Universiti Malaysia PAHANG
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THEORY OF STRUCTURES
CHAPTER 5 : THREE PIN ARCH

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Chapter 5 – Three Pin Arch

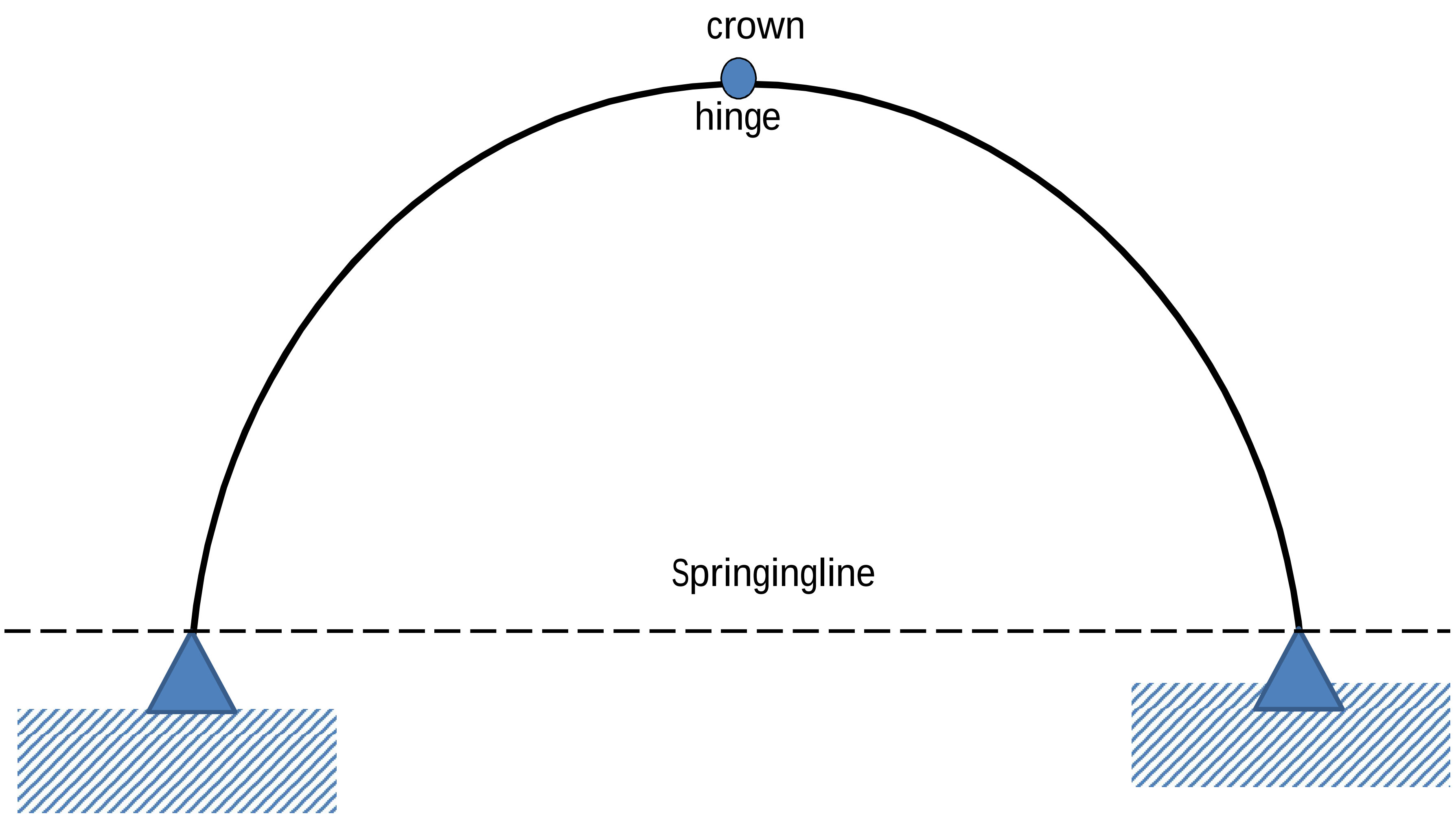
- Aims
  - Determine internal forces, shear forces and bending moments in arch member

- Expected Outcomes :
  - Able to explain the function of arch
  - Able to describe the function of arch
  - Able to determine the reaction at support for three arch structure
  - Able to determine the internal forces at any point at arch structure
  - Able to draw shear force, axial force and bending moment diagram

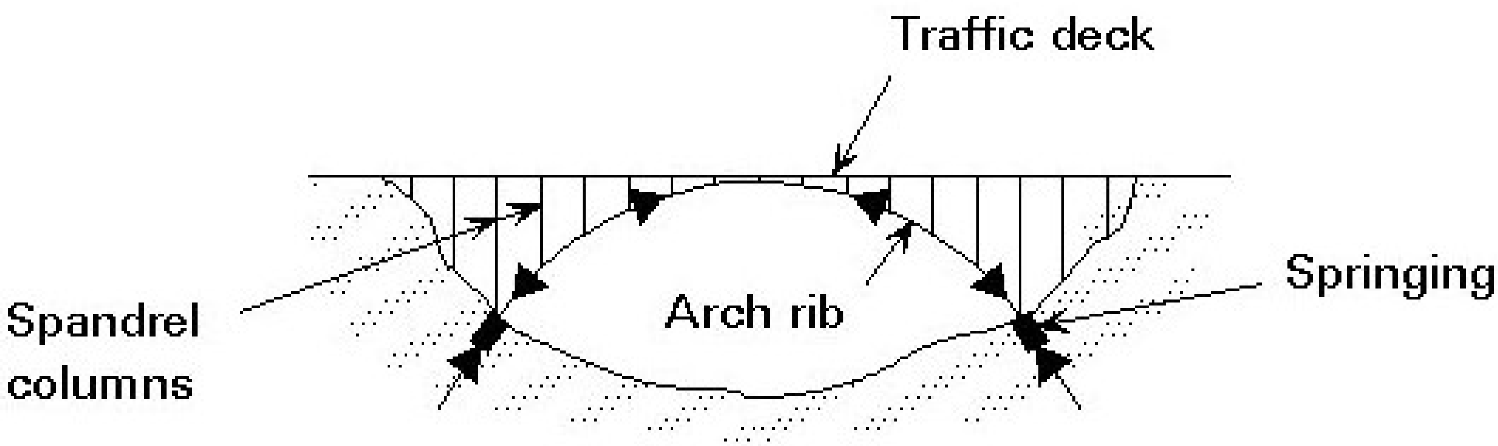
- References

  - Mechanics of Materials, R.C. Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, SI Edition by Aslam Kassimali,Cengage Learning
  - Structural Analysis, Coates, Coatie and Kong
  - Structural Analysis - A Classical and Matrix Approach, Jack C. McCormac and James K. Nelson, Jr., 4th Edition, John Wiley

