MECHANICS OF MATERIALS

Analysis of Beams for Bending

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Chapter Description

- Expected Outcomes
  - Define the elastic deformation of an axially loaded prismatic bar
  - Define the multiple prismatic bars
  - Define the principle of superposition
  - Define the Saint – Venant's principle
  - Calculate the deformations of member under axial load
  - Calculate the deformations of member for stepped composite bar
  - Analyse the deformations in systems of axially loaded bars
  - Analyse the deformations of member for statically indeterminate assemblies.
  - Calculate the deformation of member due to temperature effect.

LEARNING OUTCOME

- Calculate the reactions at support for **simply supported**, **cantilever** or **overhanging** beams.
- Analyse the beam to evaluate the **shear force** and **bending moment** due to various loadings condition using moment equations and graphical technique due to point load, UDL, triangular distributed load, external moment or combined loads.
- Draw the **shear force** and **bending moment diagram**.
- Locate the maximum shear force and bending moment and their location.

LEARNING OUTCOME

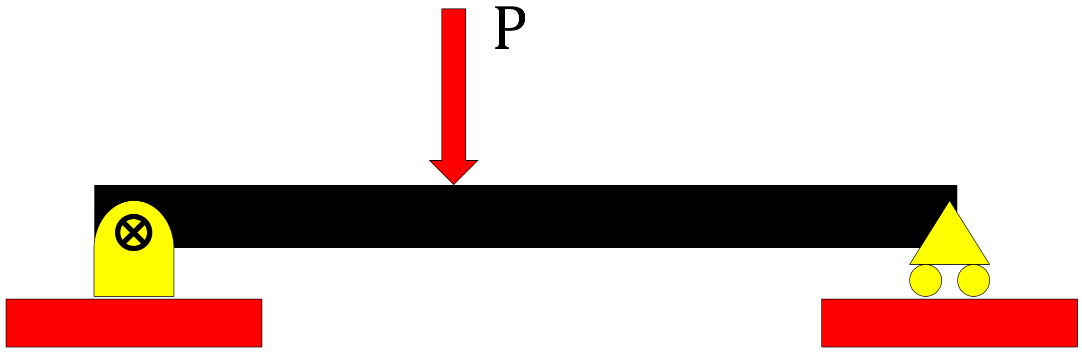
- Explain the internal moment and bending / normal stress in the beam.
- Calculate the section properties about the N.A of **symmetrical** or **unsymmetrical** 2D cross – sectional area of beam.
- Calculate the second **moment inertia** about x-x axis, I_{xx} at different cross – sectional area of beam.
- Apply and analyse the bending stress in the beam using **flexure formula** at the appropriate location(s).

5.1 INTRODUCTION

- **Beams**: Members with **support** loadings applied perpendicular to their longitudinal axis
- Beams are subjected to a variety of loading pattern:
  - Normal concentrated load
  - Inclined concentrated load
  - Uniformly distributed loads
  - Linearly distributed loads
  - Moment

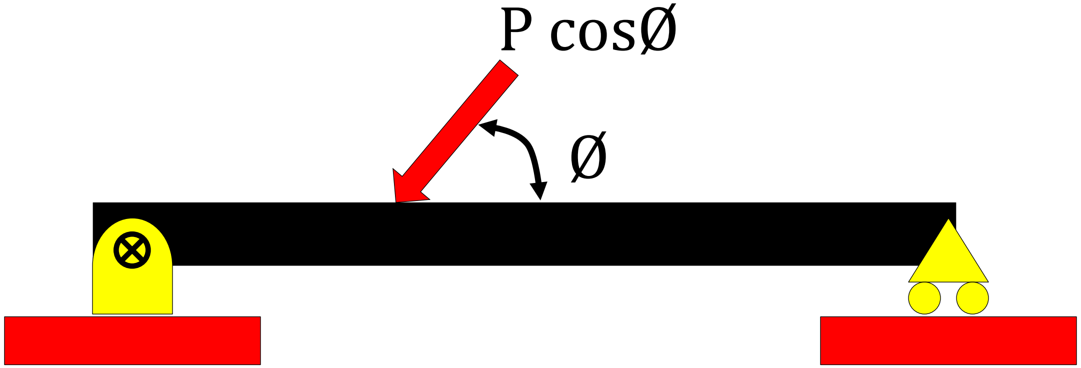
Normal Concentrated Load

A normal concentrated load is one that acts **perpendicular** (normal) to the major axis of the beam at only a point or over a very small length of the beam



