Universiti Malaysia PAHANG
Engineering • Technology • Creativity

REINFORCED CONCRETE DESIGN 1

Analysis of Section

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Lesson Outcome

At the end of this topic, students should be able to:

- Define and explain the ultimate limit state design theory
- Analyze and design for singly and doubly reinforced rectangular concrete beam

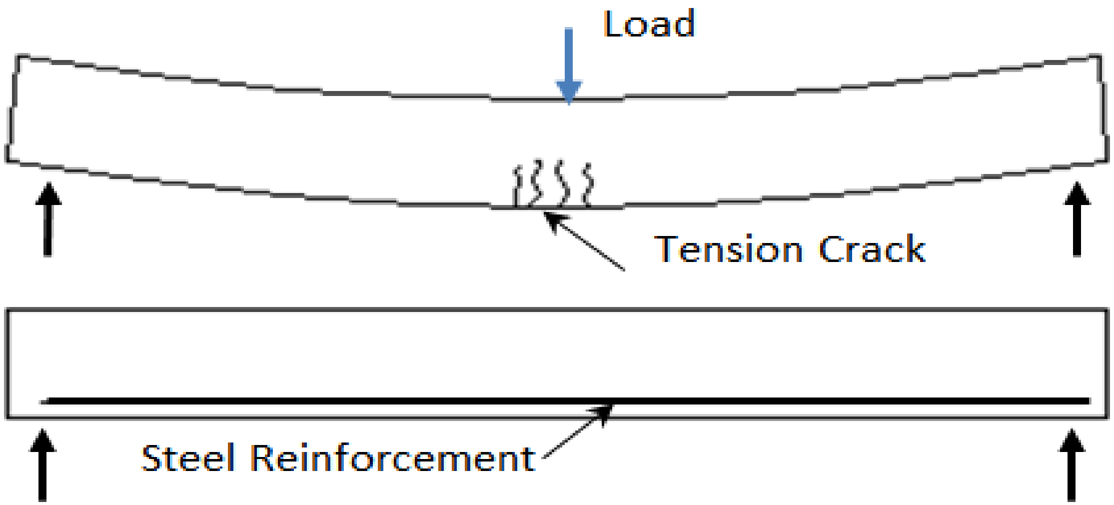
Stress – Strain Relationship

- When load is applied to a structure, the deformation occurred on the element will produce stress and strain.
- The maximum stress for concrete is assumed to be 85% of its compressive strength divided by partial safety factor of concrete.
- Whereas, the ultimate strain for concrete in compression is taken as 0.0035.

Stress – Strain Relationship

- For steel reinforcement, the maximum stress is considered as steel yield stress divided by partial safety factor of steel.
- The modulus of elasticity is taken as 200 Gpa.
- The consideration of the partial safety factor is needed in order to obtain the design strength of concrete and steel.

