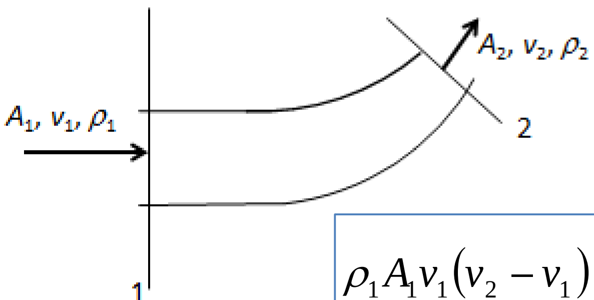
4.1 Derivation of Momentum Equation

- In mechanics, the momentum of a particle is defined as the product of its mass *m* and its velocity, *v*. $\text{Momentum} = mv$
- The particles of a fluid stream will possess momentum. In fluid motion, whenever there is change in magnitude and direction of velocity, there will be a corresponding change in momentum. The rate of momentum is given by: $\text{Rate of Momentum} = \rho Q v$
- The rate of momentum for steady flow; $(\rho_2 A_2 v_2)v_2 - (\rho_1 A_1 v_1)v_1$ or from the continuity of mass flow equation,

Change of momentum



$$\rho_1 A_1 v_1 (v_2 - v_1) = \dot{m}(v_2 - v_1) = \text{mass flow per unit time} \times \text{change of velocity}$$

- If $\rho_2 = \rho_1$ and according to continuity principle and the Newton Second Law; the rate of change between section 1 and 2 is : $\rho Q(v_2 - v_1) = F$
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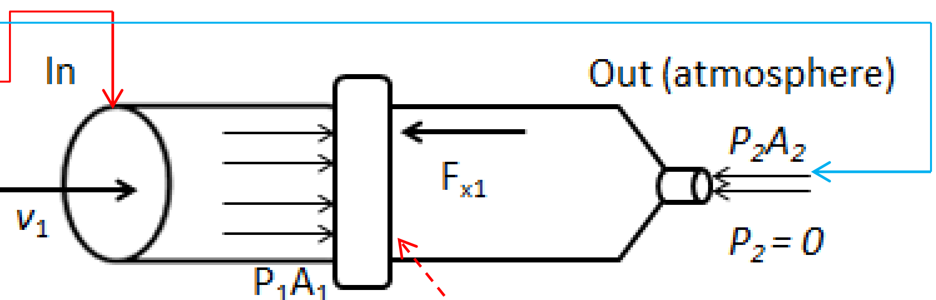
- This is the resultant force acting on the control volume in the direction of motion. For any control volume, the total force F which acts upon it in a given direction will be made up of three component forces that are :
  - ***F_1* : forces exerted in the given direction on the fluid by solid body**
  - *F_2* : forces exerted in a given direction on the fluid by body forces such as gravity
  - ***F_3* : forces exerted in the given direction on the fluid outside the control volume.**

F_3, force due to the pressure P_1 and P_2 of the fluid outside the control volume.



F_1, force exerted by the walls of the pipe

$$F = F_1 + F_3$$

$$F_R = \rho Q(v_{out} - v_{in})$$

- The force exerted by the fluid on the solid body will be equal and opposite to F_1, so that the $R = -F_1$