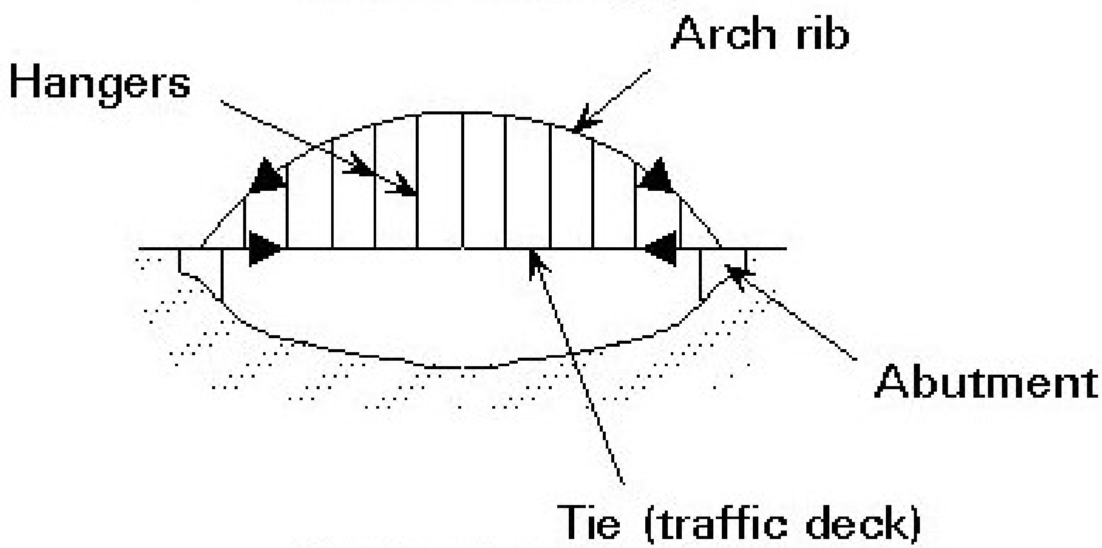
5.1 WHAT IS ARCH??



5.1 WHAT IS ARCH??



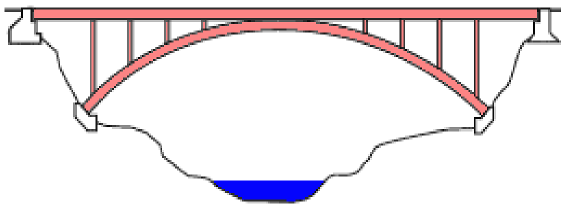
Fixed arch bridge



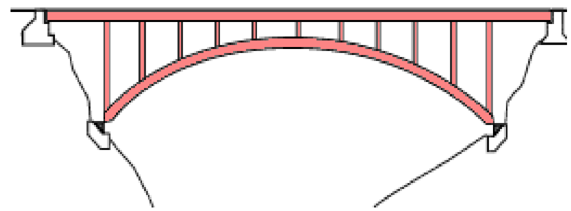
Tied arch bridge

- Arches have been used for a very long time to span large distance i.e bridges/building to carry transverse loading efficiently.
- Arch carries most of the load axially with bending moment greatly reduced due to the curvature of the arch

