MECHANICS OF MATERIALS

Transformation of Stress and Strain

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Chapter Description

- Expected Outcomes
  - Explain the concept of plane – stress transformation
  - Apply and determine the principal plane, principal stresses, maximum shear stress and location(s) of angle using general equations
  - Construct the Mohr's circle diagram
  - Apply and determine the principal plane, principal stresses, maximum shear stress and location(s) of angle using Mohr's circle.

7.1 Introduction

- The principal topics of this chapter is to deal with **more than one type of stress** exists in the member at the same time
- Applications for this combinations of stresses to thin-wall, thick-wall, filament-wound and composite pressure vessels
- To determine stresses acting on the sides of a stress element in any direction namely **transformation of stress**
- In this chapter ,methods used will be developed to determine:
  - ➢ Normal and shear stresses acting on any specific plane
  - ➢ Maximum normal and shear stresses acting at any possible orientation at point of interest

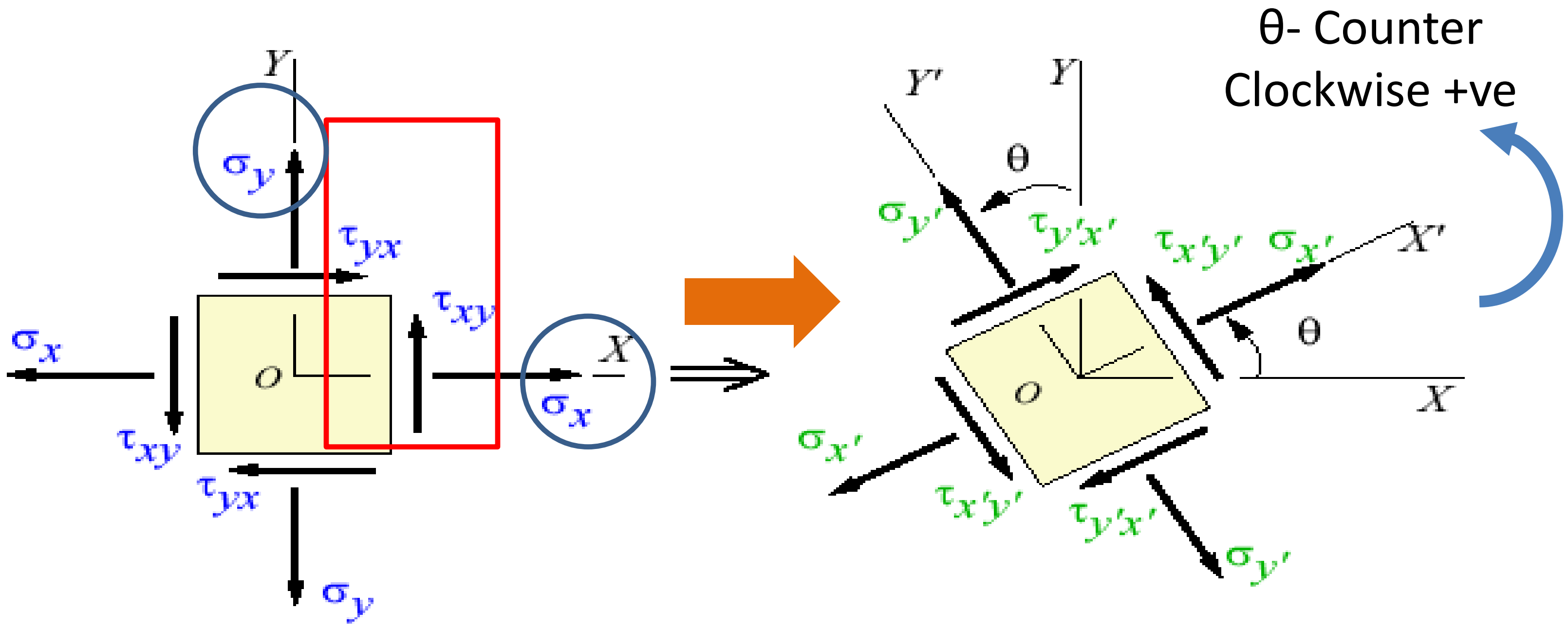
Plane Stress-transformation

- General state of stress at a point is characterized by six (6) independent normal and shear stress components
- It can be analysed in a **single plane** of a body, the material can said to be subjected to **plane stress**
- **Plane stress** - defined as two-dimensional (2D) problems which occur in a **thin plate** subjected to loading **uniformly distributed** over the thickness and parallel to the plane of plate as mention earlier
- Since the plate is thin, the stress distribution may be closely approximately by assuming the 2D stress component
- The general state of plane stress at a point represented by a combination of two normal-stress components σ_x , σ_y and one shear stress component τ_{xy}

