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THEORY OF STRUCTURES

CHAPTER 3 : MOMENT DISTRIBUTION (FOR FRAME)

PART 4

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Chapter 3 : Part 4 – Moment Distribution

- Aims
  - Determine the end moment for frame using Moment Distribution Method
- Expected Outcomes :
  - Able to do moment distribution for frame.
- References
  - Mechanics of Materials, R.C. Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, SI Edition by Aslam Kassimali,Cengage Learning
  - Structural Analysis, Coates, Coatie and Kong
  - Structural Analysis - A Classical and Matrix Approach, Jack C. McCormac and James K. Nelson, Jr., 4th Edition, John Wiley

MDM for Frame without SIDE-SWAY

- **MDM- solving indeterminate structures is a process in which the moment in the members are determined by successive approximation.**
- **Does not result in moment diagram but it provides the magnitude and sense of the internal moments at joint – to obtain the shear and bending moment.**

- **TERM USED**
  - **Fixed end moment (FEM)**
  - **Carry over factor**
  - **Stiffness or resistance to rotation of a member**

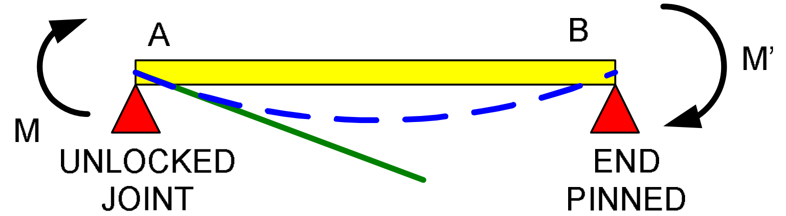
Clockwise moments are considered positive
Whereas, counterclockwise is negative

MDM for Frame without SIDE-SWAY

Summary

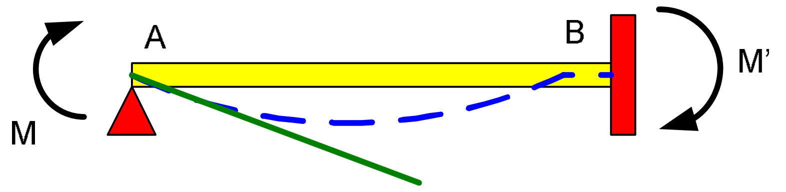
- **Stiffness for member: end is pinned equal to:**

$$K = \frac{3EI}{L}$$



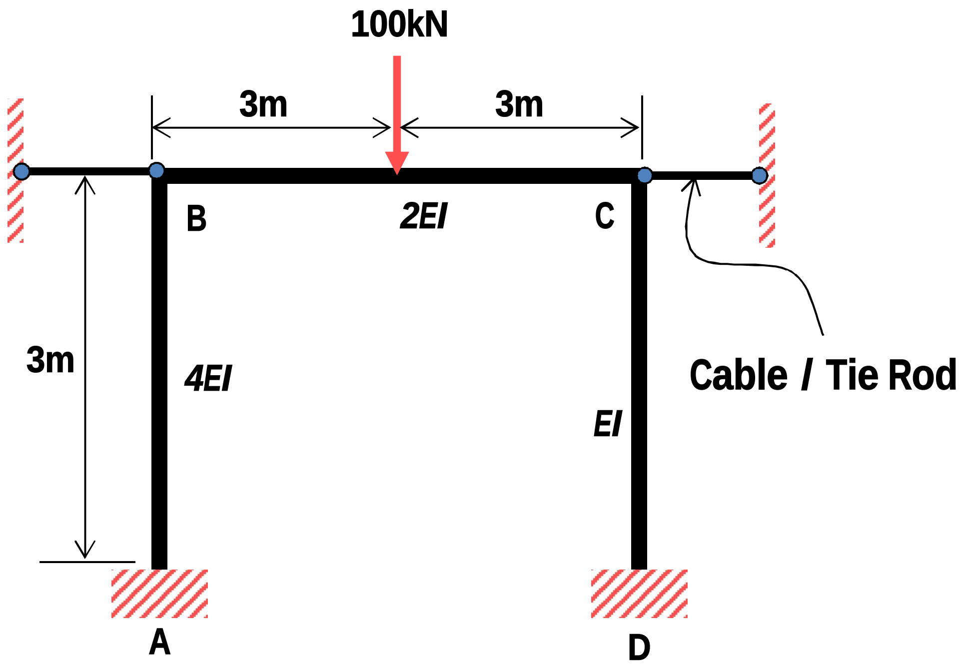
- **Stiffness for member: end is fixed equal to:**

$$K = \frac{4EI}{L}$$



EXAMPLE 1

Determine the final moment for frame ABCD shown below. Hence, Draw the SFD and BMD.EI is indicate in the figure.