Beam behaviour in bending

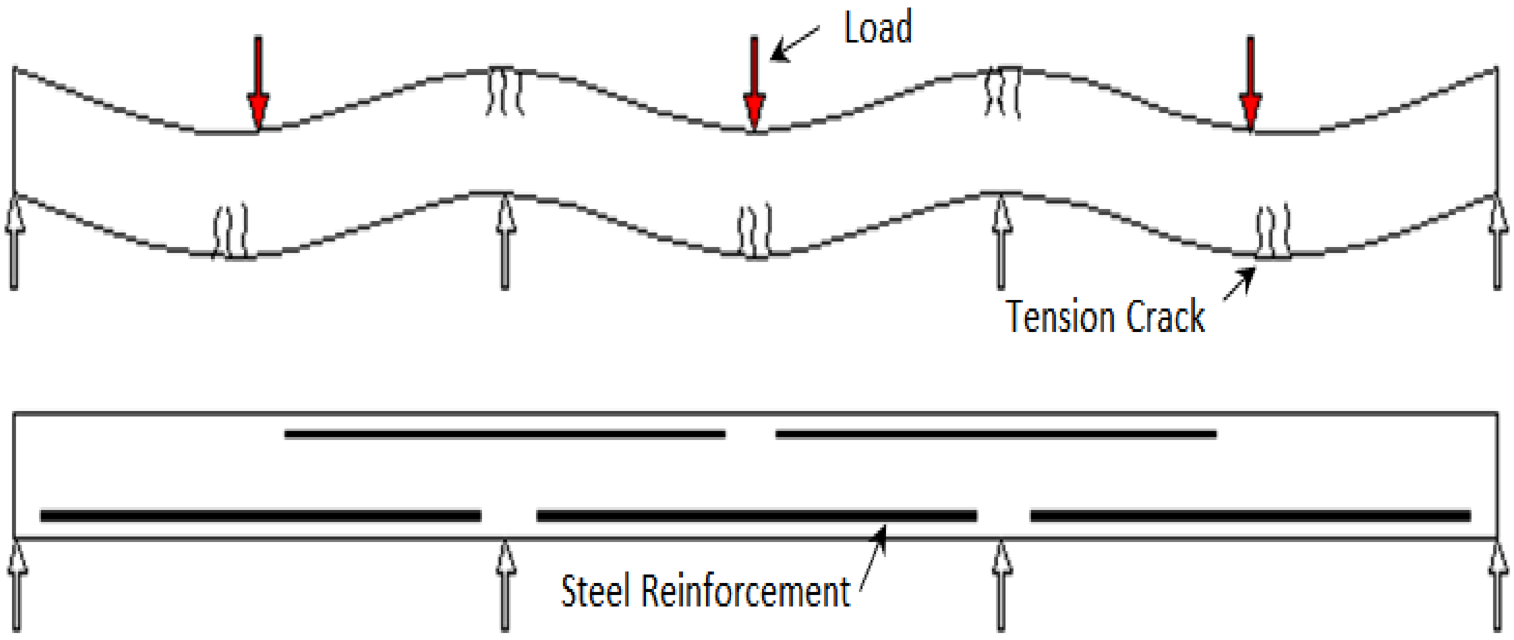


Simply supported beam

Theory of bending for reinforced concrete assumes that:

- Concrete will crack in the regions of tensile strains.
- After cracking, all the tension is carried by the reinforcement.

Beam behaviour in bending



Continuous beam

Failure modes / criteria

There are 3 types of failure modes that could occur in beam design:

1. Under reinforced
2. Balanced
3. Over reinforced

Under reinforced

- Area of steel reinforcement is very small as compared to the area of concrete
- Steel will reach its yield strength earlier than concrete

Balanced

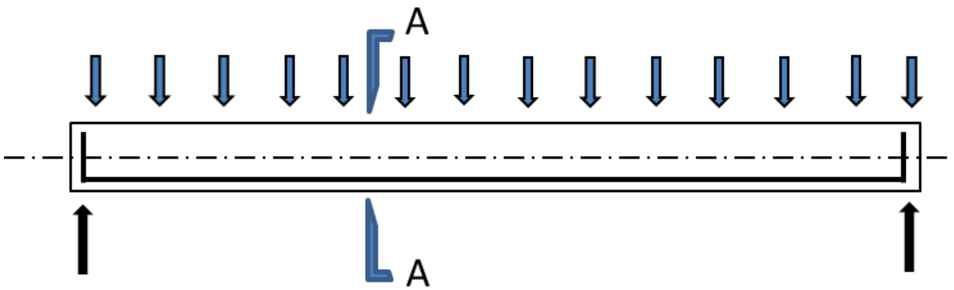
- Steel will reach its yield strength at the same time as concrete
- Ideal design