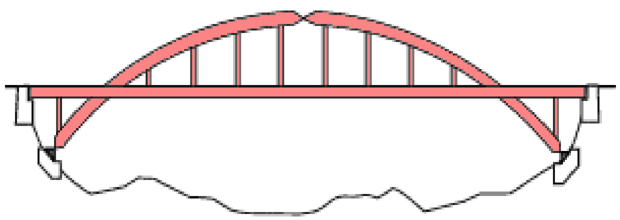
5.3 TYPES OF ARCH



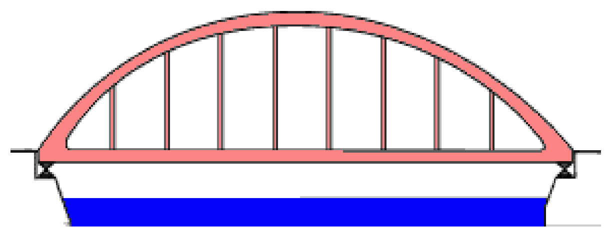
(a) Fixed arch



(a) Two-hinge arch



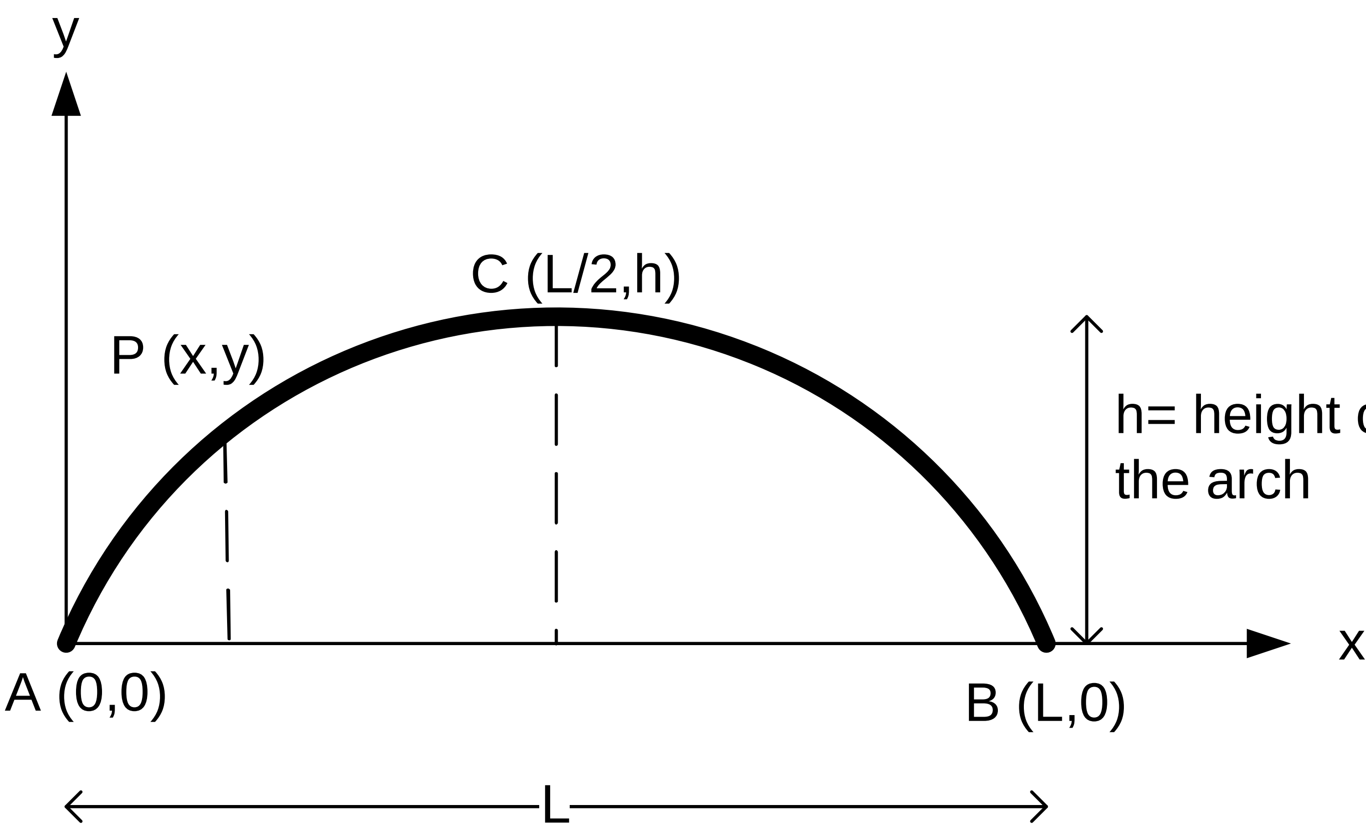
(a) Three-hinge arch



(a) Tied arch

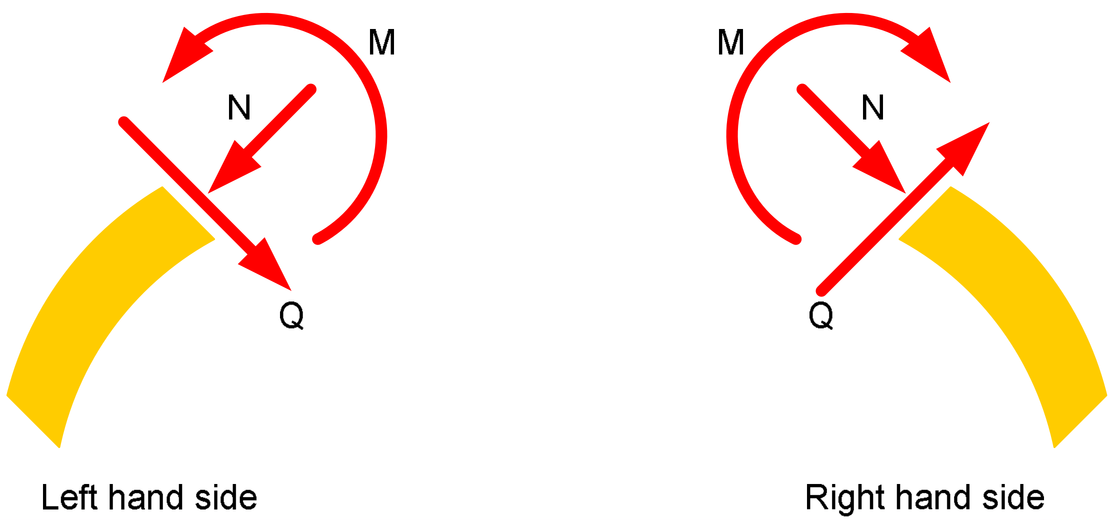
#Due to support system

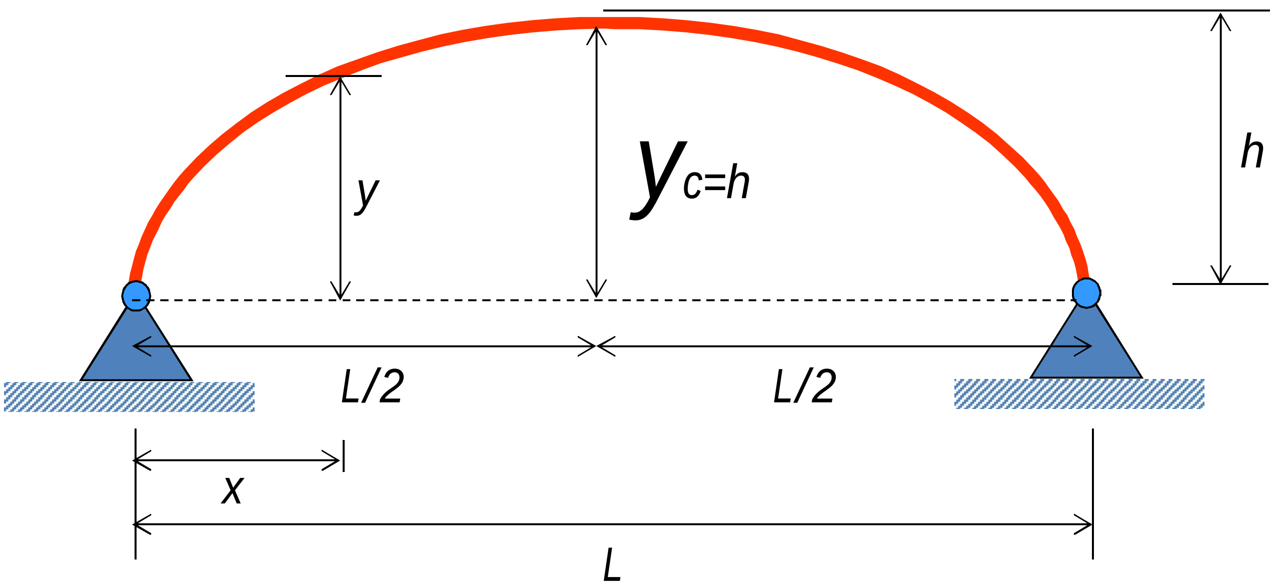
5.4 EQUATION OF PARABOLIC ARCH



The axial force, N shear force, Q and bending moment, M in the arch rib

- Shear force must be parallel to the cross section surface, whilst the axial force must be perpendicular to the shear force. The positive were shown in figure below.





$y = kx(L - x)$ ....Parabolic equation

When $x = L/2$, $y = y_c$ sub into enq.

$y_c = k(L/2)(L - L/2) = kL^2/4$

Therefore $k = 4y_c/L^2 = 4h/L^2$ then sub into eqn.