<u>Frame WITHOUT Sidesway</u>

Fixed End Moment (M^F)

$$M_{AB}^{F} = M_{BA}^{F} = M_{CD}^{F} = M_{DC}^{F} = 0$$

$$M_{BC}^{F} = -\frac{Pab^2}{L^2} = -75kNm = M_{CB}^{F} = 75kNm$$

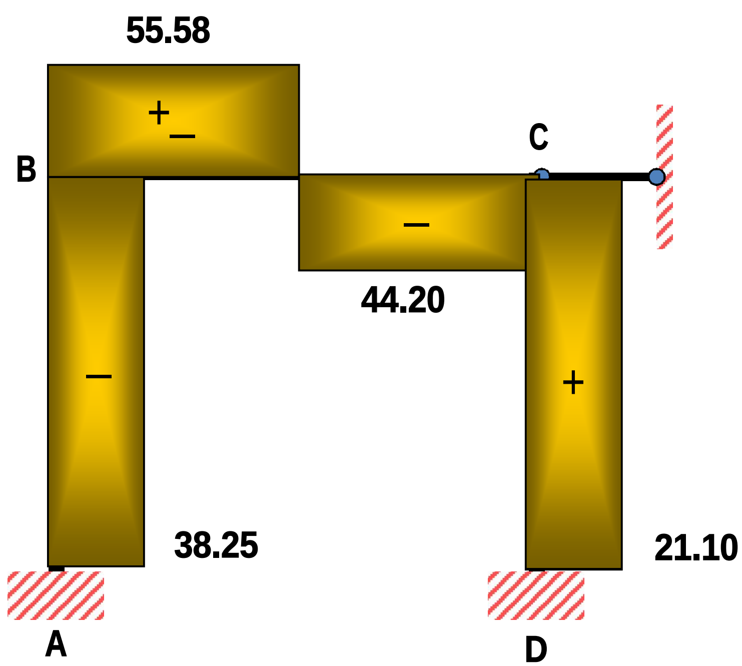
Distribution Factor (DF)

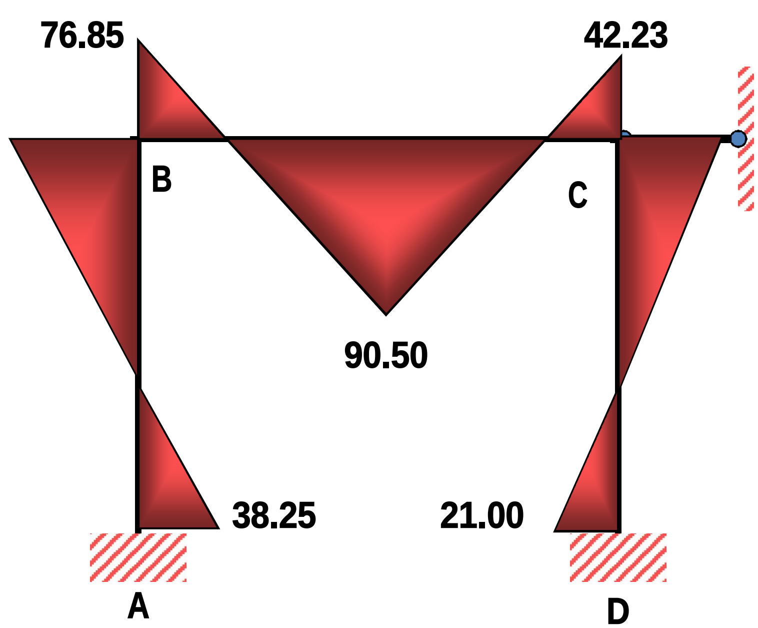
JOINT	MEMBER	K	ΣK	DF
A	AB	$\frac{4(4EI)}{3}$	$\frac{16EI}{3}+\infty$	0
B	BA	$\frac{4(4EI)}{3}$	$\frac{20EI}{3}$	0.8
	BC	$\frac{4(2EI)}{6}$		0.2
C	CB	$\frac{4(2EI)}{6}$	$\frac{8EI}{3}$	0.5
	CD	$\frac{4EI}{3}$		0.5
D	DC	$\frac{4EI}{3}$	$\frac{4EI}{3}+\infty$	0