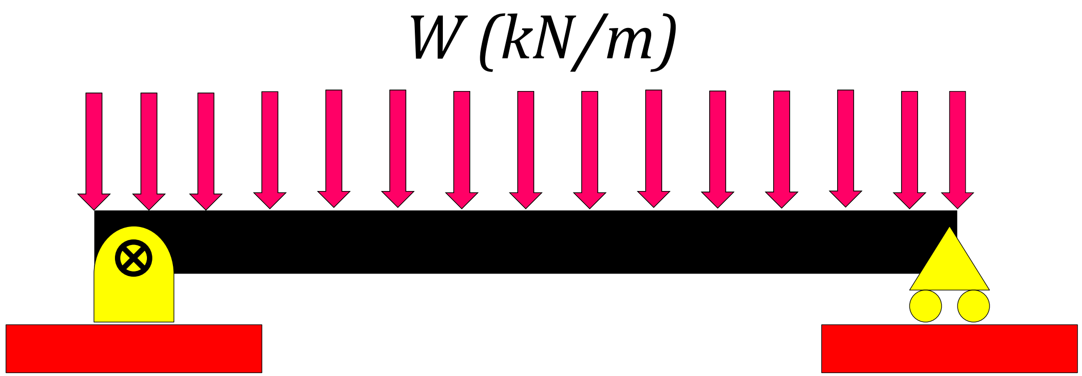
Inclined Concentrated Load

A inclined concentrated load is one that acts effectively at a point but whose line of action is at some **angle** to the main axis of the beam



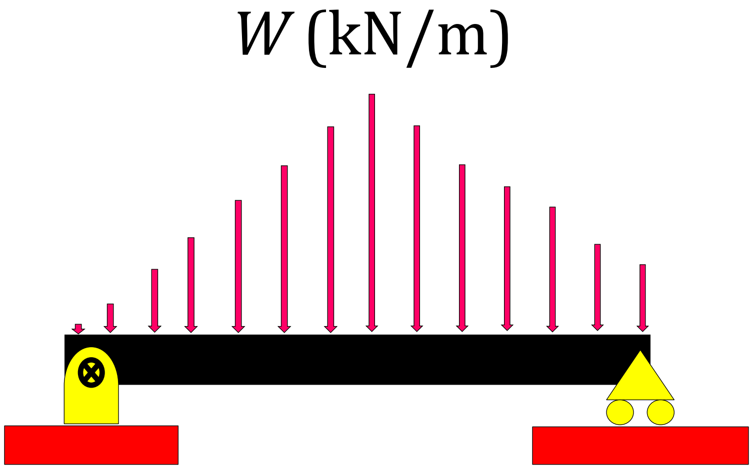
Uniformly Distributed Loads

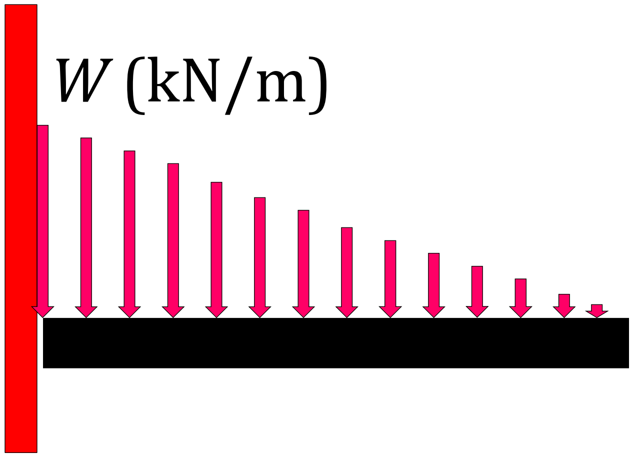
Loads of **constant magnitude** acting perpendicular to the axis of a beam over a significant part of the length of the beam