Over reinforced

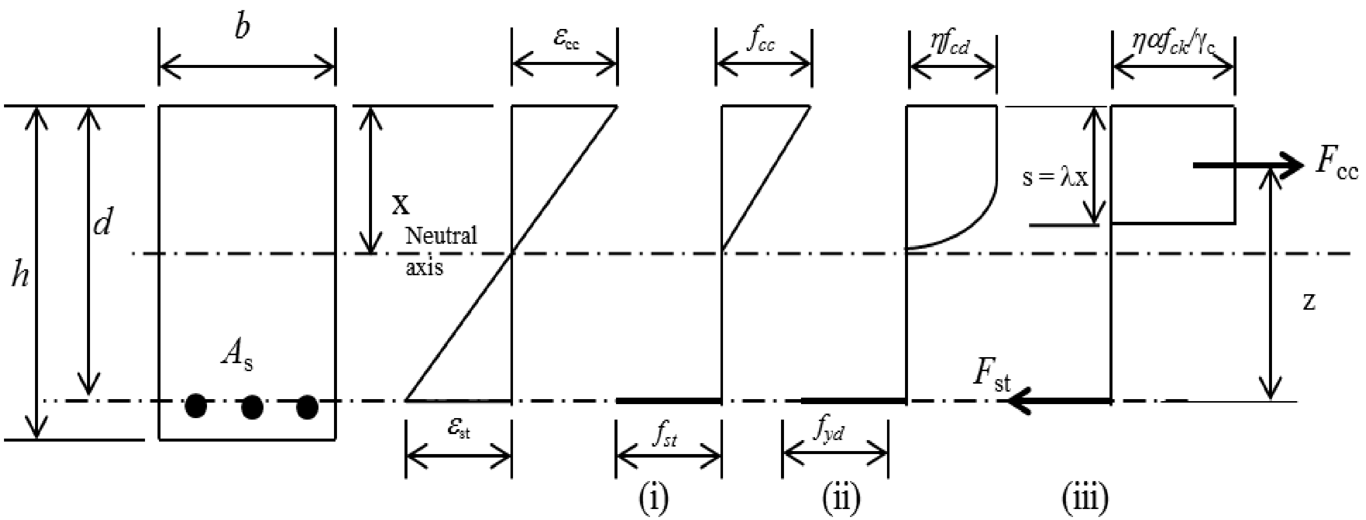
- This is strictly not allowed
- Concrete will reach its maximum strength earlier than steel
- Failure occurs caused by early failure of concrete in compression
- Failure happens without warning (abrupt of sudden failure)

Stress – strain of a section in bending

For $f_{ck} < 50$ N/mm^2 :
$\eta = 1.0$,
$\lambda = 0.8$



Longitudinal section of simply supported beam



Stress – strain of a section in bending

Notation:

h = Overall depth section
b = Breadth of section
d = Effective depth
As = Area of steel reinforcement
x = Neutral axis depth
ε_{cc} = Strain in concrete in compression
ε_{st} = Strain in steel tension
λ = Factor defining the effective height of compression zone
η = Factor defining the strength

Stress – strain of a section in bending

Notation:

f_{cc} = Stress in concrete in compression

f_{st} = Stress in steel in tension

f_{cd} = Concrete design strength

f_{yd} = Steel design strength

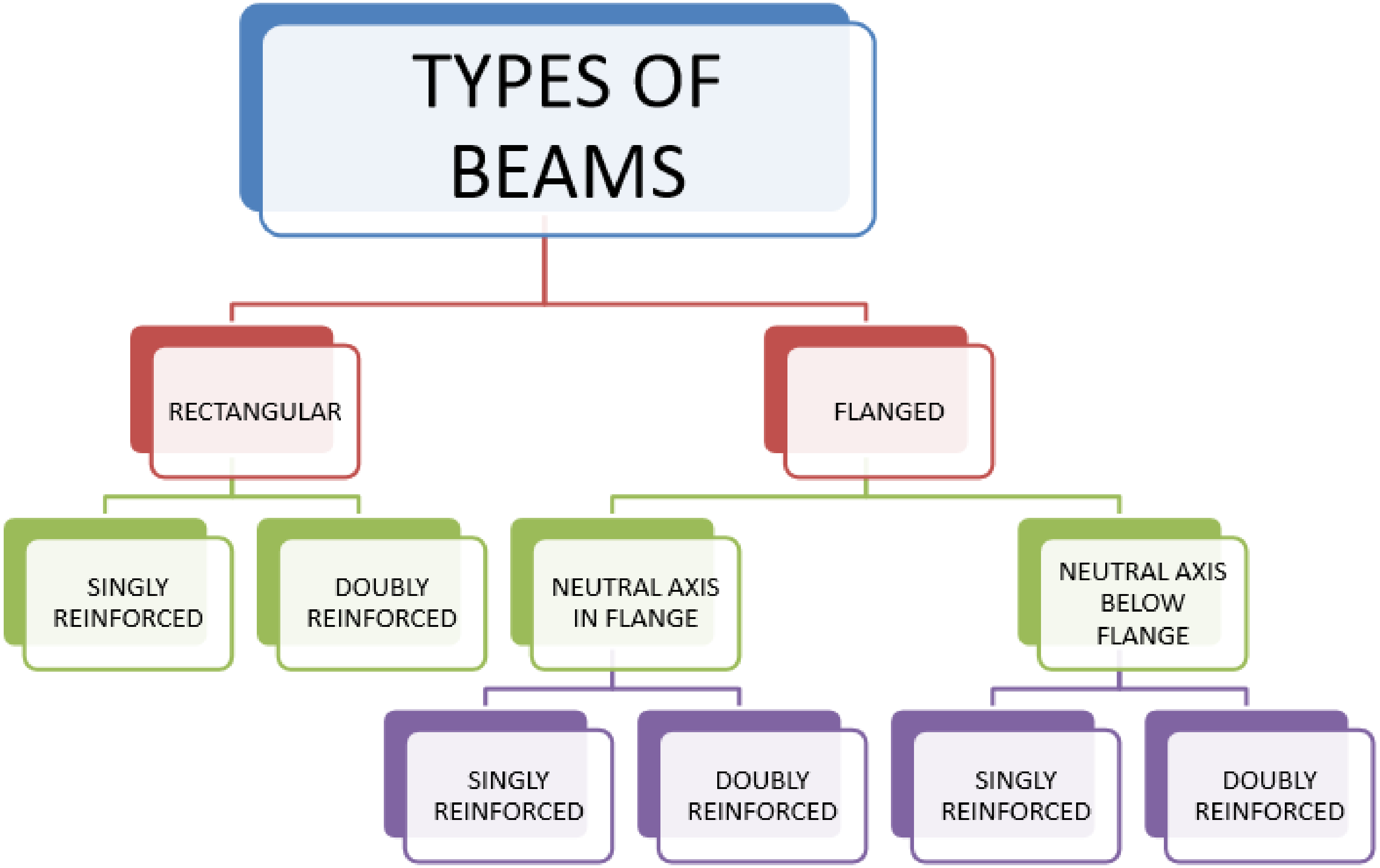
s = Stress block depth

F_{cc} = Force in concrete in compression

F_{st} = Force in steel tension

z = Lever arm

Type of beams

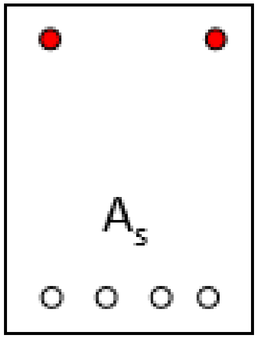


Design of Rectangular Section

There are two types of rectangular sections:

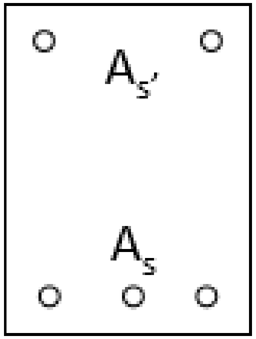
1. Singly reinforced

- Consist only tension reinforcement, As
- The top reinforcements are hanger bars (used to produce a cage-like arrangement)