Table Moment Distribution

Member	AB	BA	BC	CB	CD	DC
DF	0	0.8	0.2	0.5	0.5	0
FEM	0	0	-75	75	0	0
Bal	0	60	15	-37.5	-37.5	0
CO	30	0	-18.75	7.5	0	-18.75
Bal	0	15	3.75	-3.75	-3.75	0
CO	7.5	0	-1.88	1.88	0	-1.88
Bal	0	1.5	0.37	-0.94	-0.94	0
CO	0.75	0	-0.47	0.19	0	-0.47
Bal	0	0.38	0.1	-0.1	-0.1	0
End Moment	38.25	76.88	-76.88	42.28	-42.29	-21.10

Shear Force and Bending Moment Diagram



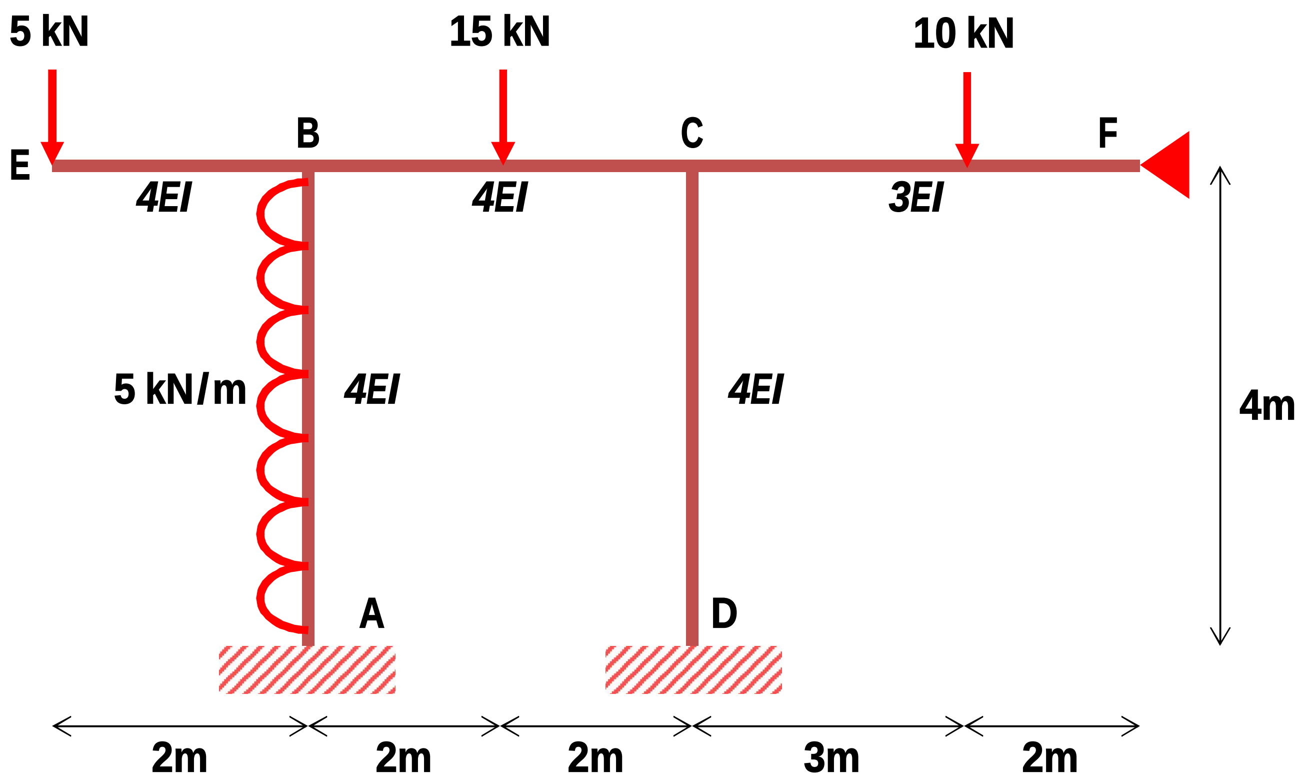


SFD

BMD

EXAMPLE 2

Determine the final moment for frame ABCDF with overhanging member BE shown below. Draw the BMD.*EI* is indicate in the figure.



