MECHANICS OF MATERIALS

Buckling of Columns

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Chapter Description

- Expected Outcomes
  - Describe the concept of columns in term of types of column, cross-sectional of shapes, industry standard application and slenderness ratio of column
  - Illustrate, explain and differentiate the failure mode of columns due to buckling
  - Explain the influence of support conditions
  - Relationship between the effective length and radius of gyration
  - Describe the compression member of long / slender column
  - Applied of the Euler formula to determine the critical load for long columns

9.1 Concept Of Stability Of Column

- A **column** in structural engineering is a vertical structural element that transmits, through compression, the weight of the structure above to other structural elements below.

- For the purpose of wind or earthquake engineering, **columns** may be designed to resist lateral forces.

- **Columns** are frequently used to support beams or arches on which the upper parts of walls or ceilings rest.

- A column is a relatively long, slender member loaded in compression.

- When a perfect column is subjected to a **compressive** axial force, the only deformation that takes place is a **shortening** of the column.
- For **low** values of F, if the column were to be deflected laterally by a force perpendicular to the column, and the lateral force were thereafter removed, the column would **return to its straight position**, even with the force F remaining in place.
- This indicates a condition of **stability**.
- If the load F were increased, there is a value of F for which, when the lateral load is removed, the column would remain in the deformed shape.
- This condition is referred to as buckling and the column is said to have failed from a structural standpoint.
- Buckling can also be described in simple terms as bending or bowing of a column due to a compressive load.

Slenderness Ratio

Slenderness ratio is a measure of how long the column is compared to its cross-section's effective width (resistance to bending or buckling). The **slenderness ratio** is the column's **effective length** divided by the **radius of gyration**.

$$s = \frac{L}{r}$$

Where $r = \sqrt{(I/A)}$

9.2 Failure Mode Of Column

What is Buckling?

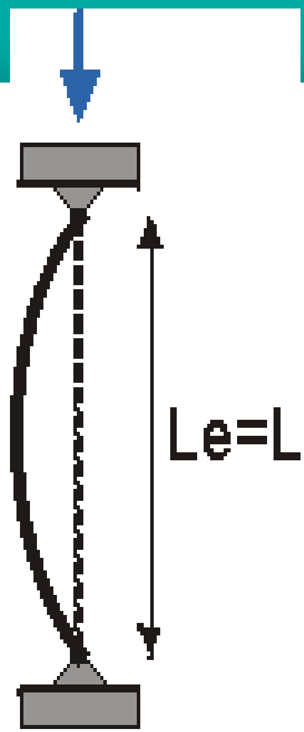
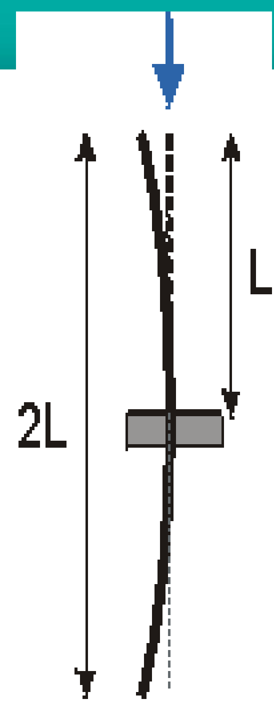
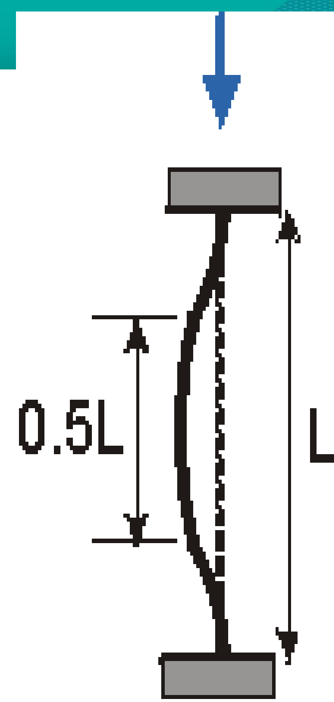
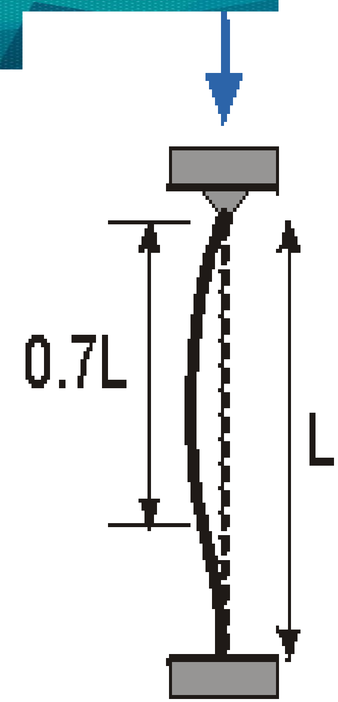
- When a slender member is subjected to an axial compressive load, it may fail by a condition called **buckling**.
- Buckling is not a failure of the material itself (as is yielding and fracture), but is due to geometric **instability of the system**.
- Note that buckling is **not dependent** on material strength.

