REINFORCED CONCRETE DESIGN 1

Design of Slab (Examples and Tutorials)

by

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Example 1: Simply supported One way slab

A rectangular reinforced concrete slab is simply-supported on two masonry walls 250 mm thick and 3.75 m apart. The slab has to carry a distributed permanent action of 1.0 kN/m^2 (excluding slab self-weight) and a variable action of 3.0 kN/m^2. The materials to be used are grade C25 concrete and grade 500 reinforcement. The slab is outside buildings which subjected to 1 hr fire resistance and design for 50 years design life. Design the slab. Assume diameter of bar = 12 mm.

Example 1: Simply supported One way slab

SLAB THICKNESS

Minimum thickness for fire resistance = 80 mm

Estimated thickness considering deflection control,

h = 3750 / 26 = 144 mm. **Use 150 mm**

DURABILITY, FIRE & BOND REQUIREMENTS

Min. cover with regard to bond, $C_{bond} = 12$ mm

Min. cover with regard to durability, $C_{min,dur} = 20$ mm

Min. required axis distance for R60 fire resistance, a = 20 mm

Min. cover with regard to fire,

$C_{min,fire} = a - \phi_{bar}/2 = 20 - 0.5(12) = 14$ mm

Allowance in design for deviation, $\Delta C_{dev} = 10$ mm

$\therefore$ Nominal cover, $C_{nom} = C_{min} + \Delta C_{dev}$

$= 20 + 10 \quad = C_{nom} \quad = 30$ mm

Use:

C_{nom} = 30 mm

Example 1: Simply supported One way slab

ACTIONS & ANALYSIS

Slab self-weight = 0.150 x 25 = 3.75 kN/m^2

Permanent load (excluding self-weight) = 1.0 kN/m^2

Total permanent action, g_k = 4.75 kN/m^2

Variable action, q_k = 3.00 kN/m^2

Design action, n_d = 1.35(4.75) + 1.5(3.00) = 10.91 kN/m^2

Consider 1 m width, $w_d = n_d$ x 1 m = 10.91 kN/m

Shear force,

$$V = w_d L/2$$
$$= 20.46 \text{ kN/m}$$

Bending moment,

$$M = w_d L^2/8$$
$$= 19.18 \text{ kNm}$$

Example 1: Simply supported One way slab

MAIN REINFORCEMENT

Effective depth:

$d = h - c_{nom} - 0.5\phi_{bar} = 150 - 30 - 0.5(12) = 114$ mm

Design moment, $M_{ED} = 19.18$ kNm

$K = M/bd^2f_{ck}$

$= (19.18 \times 10^6)/(1000 \times 114^2 \times 25) = 0.059 < K_{bal} = 0.167$

∴ Compression reinforcement is not required

$z = d\,[0.5 + \sqrt{0.25 - K/1.134}\,] = 0.94d \leq 0.95d$

$A_s = M/0.87f_{yk}z$

$= 19.18 \times 10^6/0.87 \times 500 \times 0.94 \times 114$

$= 412$ mm²/m

Main bar :

H12 -250

(452 mm²/m)

Example 1: Simply supported One way slab

Minimum and maximum reinforcement area,

$A_{s,min} = 0.26\ (f_{ctm}/f_{yk})bd = 0.26(2.60/500)bd = 0.0014bd$

$= 0.0014(1000))(114) \quad = 160\ mm^2/m$

$A_{s,max} = 0.04A_c = 0.04(1000)(150) = 6000\ mm^2/m$

Secondary bar:

H12-450

(251 mm²/m)

SHEAR

Design shear force, $V_{ED} = 20.46$ kN

Design shear resistance,

$V_{Rd,c} = [0.12k(100\rho_1 f_{ck})^{1/3}]bd$

$k = 1 + (200/d)^{1/2} \leq 2.0 = 2.32 \leq 2.0$

$\rho_1 \ = A_{s1}/bd \leq \ 0.02 = 452/1000\ x114 = 0.004 \leq 0.02$

$V_{Rd,c} = [0.12\ x\ 2.0\ (100x\ 0.004\ x\ 25)^{1/3}]\ x\ 1000\ x\ 114$

$= 58.95$ kN

Example 1: Simply supported One way slab

$V_{min} = [0.035k^{3/2} f_{ck}^{1/2}]bd = 0.035 \times 2^{3/2} \times 25^{1/2} \times 1000 \times 114 = 56.43$ kN

Thus, $V_{Rd,c}$ = 58.95 kN > V_{ED} = 20.46 kN, OK!

DEFLECTION

Percentage of required tension reinforcement,

$\rho = A_{s,req}/bd = 412 / 1000 \times 114 = 0.0036$

Reference reinforcement ratio,

$\rho_0 = (f_{ck})^{1/2} \times 10^{-3} = (25)^{1/2} \times 10^{-3} = 0.005$

Factor for structural system, K = 1.0

$$(L/d)_{basic} = K[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}(\frac{\rho_0}{\rho} - 1)^{3/2}]$$

$$(L/d)_{basic} = 1.0[11 + 10.42 + 3.88] = 25.3$$

Example 1: Simply supported One way slab

Modification factor for span less than 7 m = 1.0

Modification factor for steel area provided = $A_{s,prov}/A_{s,req}$ = 1.1 ≤ 1.5

Therefore, allowable span-effective depth ratio:

$(L/d)_{allowable}$ = 25.3 x 1 x 1.1 = 27.83

Actual span-effective depth ratio,

$(L/d)_{actual}$ = 3750/114 = 32.9 > $(L/d)_{allowable}$ =27.83, NOT OK!

