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Fluid Kinematics

by

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Chapter Description

- Aims
  - Apply and analyze Fluid Mechanics theories such as Bernoulli's Theorem, Continuity Equation in Fluid Mechanics system.

- Expected Outcomes
  - Define volume flow rate, weight flow rate, and mass flow rate and their units.
  - Define steady flow and the principle of continuity.
  - Write the continuity equation, and use it to relate the volume flow rate, area, and velocity of flow between two points in a fluid flow system.

- References

  - Douglas F.J., Gasiorek J.M., Swaffield J.A. Fluid Mechanics. Prentice Hall 4th Edition.
  - Bruce R. M., Donald F.Y and Theodore H.O. Fundamentals of Fluid Mechanics. Wiley.
  - Nakayama Y and Broucher R.F. Introduction to Fluid Mechanics. Revised. Butterworth Heinmann.

SYNOPSIS

Detail objectives:

- Define volume flow rate, weight flow rate, and mass flow rate and their units.
- Define steady flow and the principle of continuity.
- Write the continuity equation, and use it to relate the volume flow rate, area, and velocity of flow between two points in a fluid flow system.

WEEK	CHAPTER	TOPIC		
6	**3**	**Fluid Kinematics**		
		3.1	Flow Pattern	
		3.2	The Continuity Equation	
7		3.3	The Bernoulli's Theorem	
		3.4	Application Of Bernoulli And Continuity Equations	
			3.4.1	Venturi Meter
			3.4.2	Pipe Orifice

Revise

Fluid Static

- **Pressure**
- **Fluid pressure and depth**

Pascal's principle

- **Buoyancy**

Archimedes' principle

Fluid Kinematics

- **Fluid flow**

Reynolds number
Continuity Equation
Bernoulli's principle

Ideal Fluid

Consider an IDEAL FLUID

- Fluid motion is very complicated. However, by making some assumptions, we can develop a useful model of fluid behaviour. An ideal fluid is :

  - Incompressible – the density is constant
  - Irrotational – the flow is smooth, no turbulence
  - Nonviscous – fluid has no internal friction
  - Steady flow – the velocity of the fluid at each point is constant in time.

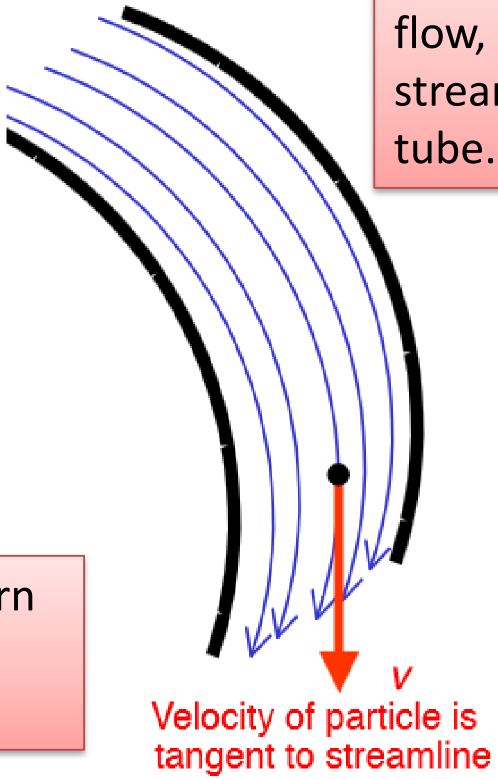
Ideal fluid

Consider the average motion of the fluid at a particular point in space and time.

An individual fluid element will follow a path called a flow line

Steady flow is when the pattern of flow lines does not change with time

Streamlines - in steady flow, a bundle of streamlines makes a flow tube.



Flow Pattern

- **Laminar** : fluid particles moving in an orderly manner and retaining the same positions in successive cross-sections.
- **Turbulent** : fluid particle occupied different relative position in successive cross sections.
- There are small fluctuation in magnitude and direction of velocity, accompanied by small fluctuation of pressure.