Effective Length

- How a column is supported governs its **buckling strength**. The effective length L_e accounts for differences in the **end supports**.

What is the effective length?

- The **effective length** is the length the column **would be if it were to buckle** as a **pinned-pinned** column.
- A dimensionless coefficient *K*, ***effective-length factor***, is used to calculate L_e

9.3 Effective Lengths for Columns with Various End Conditions				
End Condition	**Pinned-Pinned**	**Fixed-Free**	**Fixed-Fixed**	**Fixed-Pinned**
The effective length is equal to the **distance between points** in the column where **moment = 0** (between "pins"). This occurs when the **curvature of the column changes**. The fixed-free column is "**mirrored**" through the fixed end to visualize L_e=2L.				
Effective Length, L_e	**L**	**2L**	**0.5L**	**0.7L**
Relative Buckling Strength (~ 1/ L_e^2) for same L	**1**	**0.25**	**4**	**2**

Radius Of Gyration

- If all of the cross-sectional area **A** were massed a distance **r** away from the bending axis, the idealized lumped-area cross-section would have the same moment of inertia **I** as the actual cross-section if:

$$I = Ar^2$$

- Distance **r** is the **radius of gyration**. There generally two bending axes to consider, and thus two radius of gyration:

$$r_x = \sqrt{\frac{I_{xx}}{A}}; \quad r_y = \sqrt{\frac{I_{yy}}{A}}$$

9.4 Magnitude Of The Load At Which Buckling Would Occur

- Columns are long slender members subjected to an axial compressive force. **Lateral deflection** on a column is called **buckling**. The **maximum axial load** that a column can support when it is on the verge of buckling is called the **critical load P_{cr}**
- FOS is a safety margin given in design so that the member will not fail when the load is increased beyond the elastic limit or when the size in reduced.
- Normally, the factors of safety varies between 1.4 to 3

$$FOS = \frac{P_{ult}}{P_{all}} \qquad FOS = \frac{\sigma_{ult}}{\sigma_{all}}$$

USE OF THE EULER FORMULA

- **Euler Buckling Formula**

- Both ends are pinned so they can **freely rotate** and **cannot resist a moment**. The critical load P_{cr} required to buckle the pinned-pinned column is the

$$P_{cr} = \frac{\pi^2 EI}{L_e^{\,2}}$$

P_{cr} = the euler buckling load

E = Young's modulus for the materials

I = the least second moment of area of the section

L_e = effective length

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Assumptions / limitation of the Euler formula

- Axially loaded column
- Column is perfectly straight
- Isotropic and homogeneous material
- Material behaves within elastic properties
- Both ends of column support are pinned

Summary

- Long slender members subjected to an axial compressive force are called columns.
- Lateral deflection is called buckling.
- Maximum axial load a column can support when on the verge of buckling is called the critical load, Pcr.

References

- Hibbeler, R.C., Mechanics Of Materials, 9th Edition in SI units, Prentice Hall, 2013.
- Ferdinand P. Beer, E. Russell Johnston, Jr., John T. DeWolf, David F. Mazurek, Mechanics of materials 5th Edition in SI Units, McGraw Hill, 2009.

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