$$y = (4y_c/L^2)(Lx - x^2)$$

$$y = (4h/L^2)(Lx - x^2)$$ simplify

$$y = (4h/L^2)x(L - x)$$

At any point of the arch (parabolic)

$$\text{Slope} = \frac{dy}{dx}$$

but $$y = \frac{4hx}{L^2}(L - x)$$

$$= \frac{4hxL}{L^2} - \frac{4hx^2}{L^2}$$

$$= \frac{4hx}{L} - \frac{4hx^2}{L^2}$$

$$\text{Slope} = \frac{dy}{dx} = \frac{4h}{L} - \frac{2(4h)x}{L^2}$$

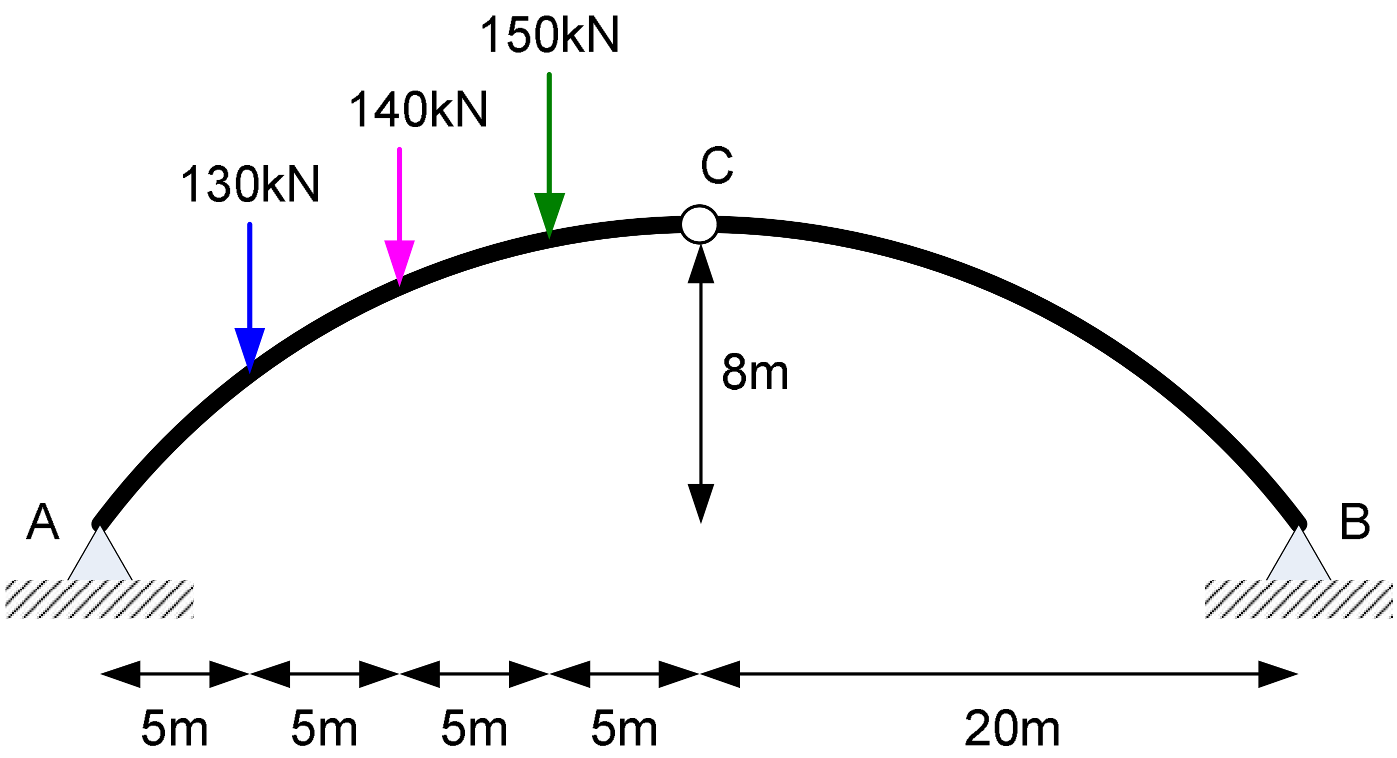
$$= \frac{4h}{L} - \frac{8hx}{L^2}$$

$$= \frac{4h}{L^2}(L - 2x)$$

Where, $\theta = \tan^{-1} \text{slope}$

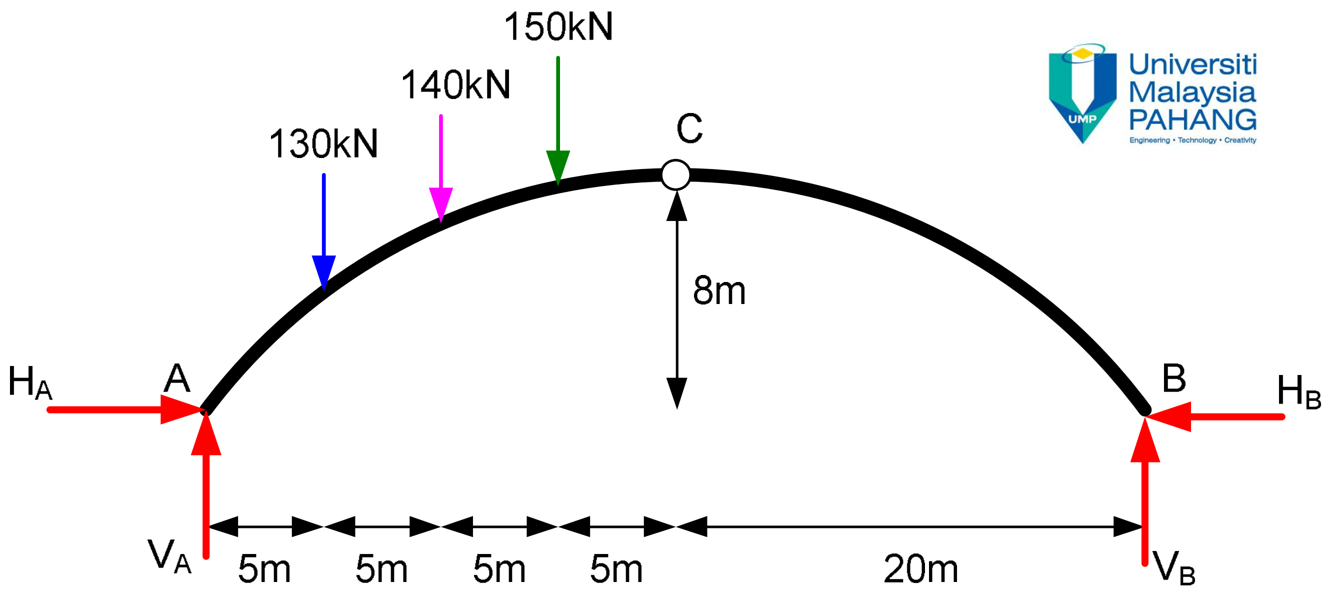
EXAMPLE 1

- Calculate the reaction at support A and B as shown in figure below.



Solution Example 1…

Draw FBD



Consider the whole structure

$$\sum M_A = 0$$

$$130(5) + 140(10) + 150(15) - V_B(40) = 0$$

$$V_B = 107.5kN$$

$$V_B = 107.5kN$$

$$\sum F_y = 0$$

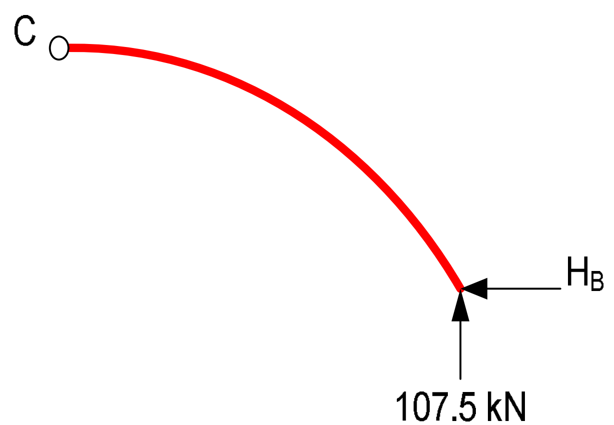
$$V_A - 130 - 140 - 150 + 107.5 = 0$$

$$V_A = 312.5kN$$

$$V_A = 312.5kN$$

Solution
Example 1...