Flow Pattern

- The Reynold's Number
- Criterion determines whether the flow is laminar or turbulent is the ratio of inertial force to viscous force acting on the particle, known as Reynold's Number.

$$\mathrm{Re} = \frac{inertia\ force\ \text{[promotes turbulent flow]}}{viscous\ force\ \text{[promotes laminar flow]}}$$

$$\mathrm{Re} = \frac{\rho \mathrm{v} l}{\mu} = \frac{\mathrm{v} l}{\upsilon}$$

  - Where ρ : density; μ : dynamic viscosity; v : velocity; l : length; v : kinematic viscosity
- In pipes; l is replaced by diameter of pipe; d.

$$\mathrm{Re} = \frac{\rho \mathrm{v} d}{\mu}$$

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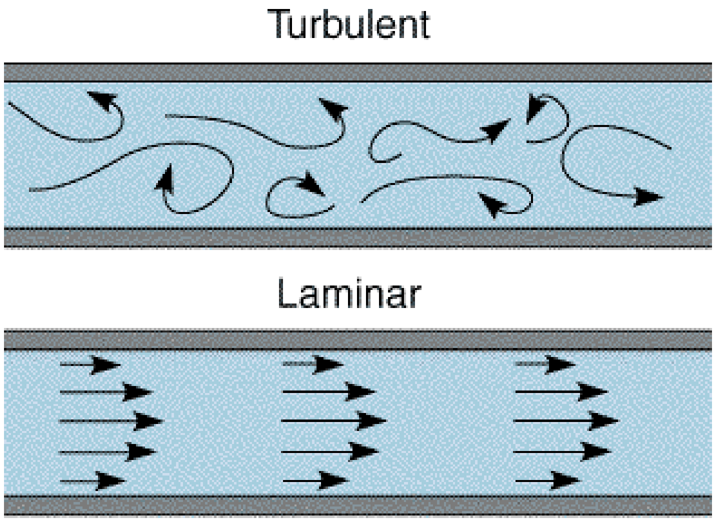
Flow Pattern

- The flow is classified based on the calculated value of Reynolds number ass follow :
  - Re < 2000 – Laminar
  - 4000 < Re > 2000 – transitional
  - Re > 4000 – Turbulent



Source : https://commons.wikimedia.org/wiki/File:Flujo-laminar-y-turbulento.gif

Fluid Flow Rate

- The quantity of fluid flowing in a system per unit time can be expressed by the following three different terms:
  - Q : The volume flow rate is the volume of fluid flowing past a section per unit time.
  - W : The weight flow rate is the weight of fluid flowing past a section per unit time.
  - $\dot{m}$: The mass flow rate is the mass of fluid flowing past a section per unit time.

Symbol	Name	Definition	SI Units	US System Unit
Q	Volume Flow Rate	$Q = Av$	m^3/s	ft^3/s
W	Weight Flow Rate	$W = \gamma Q$ $W = \gamma Av$	N/s	lb/s
M	Mass Flow Rate	$M = \rho Q$ $M = \rho Av$	kg/s	slugs/s

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Fluid Flow Rate

- The most fundamental of these terms is the volume flow rate Q, which is calculated from

$$Q = Av$$

- The weight W is related to Q by,

$$W = \gamma Q$$

- Where γ is the specific weight of the fluid. The unit of W is then

$$W = \gamma Q = \frac{\text{N}}{\text{m}^3} \times \frac{\text{m}^3}{\text{s}} = \frac{\text{N}}{\text{s}}$$

- The mass flow rate, $\dot{m}$ is related to Q by; $\dot{m} = \rho Q$

Conservation of mass, m

- The principle of conservation of mass :
  - **Matter can neither be created nor destroyed**

$$\text{mass flowing in} = \text{mass flowing out}$$
$$m_1 = m_2$$
$$\rho v_1 = \rho v_2$$
$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$
$$A_1 v_1 = A_2 v_2 = \mathrm{Q} = {}^{v}/_{t} = \text{constant}$$

- Application of the conservation of mass to steady flow in the streamtube results in the Equation of Continuity

3.2 The Continuity Equation

- The conservation of mass principle for a control volume can be expressed as: "The net mass transfer to or from a control volume (CV) during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt".

Total mass entering the CV during Δt	Total mass leaving the CV during Δt	Net change in mass within the CV during Δt

- For steady flow in a stream; $m_{in} - m_{out} = \Delta m_{CV}$

- For steady incompressible flow (ρ1= ρ2) in a stream;

$$Q_1 = Q_2$$
$$[Av]_1 = [Av]_2$$

$$m_1 = m_2$$
$$[\rho Av]_1 = [\rho Av]_2$$

3.3 The Bernoulli's theorem

Bernoulli's principle can be derived from the principle of conservation of energy.