Momentum equation for flow in a streamtube

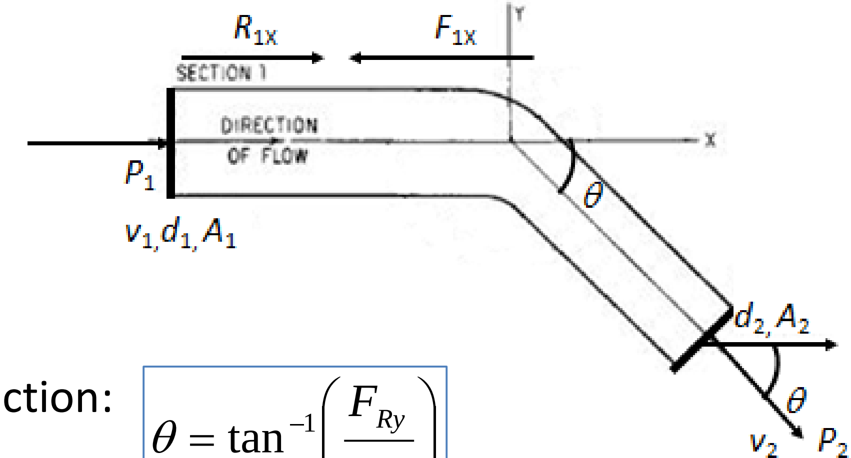
- Momentum is the quantity of motion of a moving body measured as a product of its mass and velocity.
- Rate of change in momentum of fluid in **x direction** :

$$F_x = \rho Q(v_{out} - v_{in})$$
$$F_x = \rho Q(v_{2x} - v_{1x})$$
$$F_x = \rho Q(v_2 \cos\theta_2 - v_1)$$

Force = rate of change of momentum
***F* = $\rho Q(\Delta v)$**

- Similarly **in y direction** :

$$F_y = \rho Q(v_{out} - v_{in})$$
$$F_y = \rho Q(v_{2y} - v_{1y})$$
$$F_y = \rho Q(v_2 \sin\theta_2 - v_1 \sin\theta_1)$$



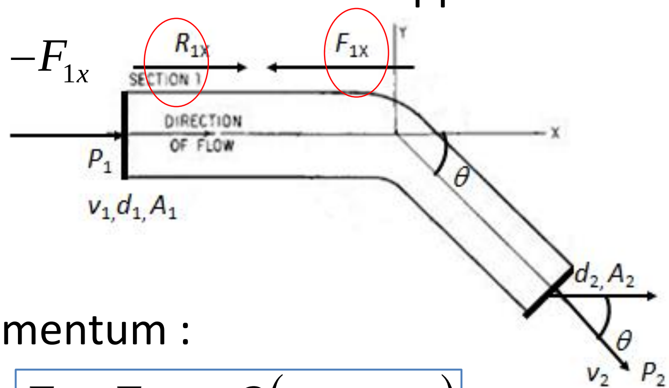
- Resultant force, $F_R = \sqrt{(F_x^{\,2} + F_y^{\,2})}$ Direction: $\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right)$
- Force acting on the fluid will be :

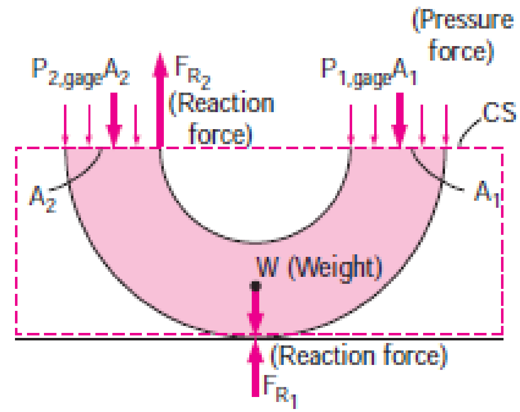
1. F_1, force exerted by the walls of the pipe
2. F_3, force due to the pressure P_1 and P_2 of the fluid outside the control volume.