Frame WITHOUT Sidesway

Fixed End Moment (M^F)

$$M_{AB}{}^{F} = -6.67 kNm$$

$$M_{BA}{}^{F} = 6.67 kNm$$

$$M_{BC}{}^{F} = -7.5 kNm$$

$$M_{CB}{}^{F} = 75 kNm$$

$$M_{CD}{}^{F} = M_{DC}{}^{F} = 0$$

$$M_{CF}{}^{F} = -4.8 kNm$$

$$M_{FC}{}^{F} = 7.2 kNm$$

$$M_{BE}{}^{F} = 5(2) = 10 kNm$$

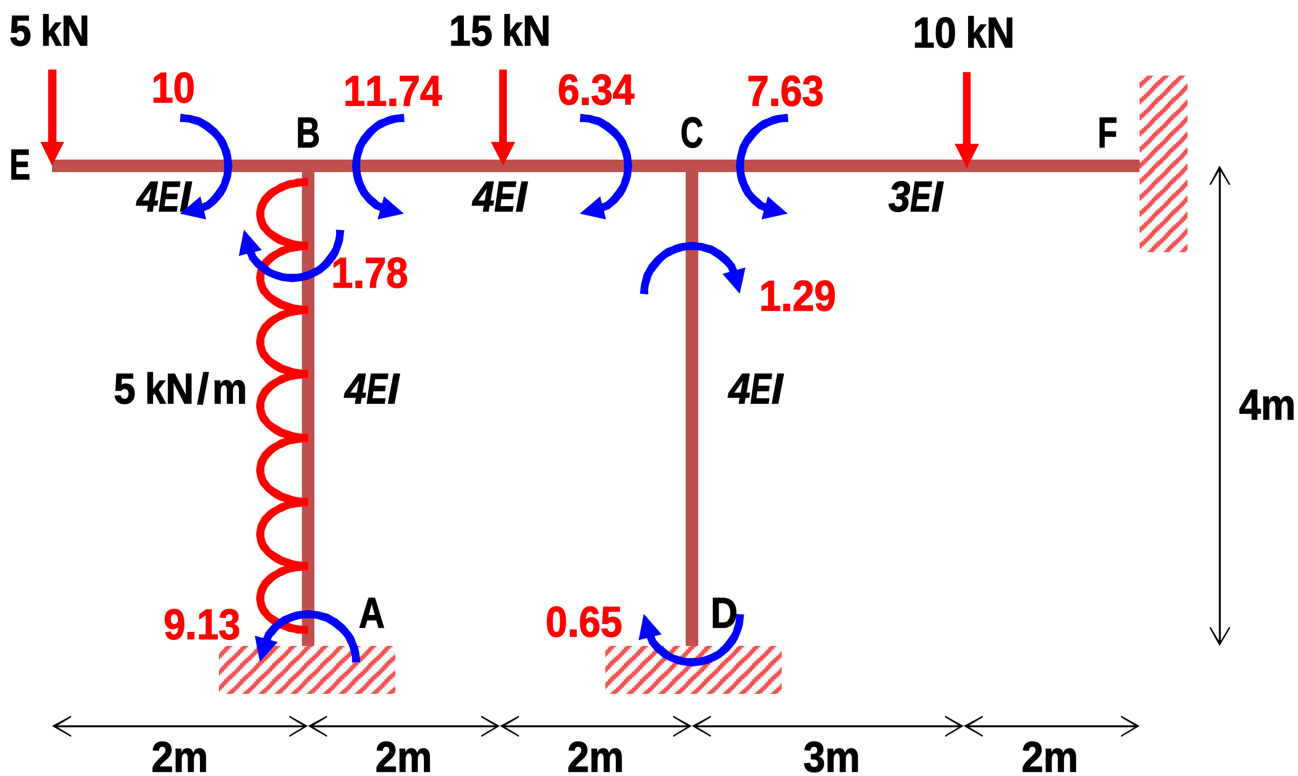
Distribution Factor (DF)