Now, consider segment CB



$$\sum M_C = 0$$

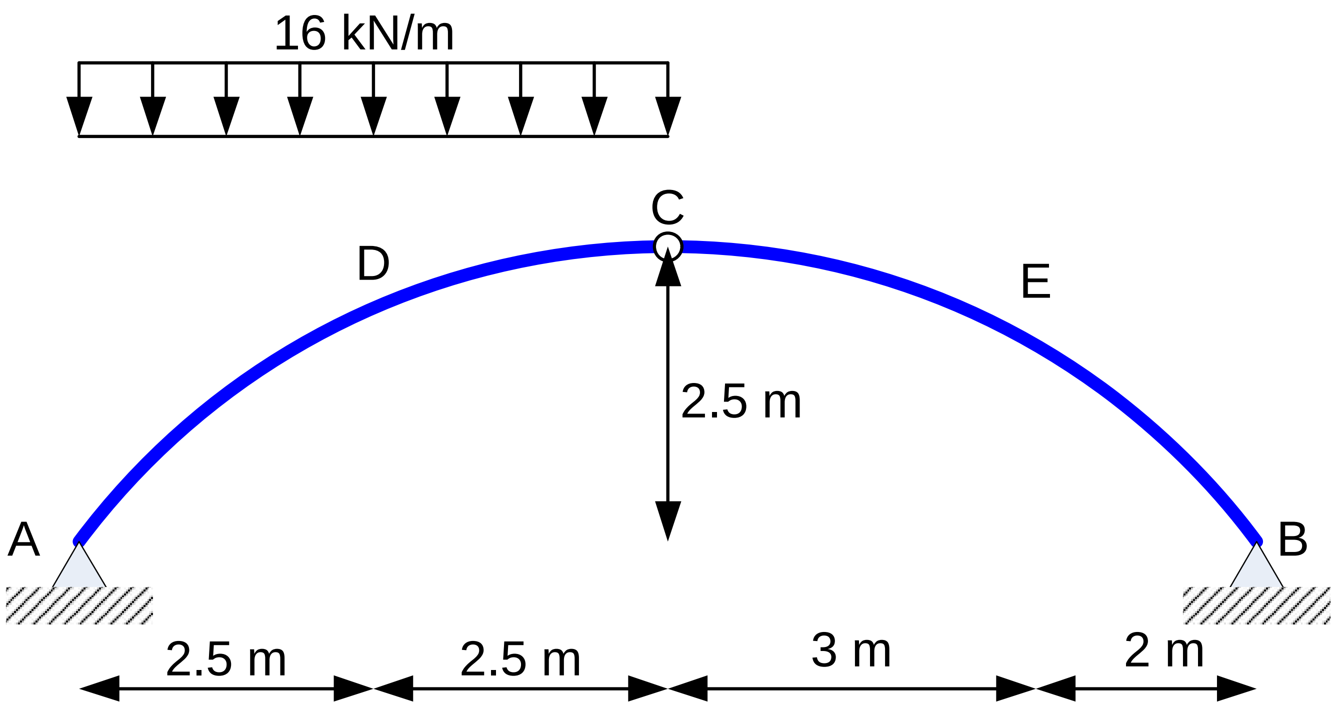
$$107.5(20) + H_B(8) = 0$$

$$H_B = 268.75kN$$

$$Hence,\ H_A = 268.75kN$$

EXAMPLE 2

- Determine the internal forces at the points D and E in the three hinge parabolic arch as shown in figure below.



Solution Example 2…

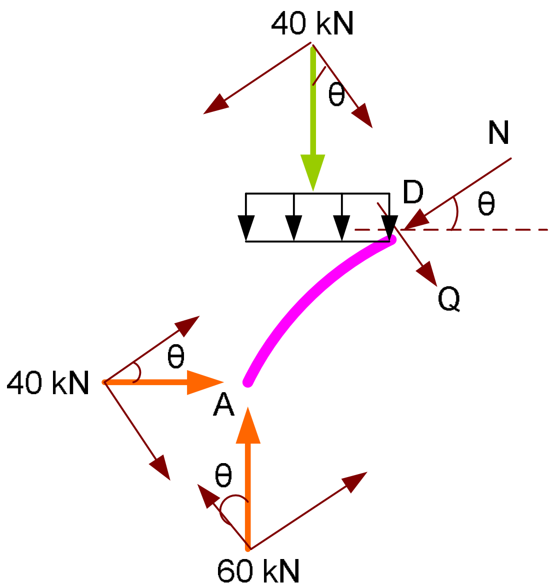
Apply equation of equilibrium, consider the whole structure then taking moment about A and B.

$$V_A = 60kN, \quad V_B = 20kN$$

Consider RHS or LHS, take moment about C is equal

$$H = 40kN$$

- Consider segment CD



$$y_D = \frac{4hx}{L^2}(L-x)$$
$$= \frac{4(2.5)(2.5)}{10^2}(10-2.5)$$
$$= 1.875m$$

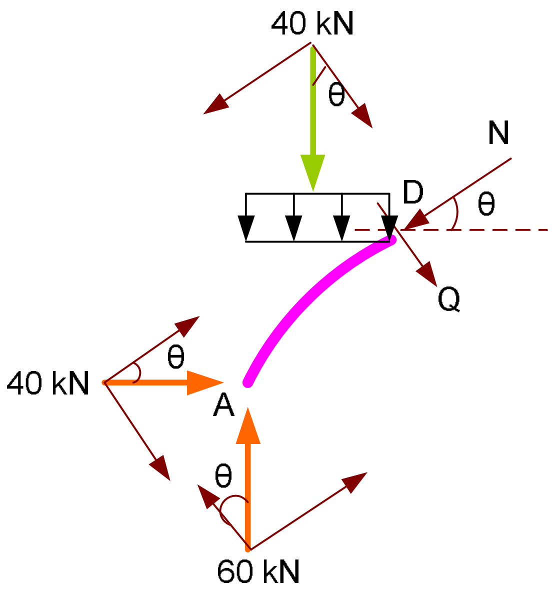
$$Slope = \frac{4h}{L^2}(L-2x)$$
$$= \frac{4(2.5)}{10^2}(10-2(2.5))$$
$$= 0.5$$