Pressure energy when fluid flowing under pressure

- Pressure energy ;

$F \times \text{distance}$

- Pressure energy per unit weight; $\dfrac{P}{\rho g}$

Kinetic energy due to its velocity

- Kinetic energy ; $\dfrac{1}{2}mv^2$
- Kinetic energy per unit weight; $\alpha \dfrac{v^2}{2g}$
-
- α = kinetic energy correction factor

Potential energy due to height above datum, z

- Potential energy;

mgz

- Potential energy per unit weight; z

- In practice flows encountered turbulent thus ignore and α is set equal to 1.
  - 1.04 < α < 1.11 for fully developed turbulent flow in round pipe
- However, some situation α is significant especially for laminar flow.
  - α = 2.0 for fully developed laminar pipe flow

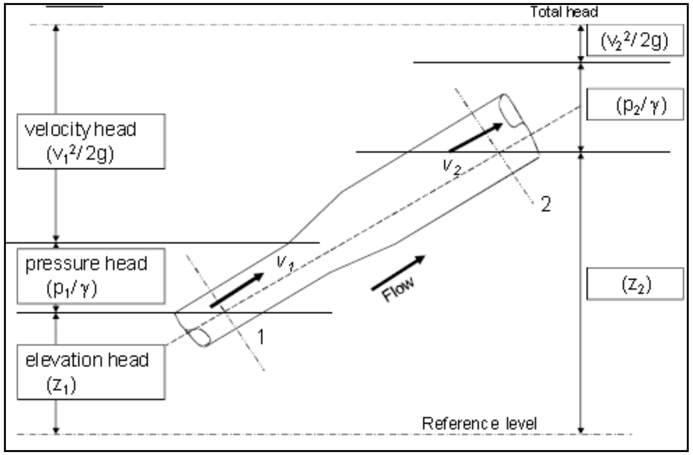
The total energy;

$$H = \frac{P}{\rho g} + \alpha \frac{v^2}{2g} + z$$

$$H = \frac{P}{\rho g} + \frac{v^2}{2g} + z$$

The Bernoulli's Theorem

- In a steady flow, the sum of all forms of energy in a fluid along a streamline is the same at all points on that streamline.
- Energy before = energy after
- Bernoulli's equation accounts for the changes in elevation head, pressure head, and velocity head between two points in a fluid flow system. It is assumed that there are no energy losses or additions between the two points, so the total head remains constant.



$$\text{Total head before}\,(H) = \text{Total head after}\,(H)$$

$$\left[\frac{P}{\rho g} + \frac{v^2}{2g} + z\right]_1 = \left[\frac{P}{\rho g} + \frac{v^2}{2g} + z\right]_2$$

$$\frac{P_1}{\rho g} + \frac{v_1^{\,2}}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^{\,2}}{2g} + z_2$$

Pressure energy per unit weight + Kinetic energy per unit weight + Potential energy per unit weight = Total energy per unit weight (H) = Constant

Bernuolli's Equation - Restriction

Valid only for incompressible fluids

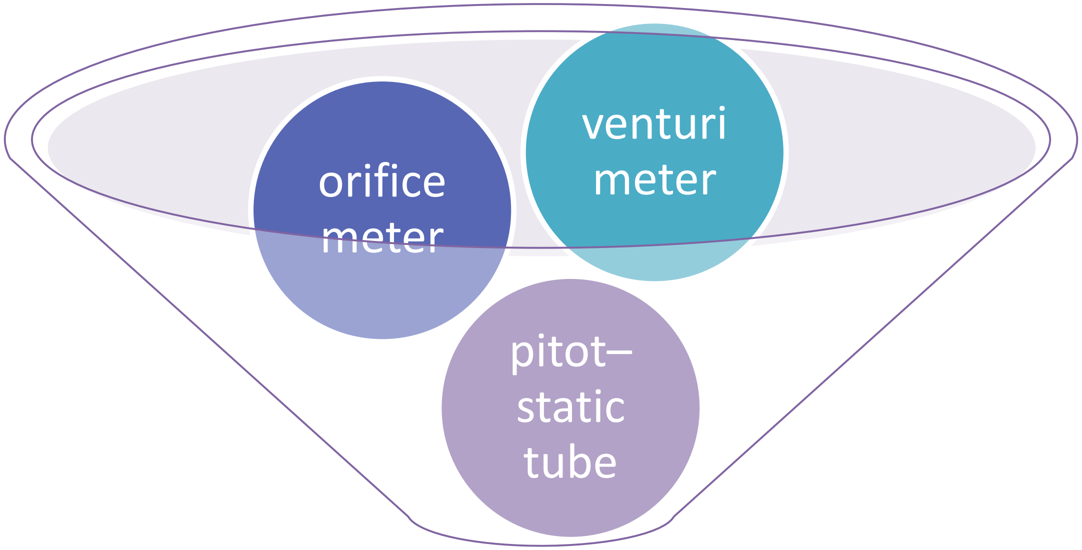
- because the specific weight of the fluid is assumed to be the same at the two sections of interest.