JOINT	MEMBER	K	ΣK	DF
B	BA	$\frac{4(4EI)}{4}$	$8EI$	0.5
	BC	$\frac{4(4EI)}{4}$		0.5
	BE	0		0
C	CB	$\frac{4(4EI)}{4}$	$\frac{49EI}{5}$	
	CD	$\frac{4(4EI)}{4}$		
	CF	$\frac{3(3EI)}{5}$		

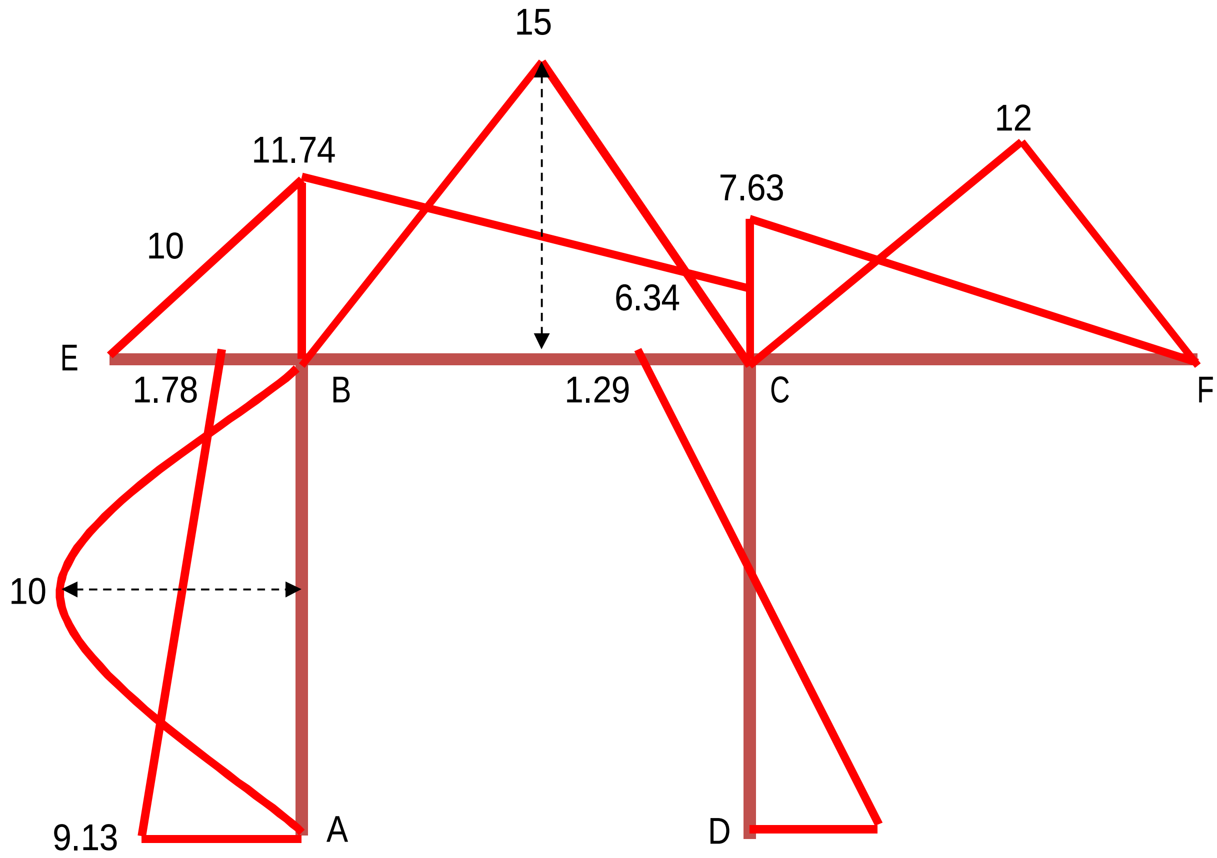
Table Moment Distribution

**Note:
Any no. divide by infinity α =0

JOINT	A	B			C			D	F	E
Member	AB	BA	BC	BE	CB	CD	CF	DC	FC	EB
DF	0	0.5	0.5	0				0	1.0	0
FEM	-6.67	6.67	-7.5	10.0	7.5	0	-4.8	0	7.2	0
Bal CO		-4.59	-4.49	0					-7.2	
Bal CO										
Bal CO										
Bal										
End Moment									0	0

