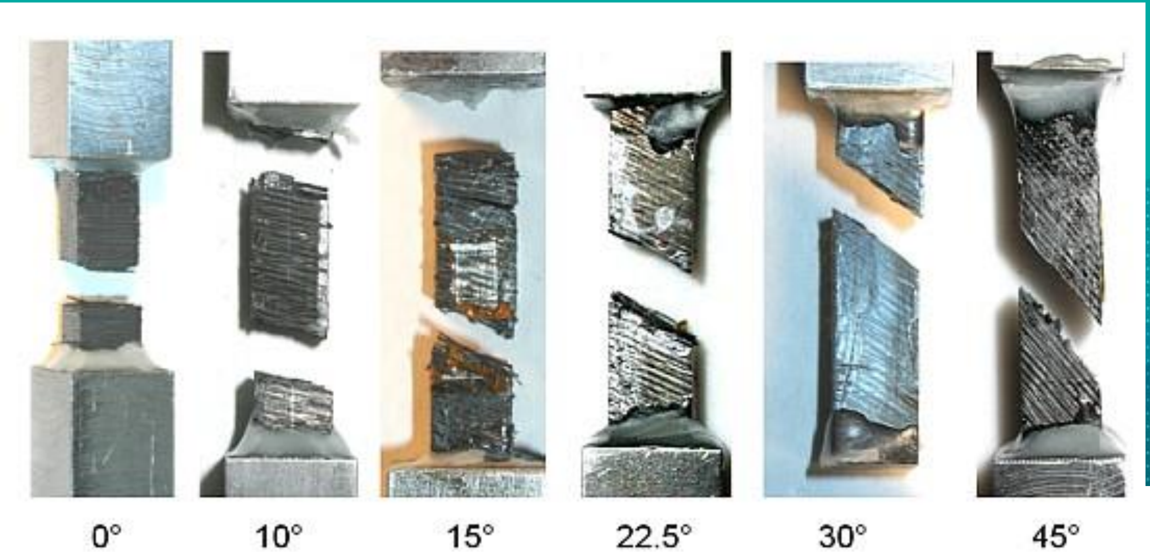
Universiti Malaysia PAHANG

BMM4893: Mechanics of Composite Materials

Chapter 4: Failure, Analysis & Design of Laminates



By M.R.M.Rejab

Learning Outcomes

By the end of the topic, students shall be able to:

1. Explain the significance of stiffness, and mechanical response of special cases of laminates
2. Analyse the failure criteria for laminates based on failure of individual lamina in a laminate
3. Design laminated structures subjected to in-plane loading

Special Cases of Laminates

Symmetric Laminates

A laminate is called symmetric if the material, angle, and thickness of plies are the same above and below the midplane. An example of symmetric laminates is $[0/30/60]_s$:

0
30
60
30
0

$$[B] = 0.$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \qquad \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

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Special Cases of Laminates

Cross-ply Laminate

Angle Ply Laminate

Antisymmetric Laminate

Balanced Laminate

Quasi-isotropic Laminate

Special Cases of Laminates: Cross-ply Laminate

A laminate is called a cross-ply laminate (also called laminates with specially orthotropic layers) if only 0 and 90° plies were used to make a laminate. An example of a cross ply laminate is a $[0/90_2/0/90]$ laminate:

For cross-ply laminates, $A_{16} = 0$, $A_{26} = 0$, $B_{16} = 0$, $B_{26} = 0$, $D_{16} = 0$, and $D_{26} = 0$; thus, Equation (4.29) can be written as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}.$$

Special Cases of Laminates: Angle ply Laminate

A laminate is called an angle ply laminate if it has plies of the same material and thickness and only oriented at +θ and –θ directions. An example of an angle ply laminate is [–40/40/–40/40]:

If a laminate has an even number of plies, then $A_{16} = A_{26} = 0$. However, if the number of plies is odd and it consists of alternating +θ and –θ plies, then it is symmetric, giving $[B] = 0$, and A_{16}, A_{26}, D_{16}, and D_{26} also become small as the number of layers increases for the same laminate thickness. This behavior is similar to the symmetric cross-ply laminates. However, these angle ply laminates have higher shear stiffness and shear strength properties than cross-ply laminates.