- **Transformation of the stress** components that are associated with a particular coordinate system into components associated with coordinate system having a **different orientation**
- When the Transformation equations are established, we should be able:
  - To obtain the magnitude of the **max. normal stress** and **max. shear stress** at a point and
  - The **orientation of the elements** upon which they act

7.2 Normal Stress (σ_x,σ_y) And Shear Stress (τ_{xy}) Sign conventions

- **Positive normal stress acts outward** from all faces and **positive shear stress acts upward on the right-hand face** of the element



Stresses at given coordinate system   Stresses transformed to another coordinate

- **Normal Stress Equations:**

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

- **Normal Shear Stress Equation:**

$$\tau_{xy}' = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \Rightarrow \tau_{xy} = \tau_{yx}$$

- **Average Normal Stress Equations:**

$$\sigma_{Ave} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

7.3 Principal Stresses And Maximum In-plane Shear Stress

- Represent the **maximum** and **minimum principal normal stress** at the point
- When the state of stress is represented by the principal stresses, **no shear stress** will act on the element
- The state of stress at the point can also be represented in terms of the **maximum in plane shear stress** – in this case an **average normal** stress will also act on the element
- The element representing the max in plane shear stress with associated average normal stresses is **oriented 45°** from the element representing the principal stresses

- **Orientation** of the planes will determine the maximum and minimum normal stress

  Orientations or Locations, σ_p

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

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- The principal plane stress oriented at θ_p can be determined form the following rule

If $\sigma_x - \sigma_y$ is positive,

θ_p indicates the orientations of σ_{p1}

If $\sigma_x - \sigma_y$ is negative,

θ_p indicates the orientations of σ_{p2}

In-Plane Principal Stresses

- The solution has **two roots**, thus we obtain the following principle stress

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\therefore \text{where } \sigma_1 > \sigma_2$$

Maximum in-plane shear stress

- Maximum In-Plane Principal Shear Stress Equations
  - Orientation of an element will determine the maximum and minimum shear stress.

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

- The solution has two roots, thus we obtain the maximum in-plane shear stress and averaged normal stress.

$$\tau_{\max \text{in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Important Observations

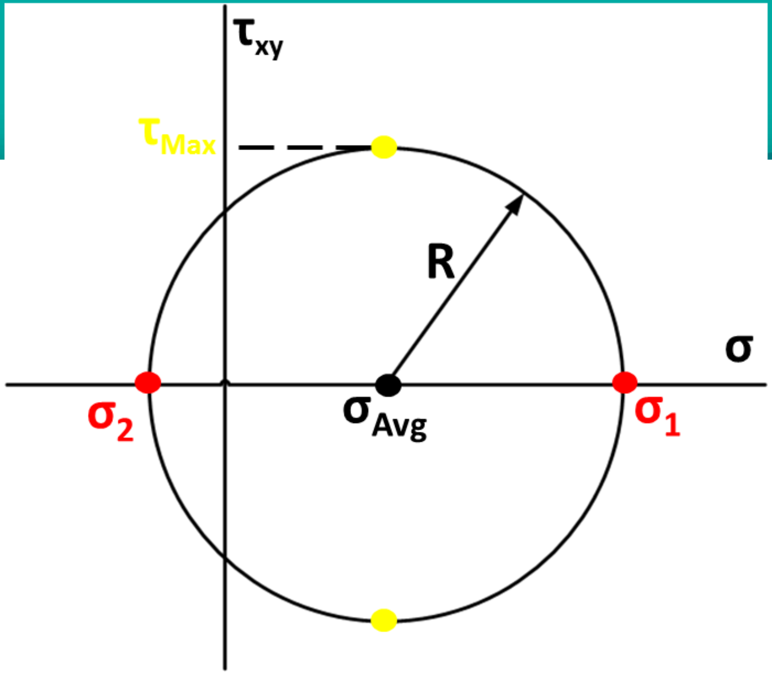
1. Principal stresses represent the **maximum** and **minimum** normal stress at the point
2. Principal stresses (σ_p) occur on mutually **perpendicular planes**
3. Shear stresses (τ) are **zero** on principal planes
4. Planes of maximum shear stress (τ) occur at **45°** to the principal plane
5. The maximum shear stress (τ_{max}) is **equal to one half** the difference of the principal stresses

7.4 Stress Transformation Using Mohr's Circle Method

- The transformation equations for plane stress can be represented in a graphical format known as **Mohr's circle**
- Useful in visualizing the **relationships** between **normal** and **shear stresses** acting on various inclined planes at a point in a stressed body
- Plane stress transformation is able to have a graphical solution that is easy to remember