Increase Area of steel provided to **566 mm^2/m (H12-200)**

Modification factor for steel area provided = $A_{s,prov}/A_{s,req}$ = 1.37 ≤ 1.5

Therefore, allowable span-effective depth ratio:

$(L/d)_{allowable}$ = 25.3 x 1 x 1.37 = 34.66 > $(L/d)_{actual}$ = 32.9 OK!

Example 1: Simply supported One way slab

CRACKING

$h = 175$ mm < 200 mm

Main bar:

$S_{max,slab} = 3h \leq 400 = 400$ mm,

Max. bar spacing = 200 < S_{max}, slab, OK

Secondary bar:

$S_{max,slab} = 3.5h \leq 450 = 450$

Max. bar spacing = 450 < S_{max}, slab, OK

Example 2: Continuous one way slab

Figure 1 shows a clear area of 12 m x 8.5 m for a hall construction in a school. The slab is supported on beams of size 225 x 500 mm spaced at 4.0 m centers. The slab thickness is to be designed as 150 mm. Given the characteristic permanent action (excluding self-weight) is 1.5 kN/m^2, characteristic variable action is 4.0 kN/m^2 with a design life of 50 years, fire exposure = REI 90, exposure class = XC1, characteristic concrete strength, f_{ck} = C25/30, high yield steel strength, f_{yk} = 500 N/mm^2, Unit weight of concrete = 25 kN/m^3. Use : Diameter of reinforcement = 10 mm