Linearly Distributed Loads

Loads of **varying magnitude** acting perpendicular to the axis of a beam over a significant part of the beam





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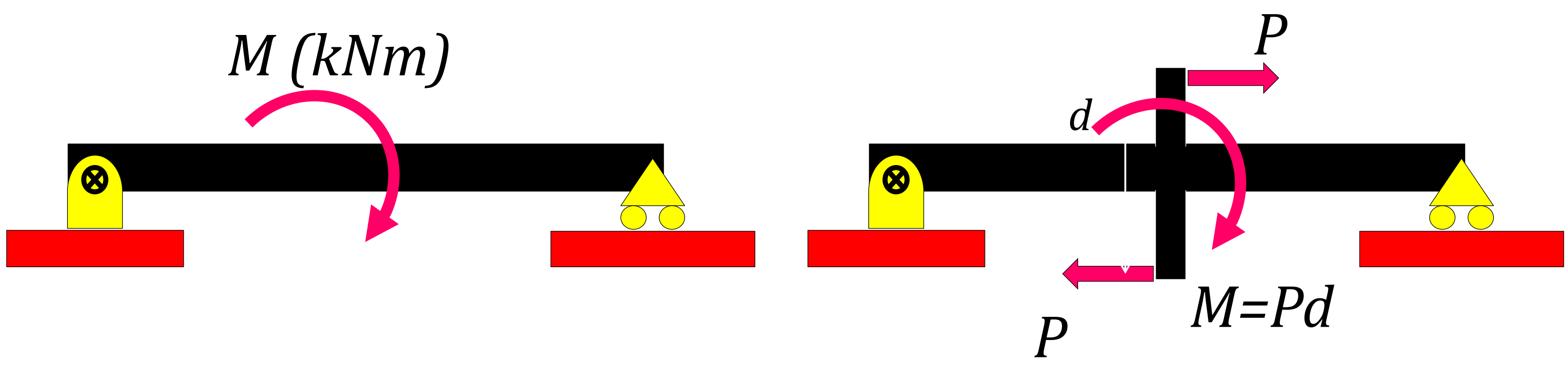
Concentrated Moment

When a **moment** act on beam at a point in a manner that tends to cause it to undergo pure **rotation**



BEAM TYPES

- Type of beam is indicated by the **types of supports** and their placement

a) Simply supported beam
b) Overhanging beam
c) Cantilever beam

Determinate beam

d) Continuous beam
e) Propped cantilever beam
f) Fixed end supports

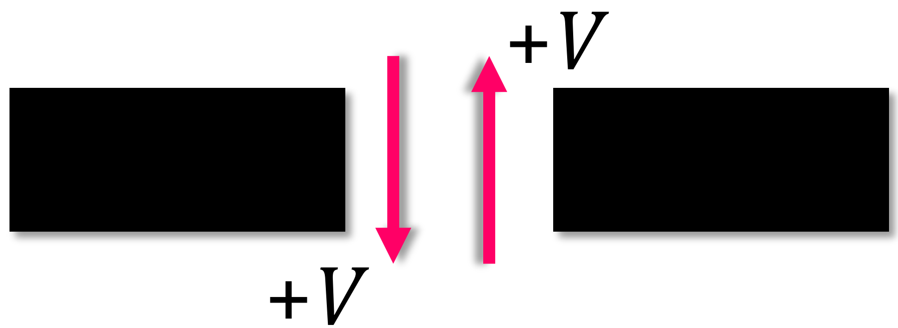
Indeterminate beam

5.2 SHEAR AND MOMENT DIAGRAMS

- Shear and moment functions can be plotted in graphs called **shear and moment diagrams**.
- Positive directions indicate the distributed load acting downward on the beam and clockwise rotation of the beam segment on which it acts.