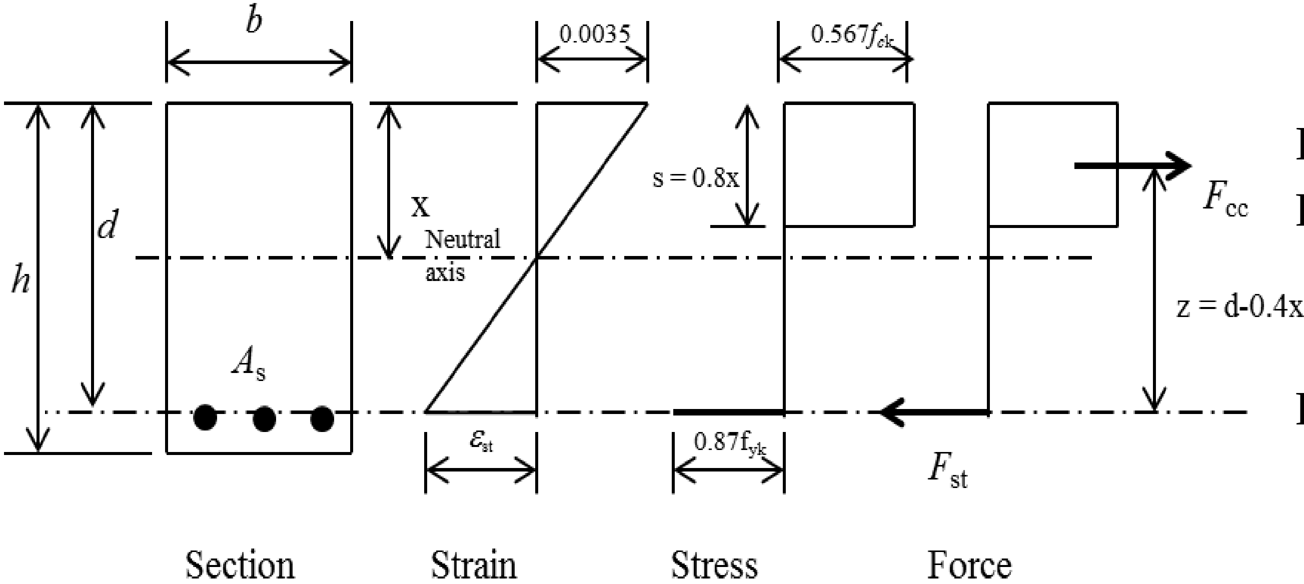


2. Doubly reinforced

- Consist of both tension, As and compression reinforcement, As’



Singly reinforced rectangular section



Internal forces,

$F_{cc} = 0.567 f_{ck}(b \times 0.8x)$

$= 0.454 f_{ck} bx$

$F_{st} = 0.78 f_{yk} A_s$

For internal forced to be in equilibrium

$F_{cc} = F_{st}$

$0.454 f_{ck} * bx = 0.87 f_{yk} As$

$x = 0.87 f_{yk} A_s / 0.454 f_{ck} b$

Singly reinforced rectangular section

Moment of resistance with respect to steel,

$$M = F_{cc}.z$$
$$= 0.454f_{ck}b\ x\ (d-0.4x)$$

Moment resistance with respect to concrete

$$M = F_{st}.z$$
$$= 0.87f_{yk}A\ (d-0.4x)$$

This equation will be used to determine the moment of resistance that can be resisted by the section with specified area of tension reinforcement.

"Higher reinforcement – higher capacity to resist larger moment"

Singly reinforced rectangular section

In design, EC2 limits x to not exceeding 0.45d in order to avoid the sudden failure exhibit by an over-reinforced section.