Example 2: Continuous one way slab

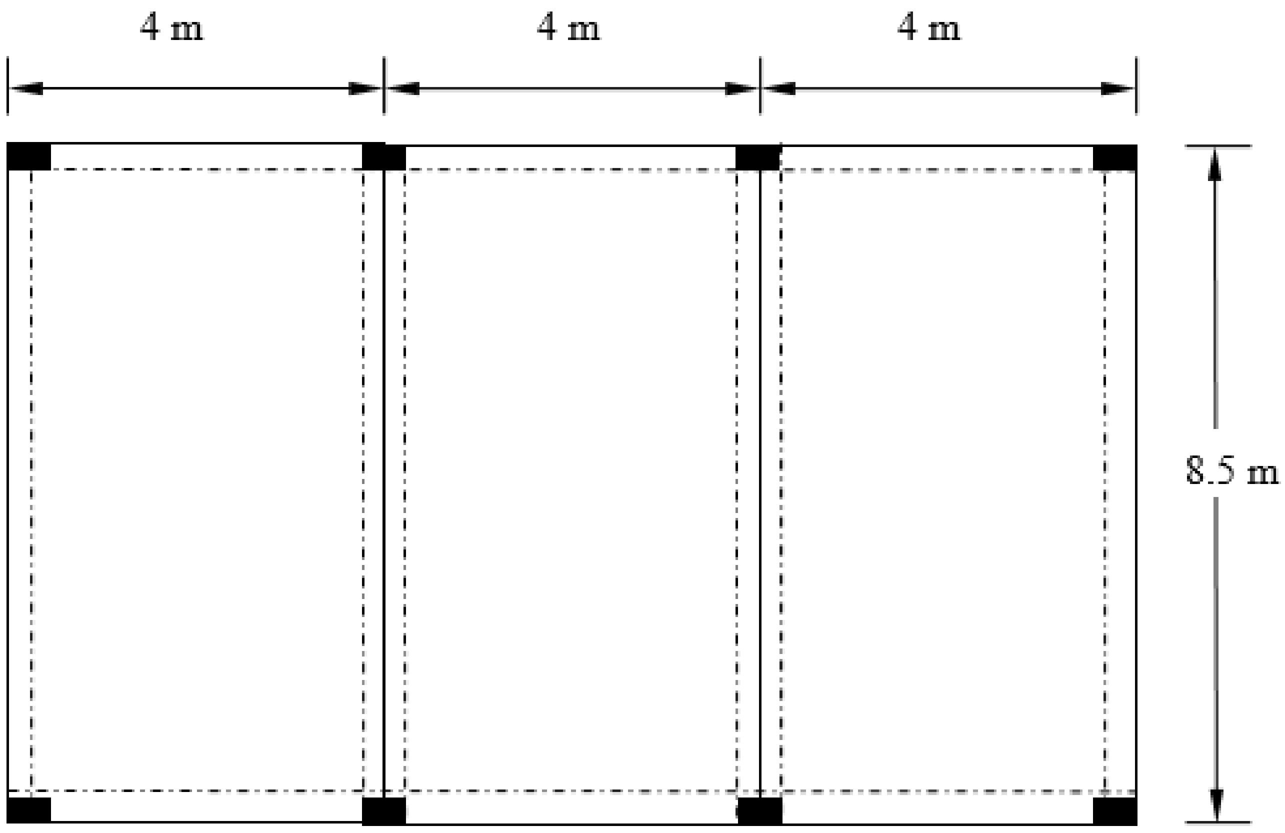


Figure 1

Example 2: Continuous one way slab

DURABILITY, FIRE & BOND REQUIREMENTS

Min. cover with regard to bond, $C_{bond} = 10$ mm

Min. cover with regard to durability, $C_{min,dur} = 15$ mm

Min. required axis distance for R90 fire resistance, a = 30 mm

Min. cover with regard to fire,

$C_{min,fire} = a - \phi_{bar}/2 = 30 - 0.5(10) = 25$ mm

Allowance in design for deviation, $\Delta C_{dev} = 10$ mm

∴ Nominal cover, $C_{nom} = C_{min} + \Delta C_{dev}$

$= 25 + 10 \quad = C_{nom} \quad = 25$ mm

Use: $C_{nom} = 25$ mm

Example 2: Continuous one way slab

ACTIONS

Slab self-weight = 0.15 x 25 = 3.75 kN/m^2

Permanent load (excluding self-weight) = 1.50 kN/m^2

Total permanent action, g_k = 5.25 kN/m^2

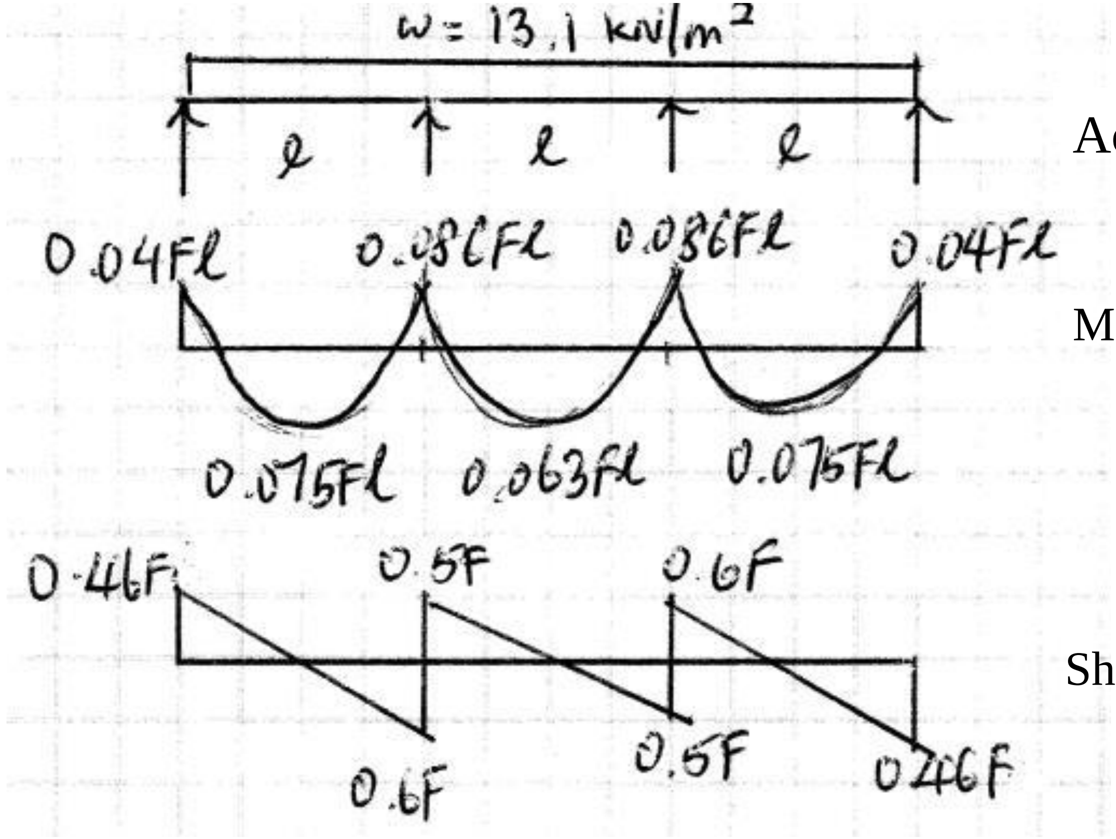
Variable action, q_k = 4.00 kN/m^2

Design action, n_d = 1.35(5.25) + (4.0) = 13.1 kN/m^2

Example 2: Continuous one way slab

ANALYSIS

Consider 1 m width of slab,



Action

Moment

Shear

F = wl = 13.1 x 4 = 52.4 kN

Example 2: Continuous one way slab

MAIN REINFORCEMENT

Effective depth:

$d_x = 150 – 35 - 0.5(10) = 110$ mm

Minimum and maximum reinforcement area,

$A_{s,min} = 0.26\ (f_{ctm}/f_{yk})bd = 0.26(2.56/500)bd = 0.0013bd$

$= 0.0013(1000))(110) \quad = 146\ mm^2/m$

$A_{s,max} = 0.04A_c = 0.04(1000)(150) = 6000\ mm^2/m$

Secondary bar:

H10 - 450

(175 mm^2/m)

At first interior support

$M = 0.086Fl = 0.086 \times 52.4 \times 4 = 18$ kNm/m

$K = M/bd^2f_{ck}$

$= (18 \times 10^6) / (1000 \times 110^2 \times 25) = 0.06 < K_{bal} = 0.167$

$\therefore$ Compression reinforcement is not required

Example 2: Continuous one way slab

$z = d\,[0.5 + \sqrt{0.25 - K/1.134}\,] = 0.94d \leq 0.95d$

$A_s = M/0.87 f_{yk} z$

$= 18 \times 10^6 / 0.87 \times 500 \times 0.94 \times 110$

$= 400\ mm^2/m$

Provide H10-175 top ($449\ mm^2/m$)