Shear force:

- It is internal forces developed in the material of a beam to balance externally applied forces in order to secure equilibrium of all parts of the beam

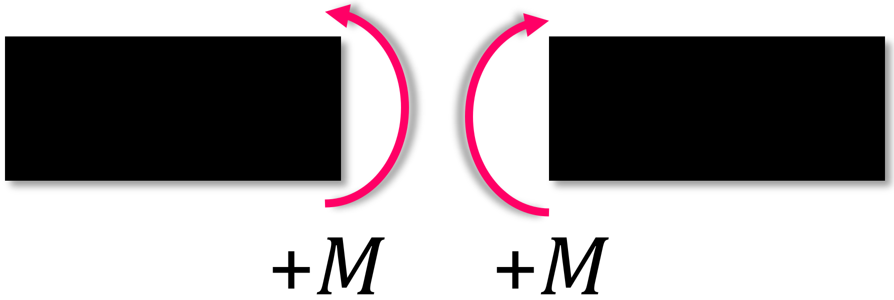


*Sign convention for shear force (*V*) – positive internal *V*

- Acts ***downward*** on right hand face of a beam
- Acts ***upward*** on the left hand face of a beam

Bending moment:

- Bending moments are internal moments developed in the material of a beam to balance the tendency for external forces to cause rotation of any part of the beam



*Sign convention for bending moment (*M*) – positive internal *M*

- Acts ***counter c/w*** on right hand face of a beam
- Acts ***clockwise*** on the left hand face of a beam

5.3 Shear & moment diagram

- The cutting plane exposes an internal **shear force** (***V***) and internal **bending moment** (***M***)
- The free body with ***V*** and ***M*** must satisfy **equilibrium**
- To develop shear force diagram (SFD) and bending moment diagram (BMD), it is necessary to determine *V* and *M* at all locations along the length of the beam
- Plotted as a function of x using **moment equations** – M(x) and **shear equations** – V(x)
- Other alternative, it can be created using **graphical technique**

Procedure for analysis

- **Support Reactions**
  - determine all the reactive forces and couple moments (if necessary) acting on the beam
- **Shear and Moment Functions**
  - Cut the beam where you wish to determine the internal force. Choose the part with the simplest FBD
- **Shear and moment Diagram**
  - Plot the SFD (V)and BMD (M) : generally show the V and M directly below the FBD of the beam

5.4 GRAPHICAL METHOD FOR CONSTRUCTING SHEAR AND MOMENT DIAGRAMS

Regions of Distributed Load

- The following 2 equations provide a convenient means for quickly obtaining the shear and moment diagrams for a beam.

Slope of the shear diagram at each point

$$\frac{dV}{dx} = -w(x)$$

-distributed load intensity at each point

Slope of moment diagram at each point

$$\frac{dM}{dx} = V$$

Shear at each point

- Integrate these areas between any two points to get change in shear and moment.

Change in shear $\Delta V = -\int w(x)dx$ -area under distributed loading

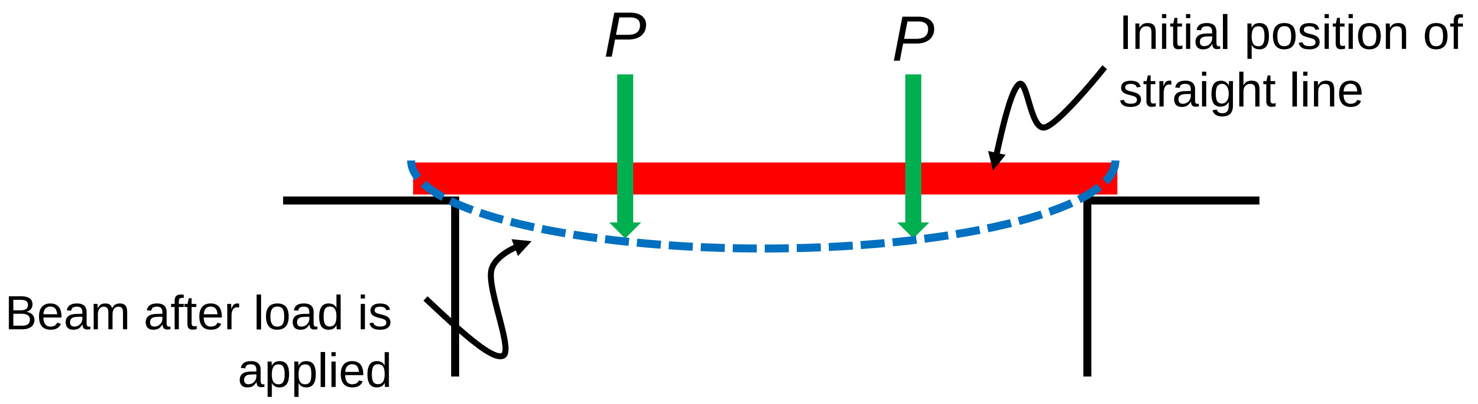
Change in moment $\Delta M = \int V(x)dx$ Area under shear diagram