7.5 Mohr's Circle

- A **graphical method** - solution for plane stress transformation
- Easy to remember
- Visualise how the normal stress and shear stress vary as the plane on which they act is oriented in **different direction**



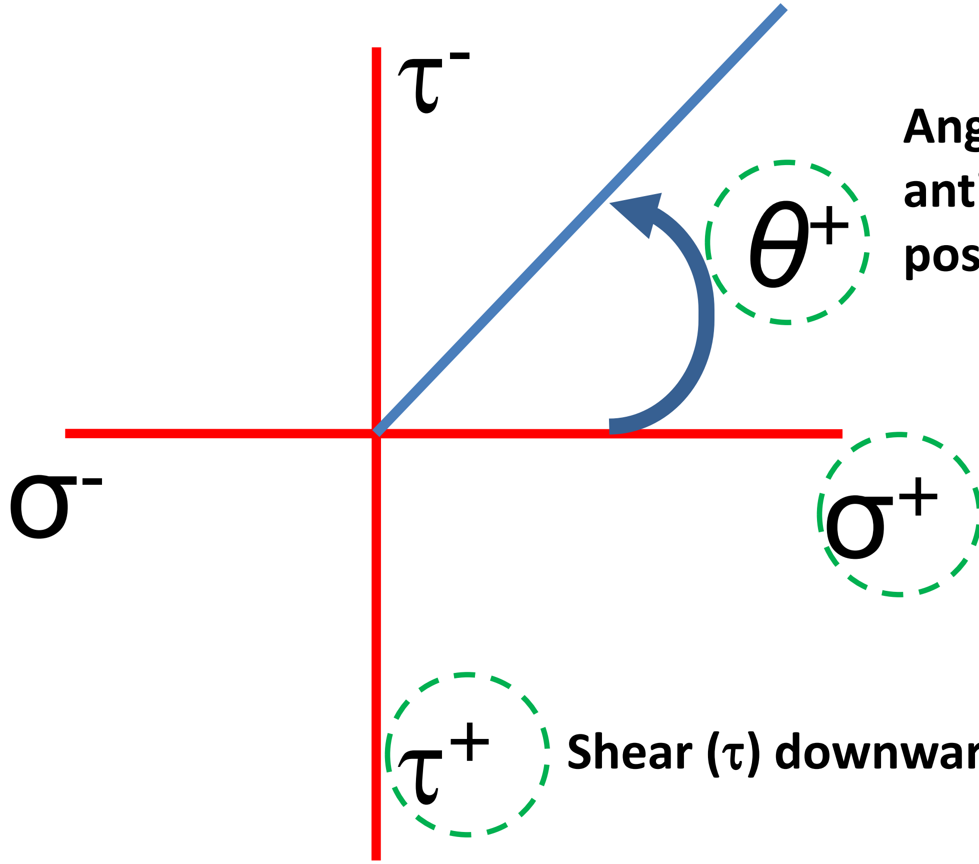
- The two principal **stresses** are shown **in red**, and the maximum shear stress is shown in orange.
- The shear stress equals the maximum shear stress when the stress element is rotated 45° away from the principal directions.

Assumption Of Mohr's Circle Diagram

- Before we discuss the procedure for constructing Mohr's circle there a several **rules** that apply

Stress Component	Plot
Normal Stresses (Horizontal Axis)	Tension (+) Compression (–)
Shear Stresses (Vertical Axis)	Clockwise Shear (+) Counter clockwise Shear (–)

Use same scales for both axes



Angle (θ) as anticlockwise is positive

Stress (σ) to the right is positive

Shear (τ) downward is positive

Procedure to construct Mohr's circle diagram

1) Draw the **appropriate and same scales** for both axes (x-y axes)
2) Determine and plot **the center (C)** of the circle with **coordinate C (σ_{ave},0)**
3) Determine and plot coordinates **point A (σ_x,τ_{xy})** and **point B (σ_y,-τ_{xy})**
4) Draw a **straight line connecting the two points (A-B)** where C by bisecting the line AB
5) Draw the **circle from C (as a radius)** 'from end to end' points A and B
6) AC as a **reference line** and **bold the line** where angle (θ) is zero
7) If the stresses on a plane making an angle (θ) with the x-plane are required, locate a point on the circle making an **angle 2θ form AC**

Absolute Maximum Shear Stress

- The **absolute maximum shear stress** and associated **average normal stress** can also be found by using Mohr's circle.

$$\tau_{\text{abs max}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} \qquad \sigma_{avg} = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

References

- Hibbeler, R.C., Mechanics Of Materials, 9th Edition in SI units, Prentice Hall, 2013.
- Ferdinand P. Beer, E. Russell Johnston, Jr., John T. DeWolf, David F. Mazurek, Mechanics of materials 5th Edition in SI Units, McGraw Hill, 2009.

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