There can be no mechanical devices between the two sections

- that would add energy to or remove energy from the system, because the equation states that the total energy in the fluid is constant.

3.4 Application of Bernoulli and Continuity Equations – Flow measurement

- Differential pressure devices :



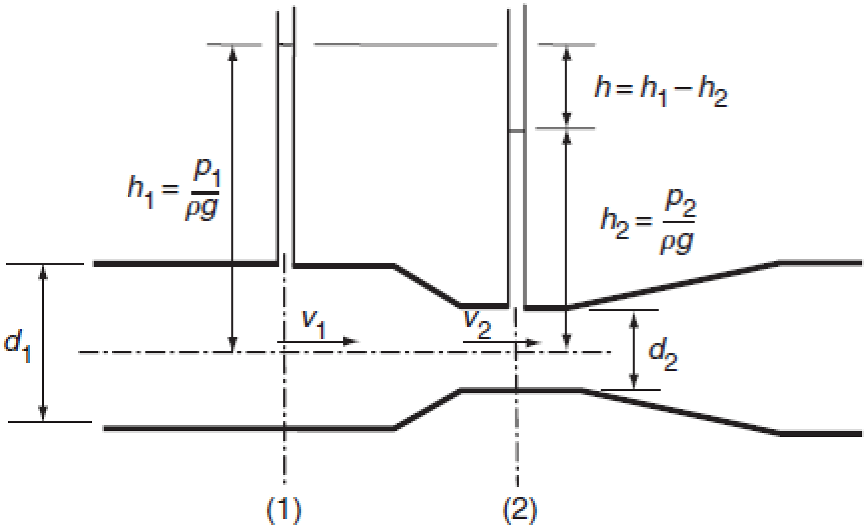
Make use of the pressure difference between two points in the flow to give an indication of the flow rate.

Bernoulli's equation is applied to obtain expressions for flow rate

3.4.1 Venturi Meter

- Consists of a section of pipe which converges to a throat
- section and then diverges back to the original pipe diameter
- Piezometers at sections (1) and (2) record the pressure heads h_1 and h_2 . The difference in the pressure heads h varies with the rate of flow.

If the fluid is an incompressible liquid, then the density will be constant and $\rho_1 = \rho_2$, thus : $Q = A_1 v_1 = A_2 v_2$



$$A_1 v_1 = A_2 v_2$$

$$\left(\frac{\pi d_1^{\ 2}}{4}\right)_1 v_1 = \left(\frac{\pi d_2^{\ 2}}{4}\right)_2 v_2$$

$$d_1^2 v_1 = d_1^2 v_2$$

$$\frac{v_1}{v_2} = \frac{d_1^2}{d_1^2}$$

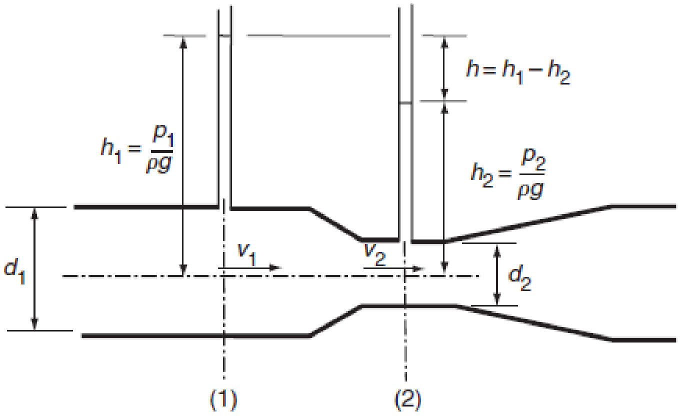
$$\frac{v_1}{v_2} = \left(\frac{d_2}{d_1}\right)^2 \quad \text{or} \quad v_2 = \left(\frac{d_1}{d_2}\right)^2 v_1$$