5.5 Introduction Flexural Formula

- Previously, you have learnt how to determine the **shear force (V_x)** and **bending moment (M_x)** at any section of a loaded beam
- In future work in structural design you will learn how to design beams capable of withstanding the effects of shear force and bending moment
- As an introduction, the **analysis on bending stresses** that result from the application of bending moment will be study

5.6 ANALYSIS ON BENDING STRESSES

- When loads are applied perpendicular to the long axis of a beam, bending moment (M) are developed **inside** the beam, causing it to bend
- The characteristically **curved shape** are shown below is evident



- It is revealed that, the fibers of the beam near its **top surface** are **shortened**
- Conversely, the fibers near the **bottom surface** are **stretched /elongated**

- Cross section of a straight beam **remains** plane when the beam deforms due to bending.
- As summary, material **above** the centroid axis will be in **compression** with the maximum compressive stress occurring at the top surface
- Material **below** the centroid axis will be in **tension** with the maximum tensile stress occurring at the bottom surface
- Along the **centroid axis** itself, there is **zero** strain and stress due to bending called as the **neutral axis** (*N.A*)

FLEXURAL FORMULA

- Resultant moment on the cross section is equal to the moment produced by the **linear normal stress** distribution about the neutral axis.
- It is shown that, the bending stress is **inversely proportional** to the **moment of inertia** of the cross-section with respect to its horizontal centroidal axis
- Theoretically expressed that:

$$\sigma = -\frac{My}{I}$$

GUIDELINES FOR APPLYING

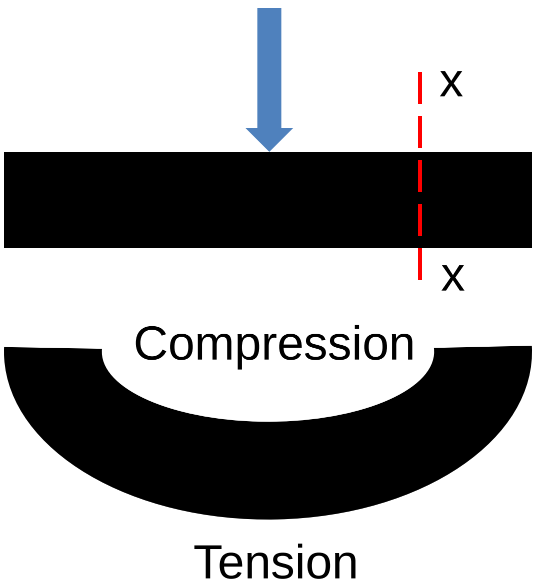
- Determine the **maximum bending moment** on the beam
- Locate the **centroid** of the cross – section of the beam
- Compute the **moment of inertia** of the cross – section with respect to its centroidal axis
- Compute the **distance** (*y*) from the centroidal axis to the top or bottom of the beam
- Compute the **bending stress** from flexure formula