At middle interior span

$M = 0.063Fl = 0.063 \times 52.4 \times 4 = 13.2\ kNm/m$

$K = M/bd^2 f_{ck}$

$= (13.2 \times 10^6) / (1000 \times 110^2 \times 25) = 0.04 < K_{bal} = 0.167$

$\therefore$ Compression reinforcement is not required

$z = d\,[0.5 + \sqrt{0.25 - K/1.134}\,] = 0.96d \leq 0.95d$

$A_s = M/0.87 f_{yk} z$

$= 13.2 \times 10^6 / 0.87 \times 500 \times 0.95 \times 110$

$= 290\ mm^2/m$

Provide H10-250 bottom ($314\ mm^2/m$)

Example 2: Continuous one way slab

Near middle of end span

M = 0.075Fl = 0.075 x 52.4 x 4 = 15.72 kNm/m

$K = M/bd^2f_{ck}$

$= (15.72 \times 10^6)/(1000 \times 110^2 \times 25) = 0.052 < K_{bal} = 0.167$

∴ Compression reinforcement is not required

$z = d\,[0.5 + \sqrt{0.25 - K/1.134}\,] = 0.95d \leq 0.95d$

$A_s = M/0.87f_{yk}z$

$= 15.72 \times 10^6/0.87 \times 500 \times 0.95 \times 110$

$= 346\ mm^2/m$

Provide
H10-200 bottom
($393\ mm^2/m$)

At outer support

M = 0.04Fl = 0.04 x 52.4 x 4 = 8.4 kNm/m

Example 2: Continuous one way slab

$K = M/bd^2f_{ck}$

$= (8.4 \times 10^6) /(1000 \times 110^2 \times 25) = 0.028 < K_{bal} = 0.167$

$\therefore$ Compression reinforcement is not required

$z = d\,[0.5 + \sqrt{0.25 - K/1.134}\,] = 0.97d \leq 0.95d$

$A_s = M/0.87 f_{yk} z$

$= 8.4 \times 10^6/0.87 \times 500 \times 0.95 \times 110$

$= 185\ mm^2/m$

Provide

H10-400 bottom

(196 mm^2/m)

Example 2: Continuous one way slab

SHEAR

Maximum design shear force, V_{ED} = 0.6F = 0.6 x 52.4 = 31.44 kN

Design shear resistance,

$V_{Rd,c} = [0.12k(100\rho_1 f_{ck})^{1/3}]bd$

$k = 1 + (200/110)^{1/2} \leq 2.0$

$= 2.35 \leq 2.0$

$\rho_1 = A_{s1}/bd \leq 0.02 = 449/1000 \text{ x}110$

$= 0.0041 \leq 0.02$

$V_{Rd,c}$ = [0.12 x 2.0 (100x 0.0041 x 25)$^{1/3}$] x 1000 x 110 = 57.35 kN

V_{min} = [0.035k$^{3/2}$ fck$^{1/2}$]bd = 0.035 x 2$^{3/2}$ x 25$^{1/2}$ x 1000 x 110 = 54.4 kN

So, $V_{Rd,c}$ = 57.35 kN > V_{ED} = 31.44 kN **OK!**

Example 2: Continuous one way slab

DEFLECTION

Percentage of required tension reinforcement,

$\rho = A_{s,req}/bd = 346 /1000 \times 110 = 0.00315$

Reference reinforcement ratio,

$\rho_0 = (f_{ck})^{1/2} \times 10^{-3} = (25)^{1/2} \times 10^{-3} = 0.005,$

Factor for structural system, K = 1.3

$$(L/d)_{basic} = K[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}(\frac{\rho_0}{\rho} - 1)^{3/2}]$$

$$(L/d)_{basic} = 1.3[11 + 11.92 + 7.20] = 39.16$$

Modification factor for steel area provided

$$= A_{s,prov}/A_{s,req} = 393/346 = 1.14 \leq 1.5$$

Example 2: Continuous one way slab

Therefore, allowable span-effective depth ratio:

$(L/d)_{allowable}$ = 39.16 x 1 x 1.14 = 44.64

Actual span-effective depth ratio:

$(L/d)_{actual}$ = 4000/110 = 36.35 < $(L/d)_{allowable}$ = 44.64, OK!

CRACKING

h = 150 mm < 200 mm

Main bar:

$S_{max,slab} = 3h \leq 400 = 400$

Max. bar spacing = 400 ≤ S_{max}, slab, **OK!**

Secondary bar:

$S_{max,slab} = 3.5h \leq 450 = 450$

Max. bar spacing = 425 < 450 mm, **OK!**

Example 3: Two way slab restrained slab

Figure 2 shows part of the second floor plan of a reinforced concrete office building. The slab carries characteristic permanent action of 1.5 kN/m^2 (excluding self-weight) of finishes, ceiling and services and characteristic variable action of 4.5 kN/m^2. The construction materials considered for the concrete consist of Grade C25, whereas, for steel reinforcement consist of Grade 500. The density of concrete is taken as 25 kN/m^3.