- Applying Bernoulli's equation to sections (1) and (2) gives:

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

$$\frac{P_1}{\rho g}+\frac{v_1{}^2}{2g}=\frac{P_2}{\rho g}+\frac{v_2{}^2}{2g}$$

$$\frac{P_1}{\rho g}-\frac{P_2}{\rho g}=\frac{v_2{}^2}{2g}-\frac{v_1{}^2}{2g}$$



- As $\frac{P_1}{\rho g}=h_1$ and $\frac{P_2}{\rho g}=h_2$ thus, $\Delta P=h$

- Thus, the pressure difference between (1) and (2),

$$h=\frac{v_2{}^2}{2g}-\frac{v_1{}^2}{2g}\ \text{rearrange gives:}$$

$$v_2{}^2-v_1{}^2=2gh\rightarrow v_2-v_1=\sqrt{2gh}$$

- Substituting $v_2 = \left(\frac{d_1}{d_2}\right)^2 v_1$ into $v_2 - v_1 = \sqrt{2gh}$ gives :

$$\left(\frac{d_1}{d_2}\right)^2 v_1 - v_1 = \sqrt{2gh}$$

$$v_1\left[\left(\frac{d_1}{d_2}\right)^2 - 1\right] = \sqrt{2gh}$$

$$v_1 = \frac{\sqrt{2gh}}{\left[\left(\frac{d_1}{d_2}\right)^2 - 1\right]}$$

thus, $$Q = A_1 \times \frac{\sqrt{2gh}}{\left[\left(\frac{d_1}{d_2}\right)^2 - 1\right]}$$

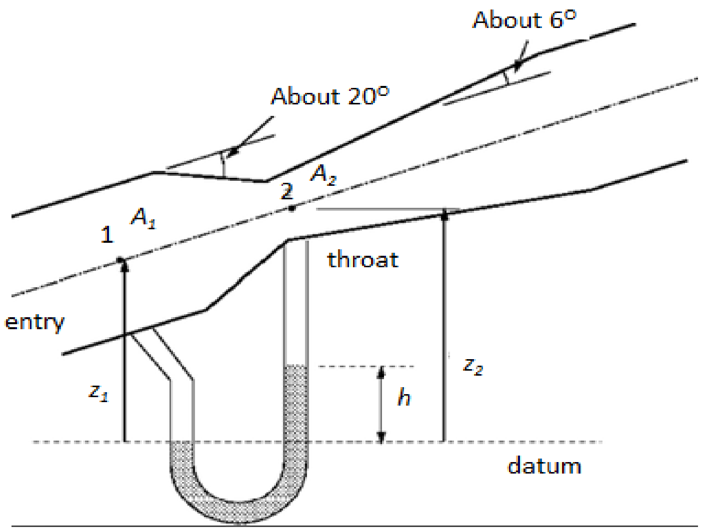
This is the theoretical volume-flow rate (Q_{theory}).

- The Q_{actual} is always slightly lesser than Q_{theory} due to viscous drag and turbulence.
- The actual flowrate is obtained by : $Q_{act} = C_d \times Q_{theory}$

- The discharge coefficient C_d for venturi meter normally range between 0.95–0.97.

Venturi meter

- Venturi meter (inclined) with the derived equations to calculate the velocity and discharge at the throat section.



$$v_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}$$

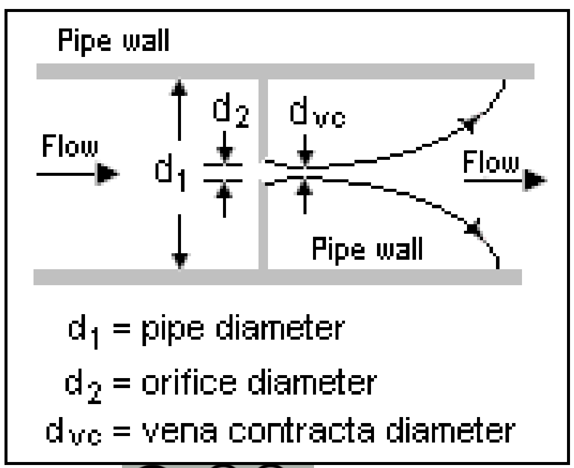
$$Q = \frac{A_2 A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}$$