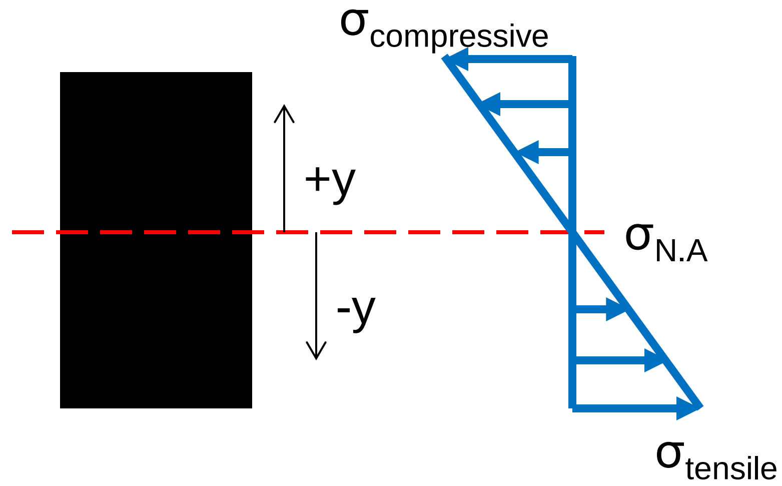
STRESS DISTRIBUTION DIAGRAM

- As mention earlier, a beam will deforms under the influence of a bending moment
- The segment assumes the characteristic ‘bent’ shape as the upper fiber are shortened and lower fiber are elongated
- The neutral axis remain zero bending stress – coincident with the centroidal axis of the x-x of the beam

STRESS DISTRIBUTION DIAGRAM

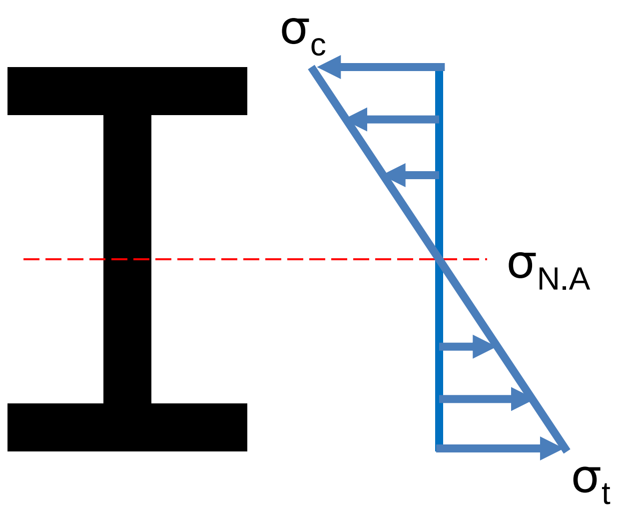
- To present the bending stress distribution diagram, we can express it by determination of bending stress at specific point located as below



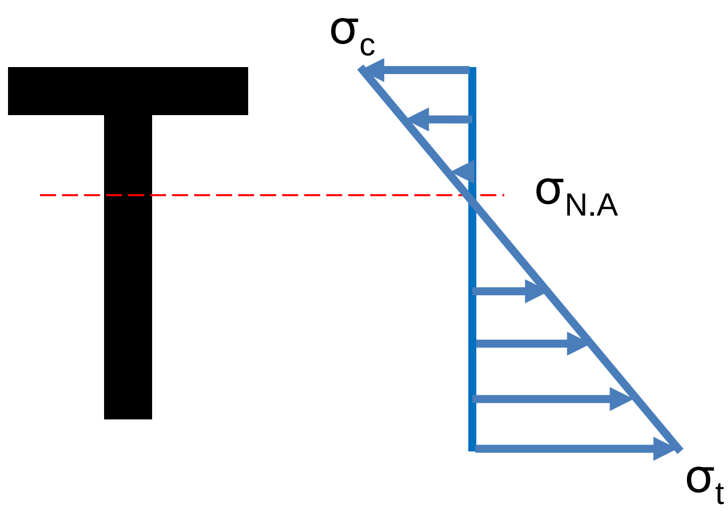


STRESS DISTRIBUTION DIAGRAM

- In general, stress distribution diagram would vary linear with distance from the neutral axis (*N.A*)



Stress distribution on **symmetrical** beam



Stress distribution on **unsymmetrical** beam

SUMMARY

- The bending stress can calculate using flexure formula
- The distance of 'y' is taken from the neutral axis to the point where to considered
- It is zero bending stress at the centroidal axis (*N.A*)of the cross-section of the beam
- It is a maximum tensile stress at the bottom surface (+σ)
- It is a maximum compressive stress at the top surface (-σ)
- The stress distribution diagram is linearly from the top to bottom

References

- Hibbeler, R.C., Mechanics Of Materials, 9th Edition in SI units, Prentice Hall, 2013.
- Ferdinand P. Beer, E. Russell Johnston, Jr., John T. DeWolf, David F. Mazurek, Mechanics of materials 5th Edition in SI Units, McGraw Hill, 2009.

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