Special Cases of Laminates: Antisymmetric Laminate

A laminate is called antisymmetric if the material and thickness of the plies are the same above and below the midplane, but the ply orientations at the same distance above and below the midplane are negative of each other. An example of an antisymmetric laminate is:

From Equation (4.28a) and Equation (4.28c), the coupling terms of the extensional stiffness matrix, $A_{16} = A_{26} = 0$, and the coupling terms of the bending stiffness matrix, $D_{16} = D_{26} = 0$:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{22} & B_{16} & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & 0 \\ B_{16} & B_{26} & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

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Special Cases of Laminates: Balanced Laminate

A laminate is balanced if layers at angles other than 0 and 90° occur only as plus and minus pairs of $+\theta$ and $-\theta$. The plus and minus pairs do not need to be adjacent to each other, but the thickness and material of the plus and minus pairs need to be the same. Here, the terms $A_{16} = A_{26} = 0$. An example of a balanced laminate is [30/40/–30/30/–30/–40]:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{26} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}.$$

Special Cases of Laminates: Quasi-Isotropic Laminate

A laminate is called quasi-isotropic if its extensional stiffness matrix $[A]$ behaves like that of an isotropic material. This implies not only that $A_{11} = A_{22}$, $A_{16} = A_{26} = 0$, and $A_{66} = \dfrac{A_{11} - A_{12}}{2}$, but also that these stiffnesses are independent of the angle of rotation of the laminate. The reason for calling such a laminate quasi-isotropic and not isotropic is that the other stiffness matrices, $[B]$ and $[D]$, may not behave like isotropic materials. Examples of quasi-isotropic laminates include $[0/\pm 60]$, $[0/\pm 45/90]_s$, and $[0/36/72/-36/-72]$.

Special Cases of Laminates

Example 5.1

A [0/±60] graphite/epoxy laminate is quasi-isotropic. Find the three stiffness matrices [*A*], [*B*], and [*D*] and show that

1. $A_{11} = A_{22}$; $A_{16} = A_{26} = 0$; $A_{66} = \dfrac{A_{11} - A_{12}}{2}$.
2. $[B] \neq 0$, unlike isotropic materials.
3. [*D*] matrix is unlike isotropic materials.

Use properties of unidirectional graphite/epoxy lamina from Table 2.1. Each lamina has a thickness of 5 mm.

Failure Criterion of Laminate

A laminate will fail under increasing mechanical and thermal loads. The laminate failure, however, may not be catastrophic. It is possible that some layer fails first and that the composite continues to take more loads until all the plies fail. Failed plies may still contribute to the stiffness and strength of the laminate. The degradation of the stiffness and strength properties of each failed lamina depends on the philosophy followed by the user.

- When a ply fails, it may have cracks parallel to the fibers. This ply is still capable of taking load parallel to the fibers. Here, the cracked ply can be replaced by a hypothetical ply that has no transverse stiffness, transverse tensile strength, and shear strength. The longitudinal modulus and strength remain unchanged.
- When a ply fails, fully discount the ply and replace the ply of near zero stiffness and strength. Near zero values avoid singularities in stiffness and compliance matrices.

The procedure for finding the successive loads between first ply failure and last ply failure given next follows the fully discounted method:

1. Given the mechanical loads, apply loads in the same ratio as the applied loads. However, apply the actual temperature change and moisture content.
2. Use laminate analysis to find the midplane strains and curvatures.
3. Find the local stresses and strains in each ply under the assumed load.
4. Use the ply-by-ply stresses and strains in ply failure theories discussed in Section 2.8 to find the strength ratio. Multiplying the strength ratio to the applied load gives the load level of the failure of the first ply. This load is called the *first ply failure* load.
5. Degrade fully the stiffness of damaged ply or plies. Apply the actual load level of previous failure.
6. Go to step 2 to find the strength ratio in the undamaged plies:
   - If the strength ratio is more than one, multiply the strength ratio to the applied load to give the load level of the next ply failure and go to step 2.
   - If the strength ratio is less than one, degrade the stiffness and strength properties of all the damaged plies and go to step 5.
7. Repeat the preceding steps until all the plies in the laminate have failed. The load at which all the plies in the laminate have failed is called the *last ply failure*.