Example 3: Two way slab restrained slab

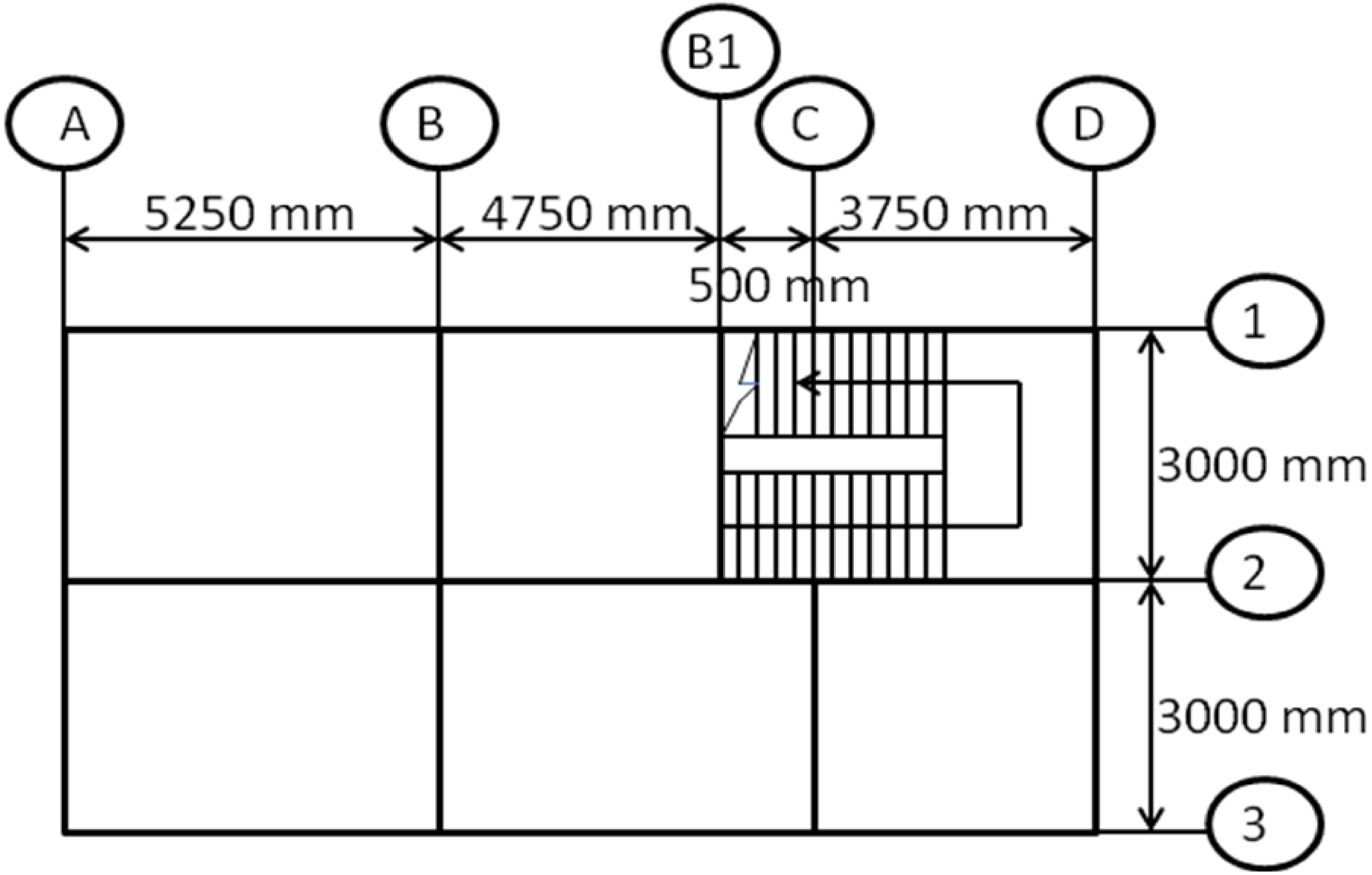


Figure 2

Example 3: Two way slab restrained slab

i) Based on the information given, calculate the area of steel required for the slab panel **C-D/2-3**. Use thickness of the slab as 150 mm, nominal cover equal to 25 mm and assume diameter of the bar is equal to 10 mm. Then, propose suitable reinforcements for this slab.

ii) Using the information obtained in (a), calculate the design shear force in the slab and check whether the slab is safe in terms of shear. Comment on your answer and give appropriate suggestions

iii) If the deflection check is passed, conduct the cracking check on the reinforcement provided for this slab. Comment on your answer and give appropriate suggestions.

Example 3: Two way slab restrained slab

ACTIONS & ANALYSIS

Slab self-weight = 0.15 x 25 = 3.75 kN/m^2

Permanent load (excluding self-weight) = 1.5 kN/m^2

Total permanent action, g_k = 5.25 kN/m^2

Variable action, q_k = 4.50 kN/m^2

Design action, n_d = 1.35(5.25) + 1.5(4.50) = 13.84 kN/m^2

L_y/L_x = 3250/3000 = 1.1 < 2.0 (Two way slab)

Case 8: Three edges discontinuous (One short edge continuous)

Short span: $M_{sx1} = \beta_{sx1} nlx^2 = 0.054 \times 13.84 \times 3^2 = 6.73$ kNm/m

Long span: $M_{sy1} = \beta_{sy1} nlx^2 = 0.044 \times 13.84 \times 3^2 = 5.48$ kNm/m

$M_{sy2} = \beta_{sy1} nlx^2 = 0.058 \times 13.84 \times 3^2 = 7.22$ kNm/m

Example 3: Two way slab restrained slab

MAIN REINFORCEMENT

Effective depth:

$d_x = 150 – 25 -0.5(10) = 120$ mm

$d_y= 150 – 25 -1.5(10) = 110$ mm

Minimum and maximum reinforcement area,

$A_{s,min} = 0.26 (f_{ctm}/f_{yk})bd = 0.26(2.6/500)bd$

$= 0.0013bd = 0.0013(1000))(120)$

$= 156\ mm^2/m$

Secondary bar:

H10 - 425

(185 mm^2/m)

$A_{s,max} = 0.04A_c = 0.04(1000)(150) = 6000\ mm^2/m$

Example 3: Two way slab restrained slab

Short span:

Mid-span:

M_{sx1} = 6.73 kNm/m

$K = M/f_{ck}bd^2$

$= (6.73 \times 10^6)/(25 \times 1000 \times 120^2) = 0.019 < K_{bal} = 0.167$

∴ Compression reinforcement not required

$z = d\,[0.5 + \sqrt{0.25 - K/1.134}\,] = 0.98d \leq 0.95d$

$A_s = M/0.87f_{yk}z$

$= 6.73 \times 10^6/0.87 \times 500 \times 0.95 \times 120$

$= 138\ mm^2/m$

Provide

H10-350 bottom

(225 mm^2/m)

Example 3: Two way slab restrained slab

Long span:

Mid-span:

M_{sy1} = 5.48 kNm/m

$K = M/f_{ck}bd^2$

$= (5.48 \times 10^6) / (25 \times 1000 \times 110^2) = 0.018 < K_{bal} = 0.167$

∴ Compression reinforcement not required

$z = d\,[0.5 + \sqrt{0.25 - K/1.134}\,] = 0.98d \leq 0.95d$

$A_s = M/0.87f_{yk}z$

$= 5.48 \times 10^6 / 0.87 \times 500 \times 0.95 \times 110$

$= 121\ mm^2/m$

Provide

H10-350 bottom

(225 mm^2/m)

Example 3: Two way slab restrained slab

Long span:

Support:

M_{sy2} = 7.22 kNm/m

$K = M/f_{ck}bd^2$

$= (7.22 \times 10^6) / (25 \times 1000 \times 110^2) = 0.024 < K_{bal} = 0.167$

∴ Compression reinforcement not required

$z = d\,[0.5 + \sqrt{0.25 - K/1.134}\,] = 0.98d \leq 0.95d$

$A_s = M/0.87f_{yk}z$

$= 7.22 \times 10^6 / 0.87 \times 500 \times 0.95 \times 110$

$= 159\ mm^2/m$

Provide

H10-350 bottom

(225 mm²/m)

Example 3: Two way slab restrained slab

SHEAR

Shear force,

Short span;

$V_{sx1} = \beta_{vs1}nlx = 0.33 \text{ x } 13.84 \text{ x } 3 = 13.7 \text{ kN/m}$

Long span;

$V_{sx1} = \beta_{vs2}nlx = 0.30 \text{ x } 13.84 \text{ x } 3 = 12.46 \text{ kN/m}$

$V_{sx2} = \beta_{vs2}nlx = 0.45 \text{ x } 13.84 \text{ x } 3 = 18.64 \text{ kN/m}$

Design shear force, $V_{ED} = 18.64$ kN

Design shear resistance,

$V_{Rd,c} = [0.12k(100\rho_1 fck)^{1/3}]bd$

$k = 1 + (200/d)^{1/2} \leq 2.0 = 2.34 \leq 2.0$

Example 3: Two way slab restrained slab

$\rho_1 = A_{s1}/bd \leq 0.02 = 159/1000 \text{ x}110 = 0.0015 \leq 0.02$

$V_{Rd,c} = [0.12 \text{ x } 2.0 \text{ } (100\text{x } 0.0015 \text{ x } 25)^{1/3}] \text{ x } 1000 \text{ x } 110$

$= 41.02 \text{ kN}$

$V_{min} = [0.035k^{3/2} \text{ } fck^{1/2}]bd = 0.035 \text{ x } 2^{3/2} \text{ x } 25^{1/2} \text{ x } 1000 \text{ x } 110$

$= 54.45 \text{ kN}$

So, $V_{Rd,c} = 54.45 \text{ kN} > V_{ED} = 18.64 \text{ kN}$, **OK!**

Example 3: Two way slab restrained slab

DEFLECTION

Percentage of required tension reinforcement,

$\rho = A_{s,req}/bd = 138 / 1000 \times 120 = 0.0012$

Reference reinforcement ratio,

$\rho_0 = (f_{ck})^{1/2} \times 10^{-3} = (25)^{1/2} \times 10^{-3} = 0.005$,

Factor for structural system, K = 1.3

$$(L/d)_{basic} = K[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}(\frac{\rho_0}{\rho} - 1)^{3/2}]$$

$$(L/d)_{basic} = 1.3[11 + 31.25 + 90.16] = 172.13$$

Modification factor for steel area provided

$= A_{s,prov}/A_{s,req} = 225/138 = 1.63 \leq 1.5$

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Example 3: Two way slab restrained slab

Therefore, allowable span-effective depth ratio:

$(L/d)_{allowable} = 172.13 \times 1 \times 1.5 = 258.2$

Actual span-effective depth ratio:

$\mathbf{(L/d)_{actual} = 3000/120 = 25 < (L/d)_{allowable}}$ **OK!**

CRACKING

h = 150 mm < 200 mm

Main bar:

$S_{max,slab} = 3h \leq 400 = 450 \leq 400$

Max. bar spacing = $350 \leq$ Smax, slab, **OK!**

Secondary bar: :

$S_{max,slab} = 3.5h \leq 450 = 525 \leq 450$

Max. bar spacing = $425 \leq 450$, **OK!**

Tutorial 1: One way continuous slab

Figure 3 shows a first floor plan of an office building. It is estimated that the 150 mm thick slab will carry 3.0 kN/m^2 variable action and 1.0 kN/m^2 from finishes and suspended ceiling. Given that the design life = 50 years, fire resistance = REI 60, exposure class = XC1, characteristic concrete strength, f_{ck} = C30/37, high yield steel strength, f_{yk} = 500 N/mm^2, unit weight of concrete = 25 kN/m^3, diameter of reinforcement = 10 mm.

Tutorial 1: One way continuous slab

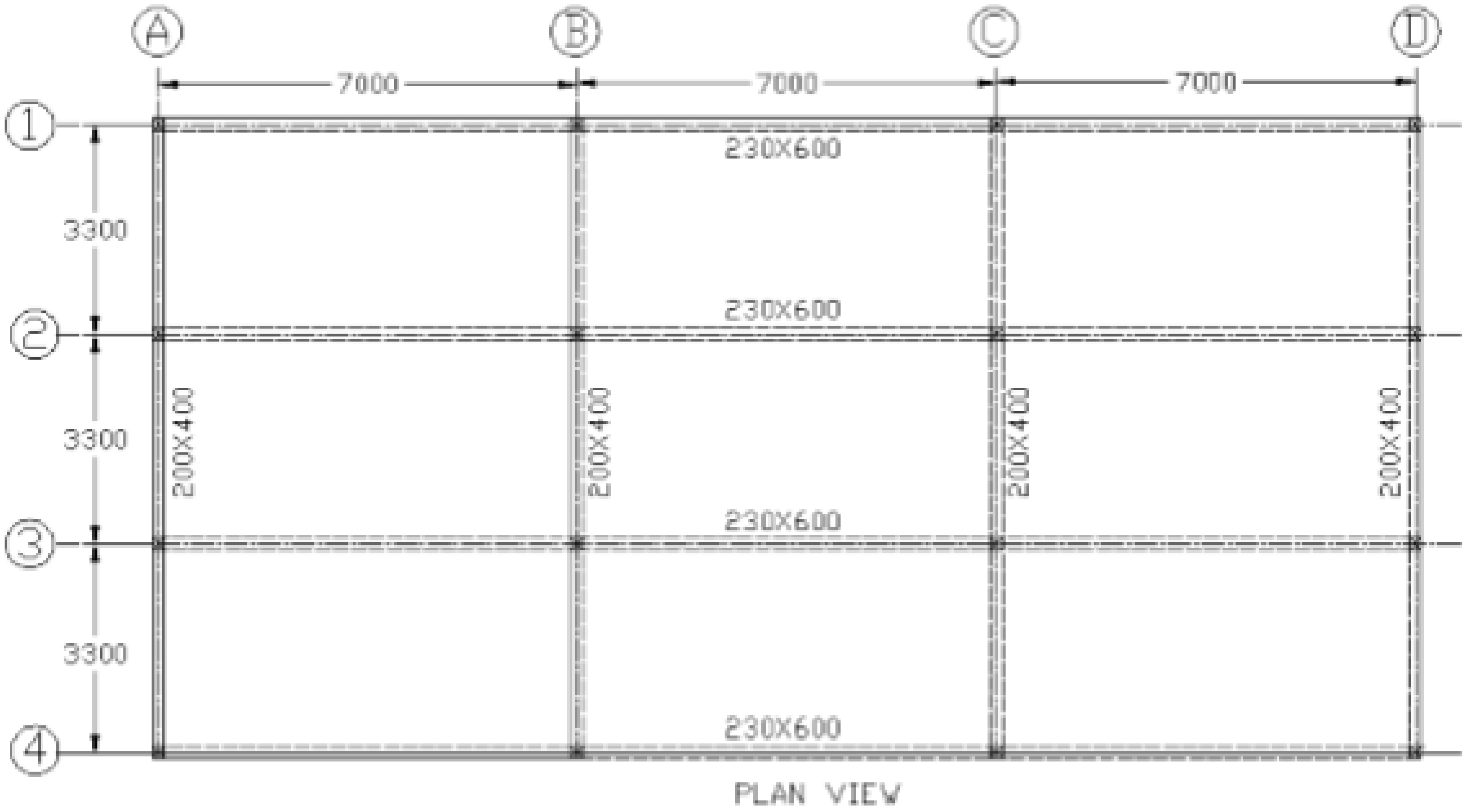

Figure 3

Tutorial 1: One way continuous slab

a) Calculate C_{nom} of the slab.

b) Calculate the loads on slab.

c) Draw the bending moment and shear force diagram.

d) Design the reinforcement and check shear.

e) Check deflection and cracking.

f) Construct the plan view detailing of the slab.

Tutorial 2: Two way simply supported slab

A renovation will be made on a double-storey house. The owner has requested to add another room measuring 5 m x 3 m. The slab will carry a variable action of 3.0 kN/m^2 and permanent action due to finishes of 1.25 kN/m^2. The characteristic material strength for concrete and steel will be used is 25 N/mm^2 and 500 N/mm^2, respectively. The slab can be considered simply supported on all four edges with corners free to lift. The slab is inside a building which is subjected to 1.5 hours fire resistance and 50 years design life.