- The force exerted by the fluid on the bend is opposite to the resultant force, so the

Force acting on the fluid is $F_1 + F_3$

$$R_{1x} = -F_{1x}$$





- The rate of change of momentum :

$$F_1 + F_3 = \rho Q(v_{out} - v_{in})$$

- Resolving in x direction,

$$F_{1x} + F_{3x} = \rho Q(v_{2x} - v_{1x})$$
$$-R_x + F_{3x} = \rho Q(v_{2x} - v_{1x})$$
$$R_x = F_{3x} - \rho Q(v_{2x} - v_{1x})$$
$$R_x = F_{3x} - \rho Q(v_2 \cos\theta - v_1)$$
$$R_x = [P_1A_1 - P_2A_2\cos\theta] - \rho Q(v_2\cos\theta - v_1)$$

Resolving in y direction,

$$F_{1y} + F_{3y} = \rho Q(v_{2y} - v_{1y})$$
$$-R_y + F_{3y} = \rho Q(v_{2y} - v_{1y})$$
$$R_y = F_{3y} - \rho Q(v_{2y} - v_{1y})$$
$$R_y = [P_1A_1 - (-P_2A_2\sin\theta)] - \rho Q(-v_2\cos\theta - 0)$$
$$R_y = [P_2A_2\sin\theta] + \rho Q(v_2\cos\theta)$$

- Resultant force: $F_R = \sqrt{\left(F_x^{\ 2} + F_y^{\ 2}\right)}$ Direction : $\theta = \tan^{-1}\left(\dfrac{F_{Ry}}{F_{Rx}}\right)$

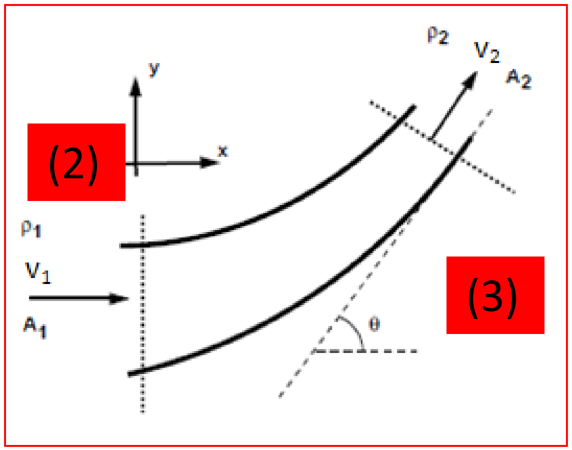
Step in Analysis with Momentum Equation

1. **Draw a control volume** : Based on the problem, selecting the stream between two gradually varied flow sections as the control volume;
2. **Decide on co-ordinate axis system** : Determining the directions of co-ordinate axis, magnitudes and directions of components of all forces and velocities on each axis.
3. **Plotting diagram for computation** : Analyzing the forces on control volume and plotting the directions of all forces on the control volume.
4. **Writing momentum equation and solving it (total force and pressure force)** : Substituting components of all forces and velocities on axes into momentum equation and solving it. All the pressures are relative to the relative pressure.

Momentum equation for flow in a streamtube

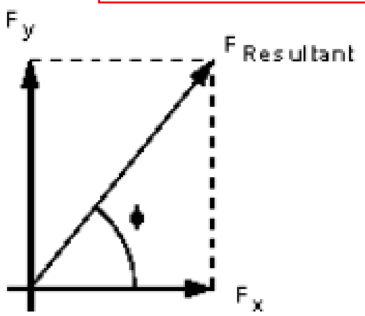
- At the inlet the velocity vector, v_1, makes an angle, θ_1, with the x-axis, while at the outlet v_2 make an angle θ_2

(1)



Force = rate of change of momentum
***F*= *ρQ*(Δ*v*)**

(4)



$$F_{resultant} = \sqrt{F_x^2 + F_y^2}$$

Force in the x-direction, Fx	Force in the y-direction, F_y
$F_x = \rho Q(v_{out} - v_{in})$ $F_x = \rho Q(v_{2x} - v_{1x})$ $F_x = \rho Q(v_2 \cos\theta_2 - v_1 \cos\theta_1)$	$F_y = \rho Q(v_{out} - v_{in})$ $F_y = \rho Q(v_{2y} - v_{1y})$ $F_y = \rho Q(v_2 \sin\theta_2 - v_1 \sin\theta_1)$

the angle which this force acts at is given by : $\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$