Design of a Laminated Composite

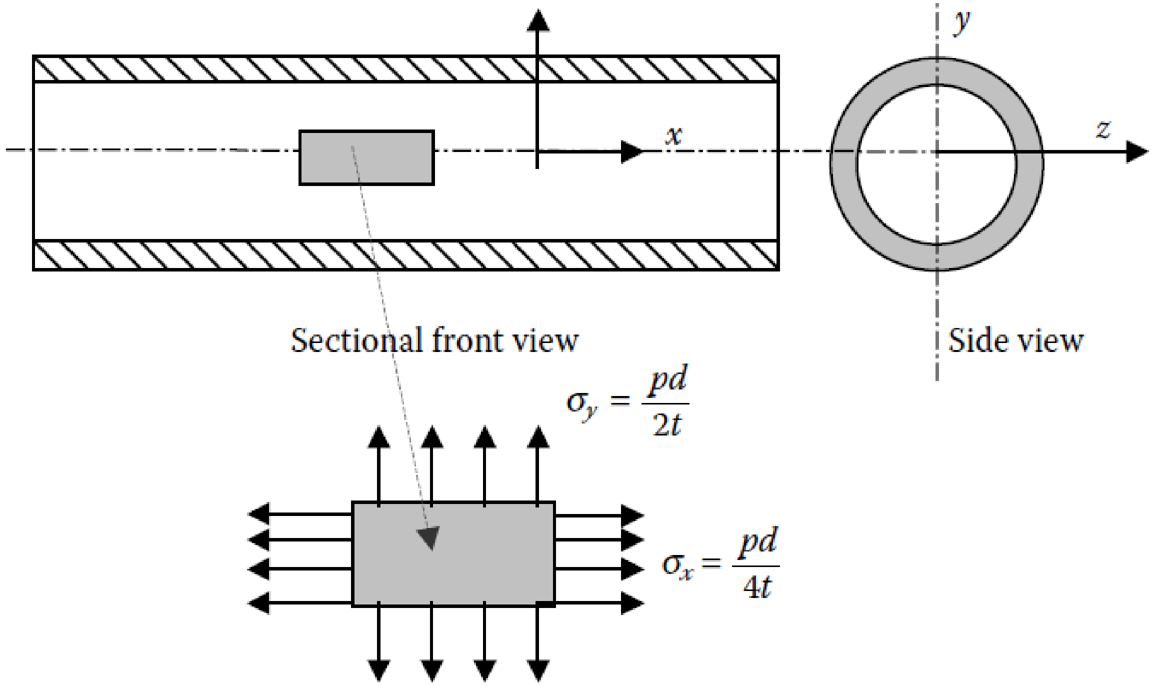
Design of laminated composites includes constraints on optimizing and constraining factors such as

- Cost
- Mass as related to aerospace and automotive industry to reduce energy cost
- Stiffness (to limit deformations) as related to aircraft skins to avoid buckling
- Thermal and moisture expansion coefficients as related to space antennas to maintain dimensional stability

Design of a Laminated Composite

Example 5.7

A 6-ft-long cylindrical pressure vessel (Figure 5.2) with an inner diameter of 35 in. is subjected to an internal gauge pressure of 150 psi. The vessel operates at room temperature and curing residual stresses are neglected. The cost of a graphite/epoxy lamina is 250 units/lbm and cost of a glass/epoxy lamina is 50 units/lbm. The following are other specifications of the design:



Design of a Laminated Composite

1. Only 0°, +45°, –45°, +60°, –60°, and 90° plies can be used.
2. Only symmetric laminates can be used.
3. Only graphite/epoxy and glass/epoxy laminae, as given in Table 2.2, are available, but hybrid laminates made of these two laminae are allowed. The thickness of each lamina is 0.005 in.*
4. Calculate specific gravities of the laminae using Table 3.3 and Table 3.4 and fiber volume fractions given in Table 2.2.
5. Neglect the end effects and the mass and cost of ends of the pressure vessel in your design.
6. Use Tsai–Wu failure criterion for calculating strength ratios.
7. Use a factor of safety of 1.95.

$$F = \frac{A}{B} + \frac{C}{D},$$

A = mass of composite laminate
B = mass of composite laminate if design was based only on minimum mass
C = cost of composite laminate
D = cost of composite laminate if design was based only on minimum cost

Other Mechanical Design Issues

- Sandwich Composites
- Long Term Environmental Effects
- Interlaminar Stresses
- Impact Resistance
- Fatigue Resistance
- Fracture Resistance

Thank you

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