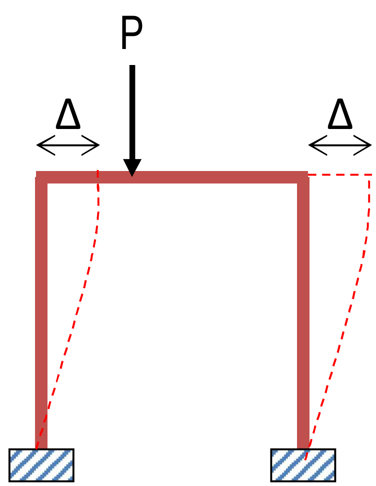

5 kN
15 kN
10 kN
10
11.74
6.34
7.63
E
B
C
F
4EI
4EI
3EI
1.78
1.29
5 kN/m
4EI
4EI
4m
9.13
A
0.65
D
2m
2m
2m
3m
2m

BMD

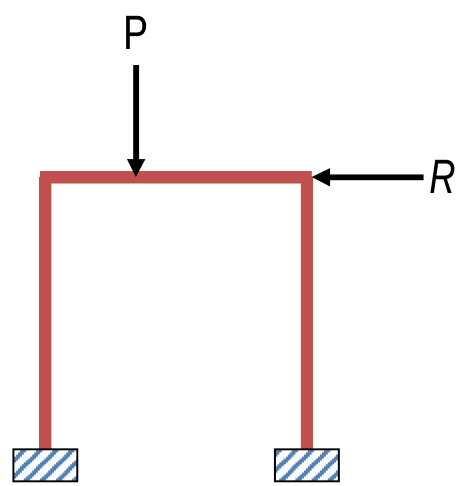


MDM for Frame with SIDE-SWAY

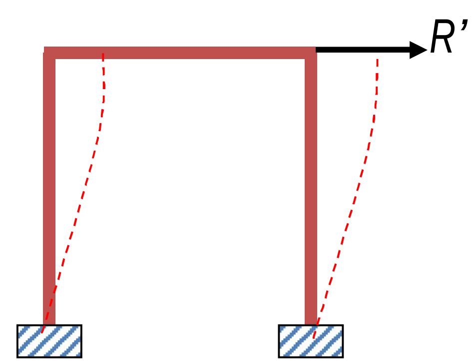
- The frames that are non-symmetrical or subjected to non-symmetrical loadings have a tendency to SIDE-SWAY
- Application of this technique is illustrated as below.



Horizontal displacement are EQUAL



Virtual Prop Force (No side-sway)

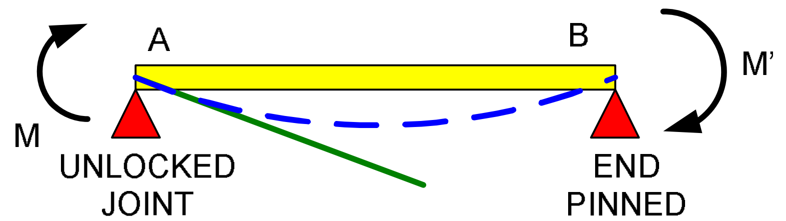


Virtual Prop Force is REMOVED (side-sway)