Tutorial 2: Two way simply supported slab

a) If the slab thickness is taken as 150 mm, calculate and design the main reinforcement for the slab. Use concrete nominal cover equal to 30 mm.

b) Perform all checking on shear, deflection and cracking of the slab. Comment on your answer and give appropriate suggestions.

c) Construct the detail reinforcement obtained in (a) for the slab.

Tutorial 3: Two way restrained slab

Figure 4 shows a first floor plan of a reinforced concrete office building. During construction, slabs and beams are cast monolithically. The overall thickness of the slab is 150 mm. Based on the properties and design data provided, design slab panel S1.

Char. permanent action (excluding self-weight)	= 1.0 kN/m^2
Characteristic variable action	= 3.5 kN/m^2
Design life	= 50 years
Fire exposure	= R60
Exposure class	= XC1
Characteristic concrete strength, f_{ck}	= C30/37
High yield steel strength, f_{yk}	= 500 N/mm^2
Unit weight of concrete	= 25 kN/m^3

Tutorial 3: Two way restrained slab

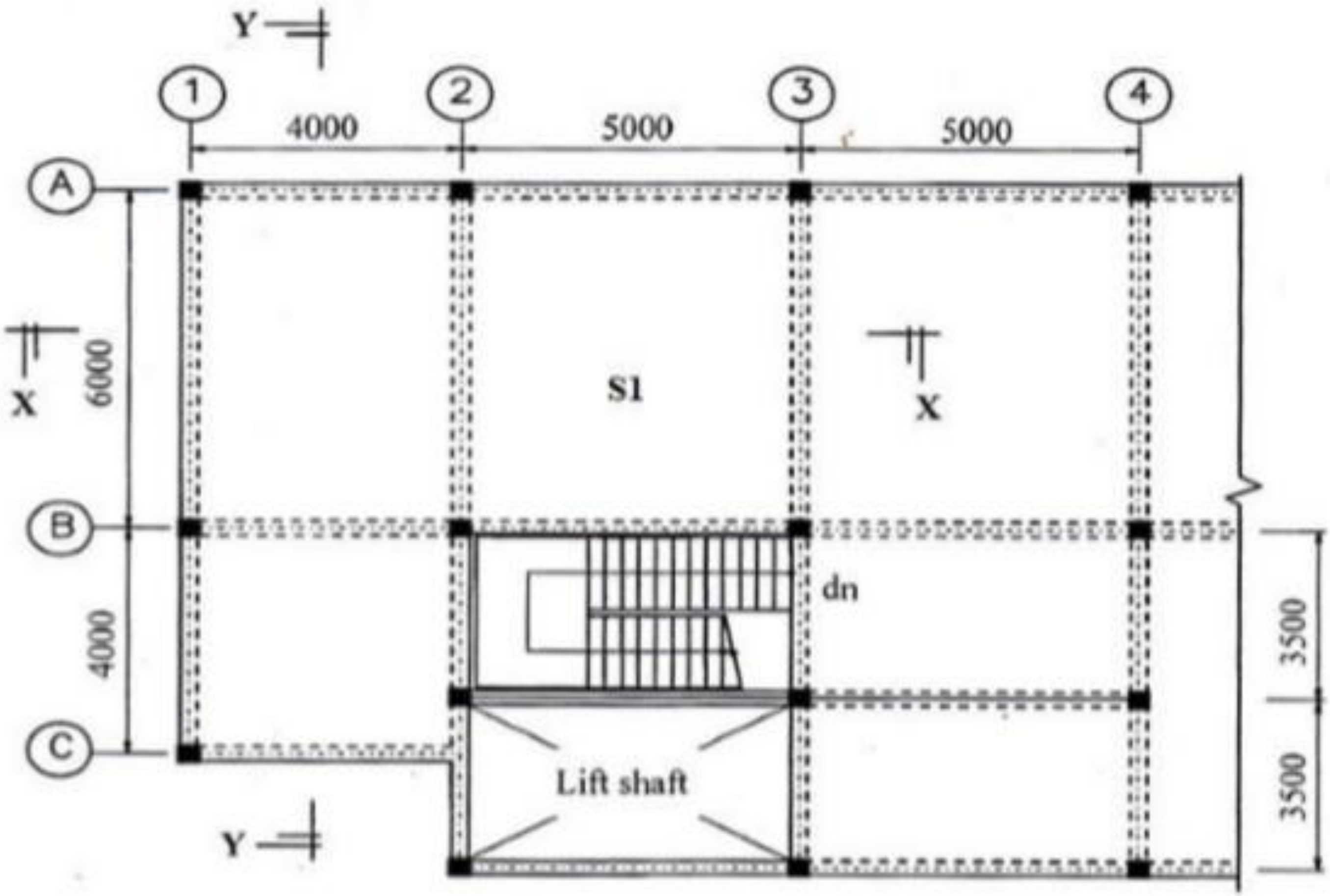

Figure 4

Tutorial 3: Two way restrained slab

a. Determine the nominal cover, c_{nom} for the slab.

b. Calculate the design action carried by the slab.

c. Calculate and propose the main reinforcement.

d. Check the slab for shear, deflection and crack requirement.

e. Sketch the detailing for the slab.

End of Examples and Tutorials