


Answer:

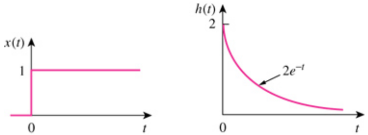
$$y(t) = \begin{cases} 3(1 - e^{-t}, & 0 \le t \le 1 \\ 3(e - 1)e^{-t}, & t \ge 1 \\ 0, & \text{elsewhere} \end{cases}$$

Problem 15.43, pg. 713 (Alexander & Sadiku, 2009)

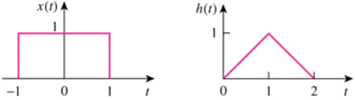
Find $y(t) = x(t) * h(t)$ for each paired $x(t)$ and $h(t)$ in Fig. 15.37.



(a)



(b)



(c)

Problem 15.43, pg. 713 (cont.)

Answer:

(a) $$y(t) = \begin{cases} \frac{1}{2}t^2, & 0 < t < 1 \\ -\frac{1}{2}t^2 + 2t - 1, & 1 < t < 2 \\ 1, & t > 2 \\ 0, & \text{otherwise} \end{cases}$$

(b) $y(t) = 2(1 - e^{-t}), \quad t > 0$

(c) $$y(t) = \begin{cases} \frac{1}{2}t^2 + t + \frac{1}{2}, & -1 < t < 0 \\ -\frac{1}{2}t^2 + t + \frac{1}{2}, & 0 < t < 2 \\ \frac{1}{2}t^2 - 3t + \frac{9}{2}, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$$

List of References

1. C.K. Alexander and M.N.O. Sadiku (2009), Fundamentals of Electric Circuits 4th ed., New York: McGraw-Hill.