$$\theta = \tan^{-1}(0.5)$$
$$= 26.57^o$$

Resolving in Q direction

$$Q + 40\sin(26.57) - 60\cos(26.57) + 40\cos(26.57) = 0$$
$$Q = -0.003kN$$

- Consider segment CD



$$y_D = \frac{4hx}{L^2}(L - x)$$

$$= \frac{4(2.5)(2.5)}{10^2}(10 - 2.5)$$

$$= 1.875m$$

$$Slope = \frac{4h}{L^2}(L - 2x)$$

$$= \frac{4(2.5)}{10^2}(10 - 2(2.5))$$

$$= 0.5$$

$$\theta = \tan^{-1}(0.5)$$

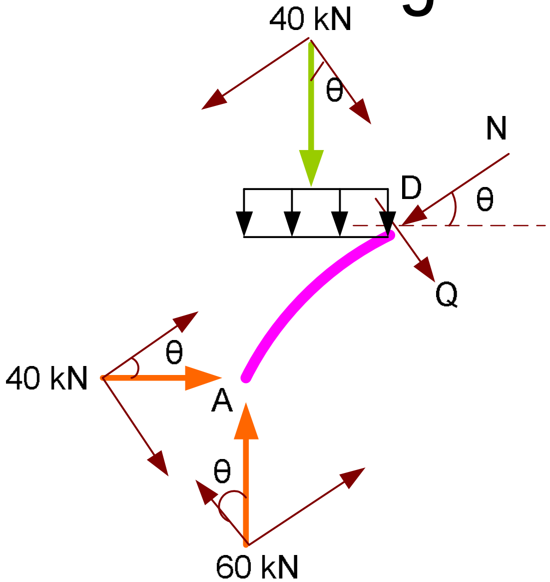
$$= 26.57^o$$

Resolving in N direction

$$N - 40\cos(26.57) - 60\sin(26.57) + 40\sin(26.57) = 0$$

$$N = 44.72kN$$

- Consider segment CD



$$\theta = \tan^{-1}(0.5)$$
$$= 26.57^o$$

Bending moment

$$\sum M_D = 0$$

$$-M_D - 40(\frac{2.5}{2}) + 60(2.5) - 40(1.875) = 0$$

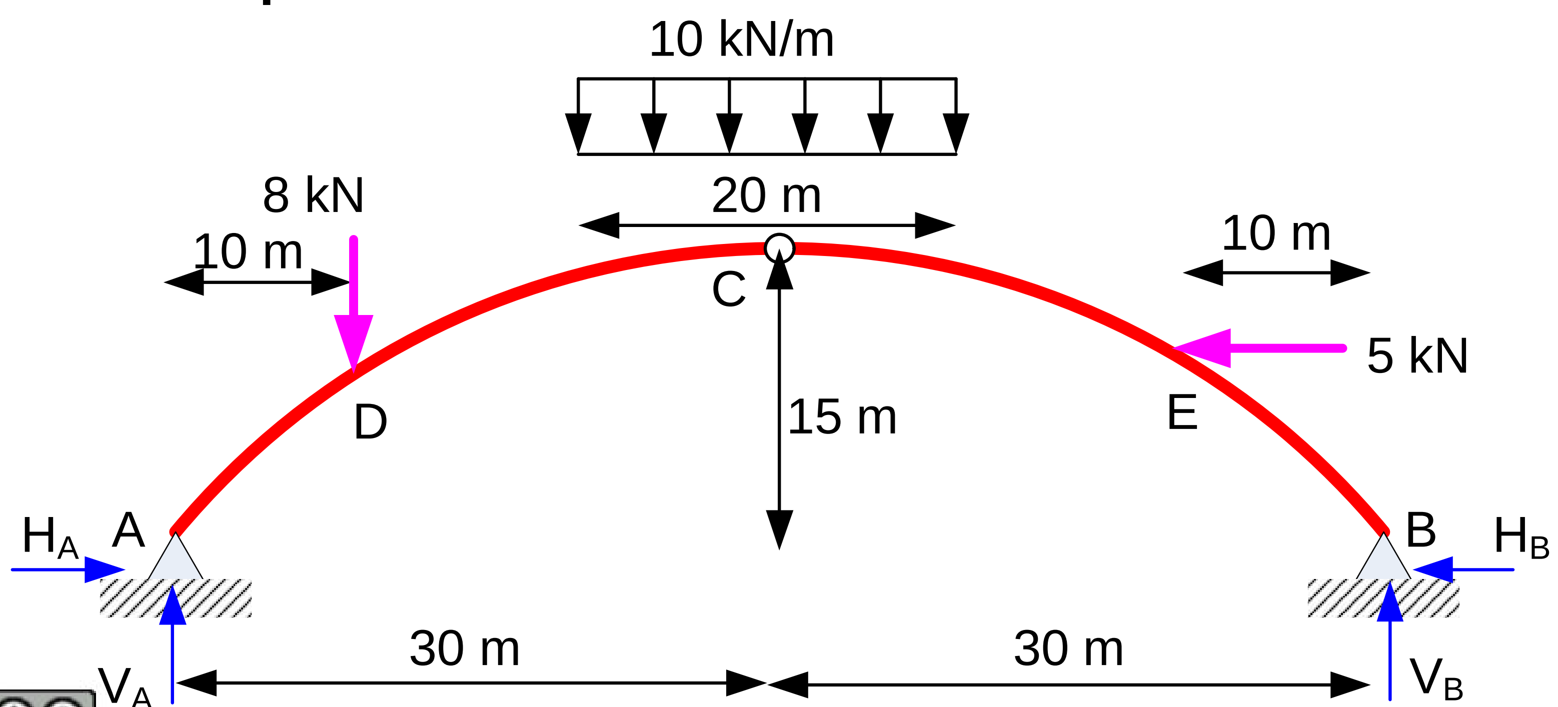
$$M_D = 25kNm$$

$$y_D = \frac{4hx}{L^2}(L-x)$$
$$= \frac{4(2.5)(2.5)}{10^2}(10-2.5)$$
$$= 1.875m$$

$$Slope = \frac{4h}{L^2}(L-2x)$$
$$= \frac{4(2.5)}{10^2}(10-2(2.5))$$
$$= 0.5$$

EXAMPLE 3

- Determine the bending moment at 25 m from the right hand support B and axial and shear force at point D and E.



Solution
Example 3...