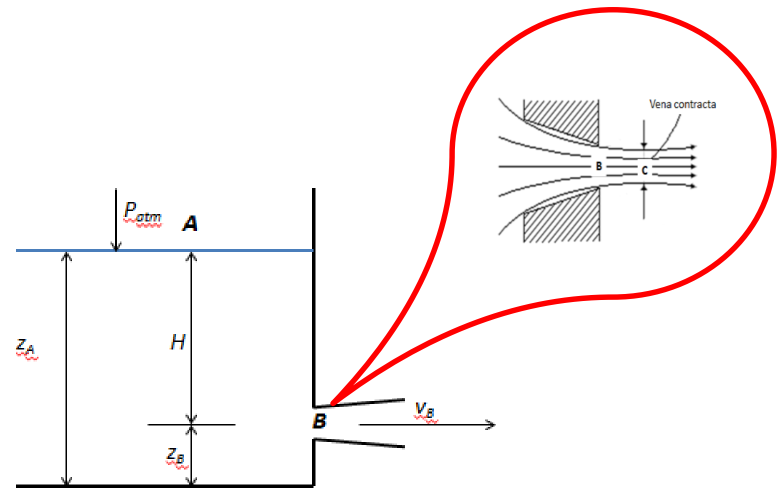
- The discharge formula given above is the theoretical discharge. Thus in order to get the actual discharge, the discharge coefficient C_d must be known.

$$Q_{act} = C_d \times Q_{theory}$$

3.4.2 Pipe orifice

- An orifice meter operates exactly the same as the venturi meter.
- Cheap compared to the venturi meter, however there is substantial energy losses.
- Consists of a sharp-edged orifice plate which is inserted between the connecting flanges of a pipeline.
- The arrangement : consists of flat circular plate which has a circular hole. Diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter



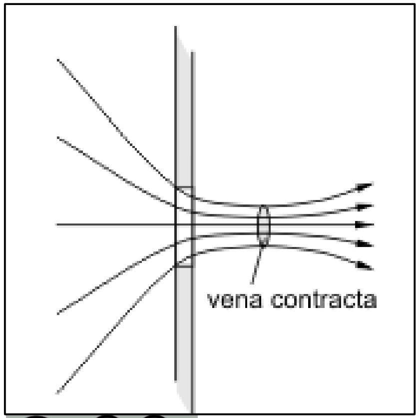


- The section at C is called Vena Contracta, and C_c is the coefficient of contraction.
- Actual area of jet at C is, C_c x A. a sharp edge orifice has coefficient of contraction $C_c = 0.64$.
- The value of coefficient C_d, C_v, C_c are determined experimentally and normally the C_d ranges from 0.6 to 0.65 for most orifice meters.

$$C_d = \frac{\text{actual measured discharge}}{\text{theoretical discharge}} = \frac{Q_{act}}{Q_{theory}}$$

$$C_v = \frac{\text{velocity at vena contracta}}{\text{theoretical velocity}} = \frac{v_c}{v_{theory}}$$

$$C_c = \frac{\text{area of jet at vena contracta}}{\text{area of the orifice}} = \frac{A_c}{A_o}$$



- Assuming no energy loss, thus

$$\frac{P_A}{\rho g}+\frac{v_A^2}{2g}+z_A=\frac{P_B}{\rho g}+\frac{v_B^2}{2g}+z_B$$

- As A is open to atmosphere, and water is still, h= zA – zB, thus: $h=\frac{v_B^2}{2g}$

- Rearranging gives; $v=\sqrt{2gh}$ this theory is known as Torricelli's theorem. "the velocity of the issuing jet is proportional to √h, where h is the head producing flow.

- As with the venturi meter, the actual flow rate Q is given by:

$$Q=A_1\times\frac{\sqrt{2gh}}{\left[\left(\frac{d_1}{d_2}\right)^2-1\right]}$$ or $$Q=A\sqrt{2gH}$$

- Again, the actual discharge flowing through the pipe orifice is : $Q_{act}=C_d A\sqrt{2gH}$