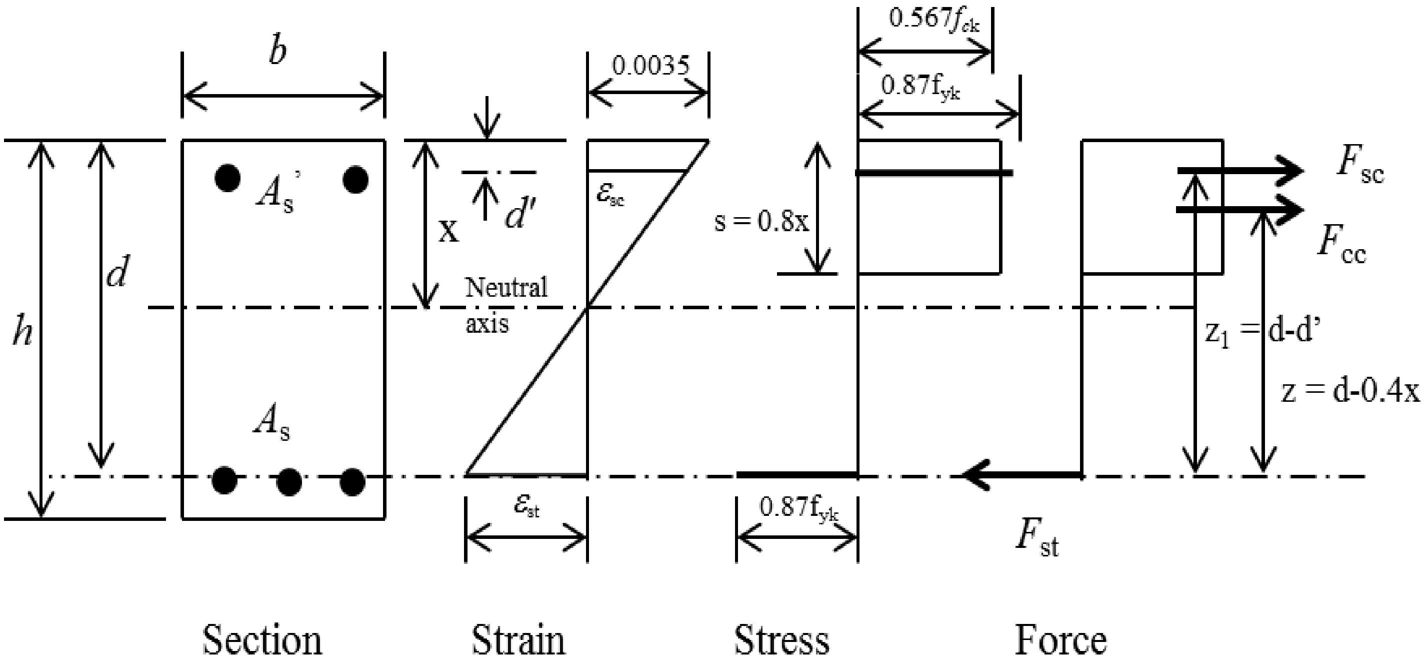
$$M_{bal} = [0.454 f_{ck} b\,(0.45d)].[d - 0.4(0.45d)]$$
$$= [0.454 f_{ck} b\,(0.45d)].[0.82d]$$
$$= 0.167 f_{ck} bd^2$$
$$= K_{bal} f_{ck} bd^2 \quad \text{where} \quad K_{bal} = 0.167$$

NOTE:

If $K \leq K_{bal}$ = only tension reinforcement is required (singly reinforced)

If $K > K_{bal}$ = both tension and compression reinforcement is required (doubly reinforced)

Doubly reinforced rectangular section



Doubly reinforced rectangular section

In equilibrium,

$$F_{st} = F_{cc} + F_{sc}$$

$$x = (0.87f_{yk} A_s - 0.87f_{yk} A_{s'}) / 0.45f_{ck}b$$

Moment about F_{st}

$$M = F_{sc} .z_1 + F_{cc} .z$$

Ultimate moment of resistance at x = 0.45d for doubly reinforced section

$$M = 0.87f_{yk} A_{s'} (d - d') + M_u$$

Design for rectangular section

The calculation for beam design is based on the Eurocode 2 design guideline as stated on Section 6.1: MS EN 1992 – 1 – 1: 2010.

Examples and Tutorials for Singly and Doubly Rectangular Section