- Calculate Y_E

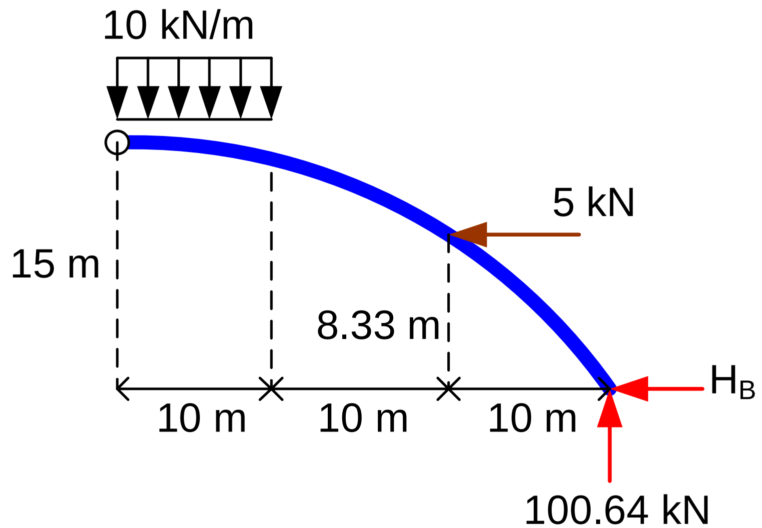
$$y_E = \frac{4(15)(10)}{60^2}(60-10)$$
$$= 8.33m$$

- Consider whole structure

$$\sum M_A = 0$$
$$-V_B(60) + 8(10) + 10(20)(10+20) - 5(8.33) = 0$$
$$V_B = 100.64kN$$

$$\sum F_y = 0$$
$$V_A + 100.64 - 8 - 10(20) = 0$$
$$V_A = 107.36kN$$

- Consider RHS of the arch, taking moment at C is equal to zero.



$$\sum M_C = 0$$

$$-100.64(30) + 10(10)(5) + 5(15-8.33) + H_B(15) = 0$$

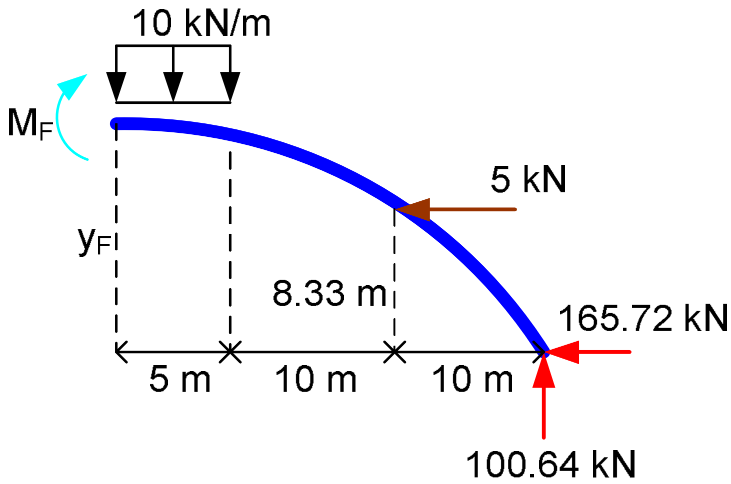
$$H_B = 165.72kN$$

$$\sum F_x = 0$$

$$H_A - 165.72 - 5 = 0$$

$$H_A = 170.72kN$$

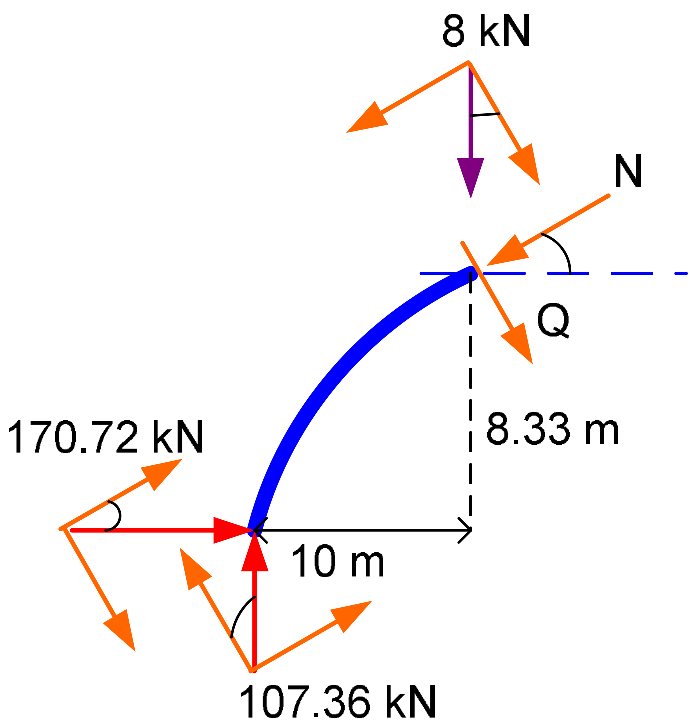
- Calculate the bending moment at 25 m from support , say point F



$$y_F = \frac{4(15)(25)}{60^2}(60-25)$$
$$= 14.58m$$

$$\sum M_F = 0$$
$$M_F - 100.64(25) + 165.72(14.58)$$
$$+ 5(14.58 - 8.33) + 10(5)(2.5) = 0$$
$$M_F = -56.45kNm$$

- At point D (with point load)



Resolving in Q direction

$$Q + 170.72\sin(33.67) - 107.36\cos(33.67) + 8\cos(33.67) = 0$$

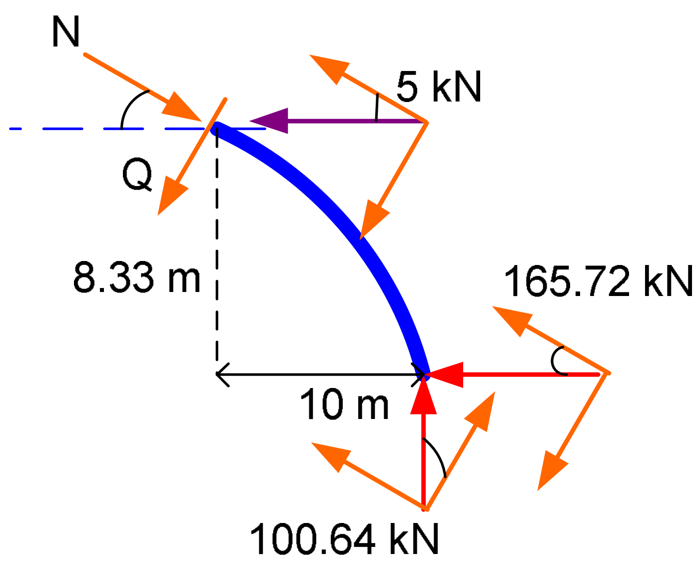
$$Q = -12.03kN$$

Resolving in N direction

$$N - 170.72\cos(33.67) - 107.36\sin(33.67) + 8\sin(33.67) = 0$$

$$N = 197.16kN$$

- At point F (with point load)



Resolving in Q direction

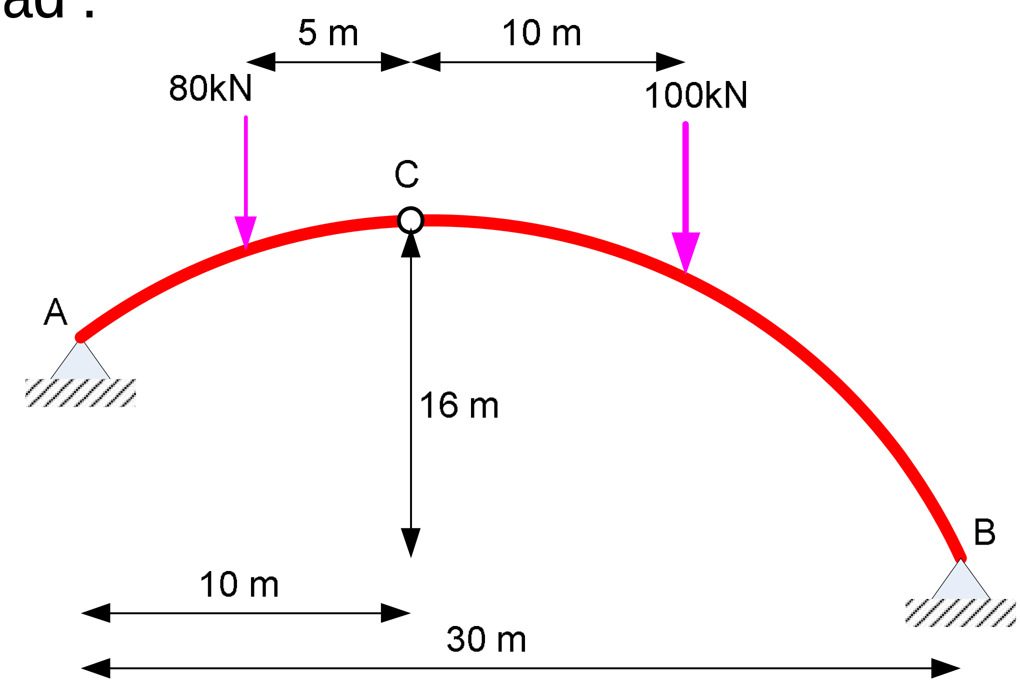
$$Q + 100.64\cos(33.67) - 165.72\sin(33.67)$$
$$- 5\sin(33.67) = 0$$
$$Q = 10.96kN$$

Resolving in N direction

$$N - 100.64\sin(33.67) - 165.72\cos(33.67)$$
$$- 5\cos(33.67) = 0$$
$$N = 197.87kN$$

EXAMPLE 4

Determine the reactions at supports and bending moment under the load :



Solution
Example 4…

$$\frac{l_1}{l_2} = \sqrt{\frac{r_1}{r_2}}$$

$$\frac{20}{40} = \sqrt{\frac{r_1}{16}}$$

$$\therefore r_1 = 4m$$

$$y_A = 16 - 4 = 12m$$

Or…

$$y_A = \frac{4hx}{L^2}(L - x)$$

$$y_A = \frac{4(16)(10)}{40^2}(40 - 10)$$

$$y_A = 12m$$

- Consider the whole structure. Taking moment at A is equal zero

$$\sum M_A = 0$$

$$80(5) + 100(20) - V_B(30) + H_B(12) = 0$$

$$30V_B - 12H_B = 2400 \;........................(i)$$

- Consider RHS, taking moment at C is equal to zero

$$\sum M_C = 0$$

$$100(10) - V_B(20) + H_B(16) = 0$$

$$20V_B - 16H_B = 1000 \;......................(ii)$$

- Resolving by using calculator

$$V_B = 110kN$$

$$H_B = 75kN$$

- Apply static equation

$$\sum F_y = 0$$

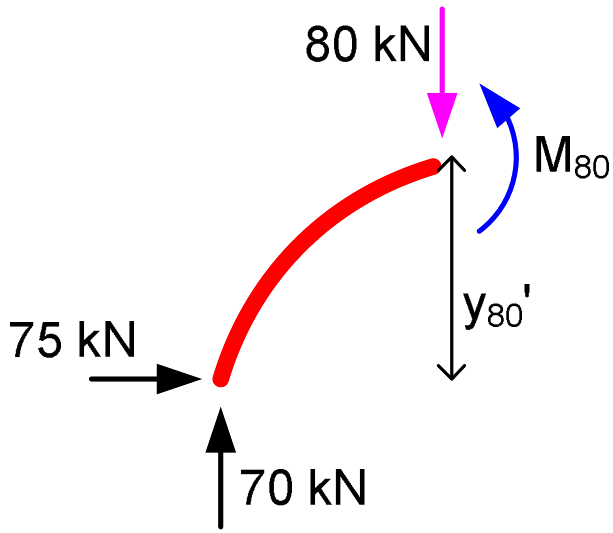
$$V_A + 110 - 80 - 100 = 0$$

$$V_A = 70kN$$

$$\sum F_x = 0$$

$$H_A = H_B = 75kN$$

- Consider LHS, taking moment under load 80kN



$$y_{80} = \frac{4hx}{L^2}(L-x)$$

$$= \frac{4(16)(15)}{40^2}(40-15)$$

$$= 15m$$

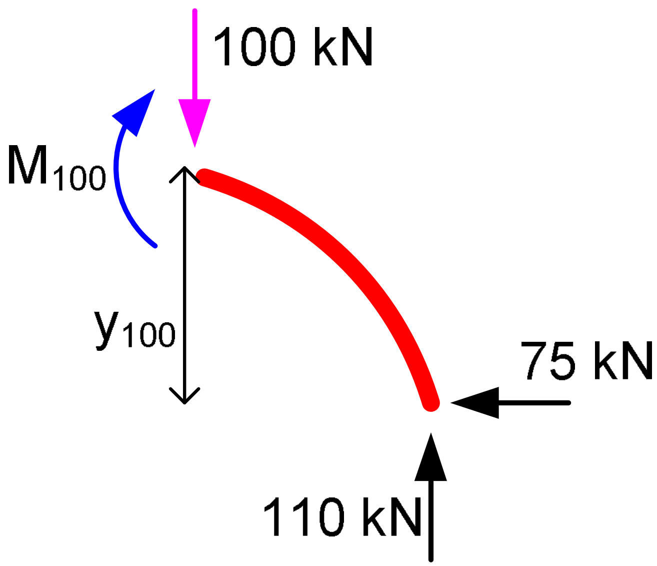
$$y'_{80} = 15-12 = 3m$$

$$\sum M_{80} = 0$$

$$-M_{80} + 70(5) - 75(3) = 0$$

$$M_{80} = 125kNm$$

- Consider RHS, taking moment under load 100kN



$$y_{100} = \frac{4hx}{L^2}(L-x)$$
$$= \frac{4(16)(10)}{40^2}(40-10)$$
$$= 12m$$

$$\sum M_{100} = 0$$
$$M_{100} + 75(12) - 110(10) = 0$$
$$M_{100} = 200kNm$$

THANKS

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