Summary

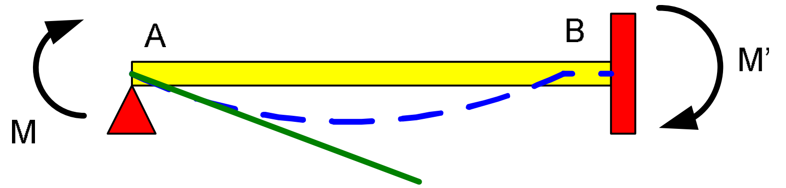
- **Stiffness for member: end is pinned equal to:**

$$K = \frac{3EI}{L}$$



- **Stiffness for member: end is fixed equal to:**

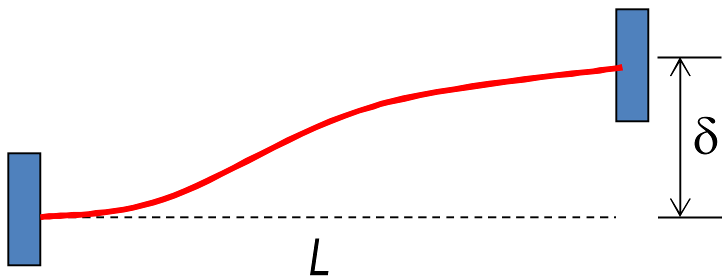
$$K = \frac{4EI}{L}$$



Summary

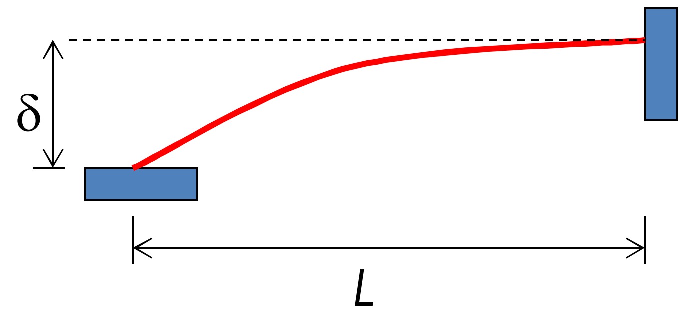
- Moment produced at each member when one end of member is displaced relative to other and both ends are FIXED

$$M = \frac{6EI\delta}{L^2}$$



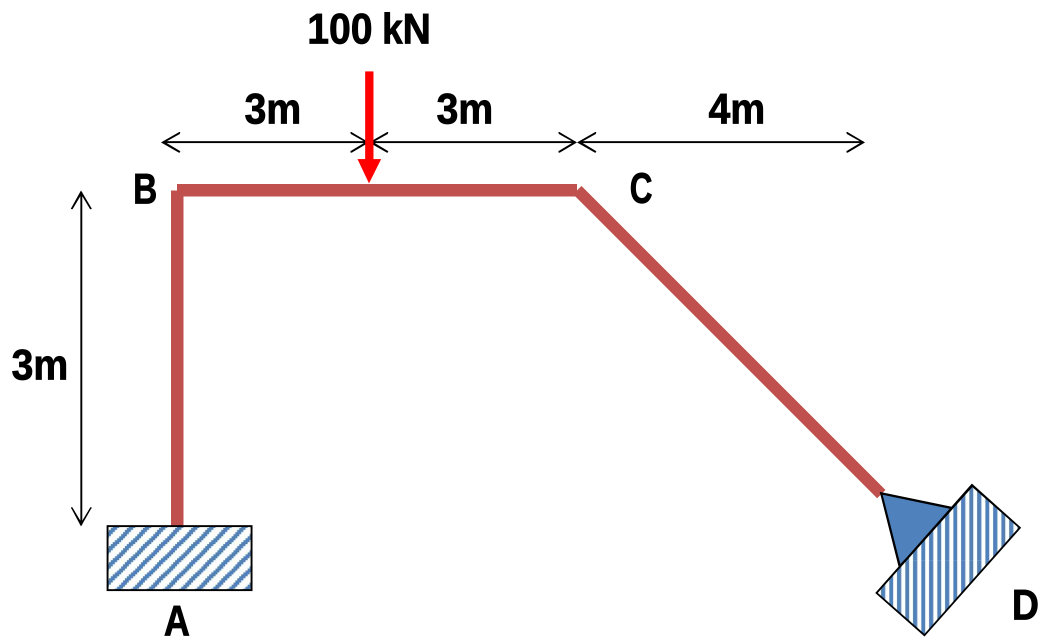
- Moment produced at the near end of a member if the remote end is displaced relative to the near end with remote end held in position but allowed to rotate

$$M = \frac{3EI\delta}{L^2}$$

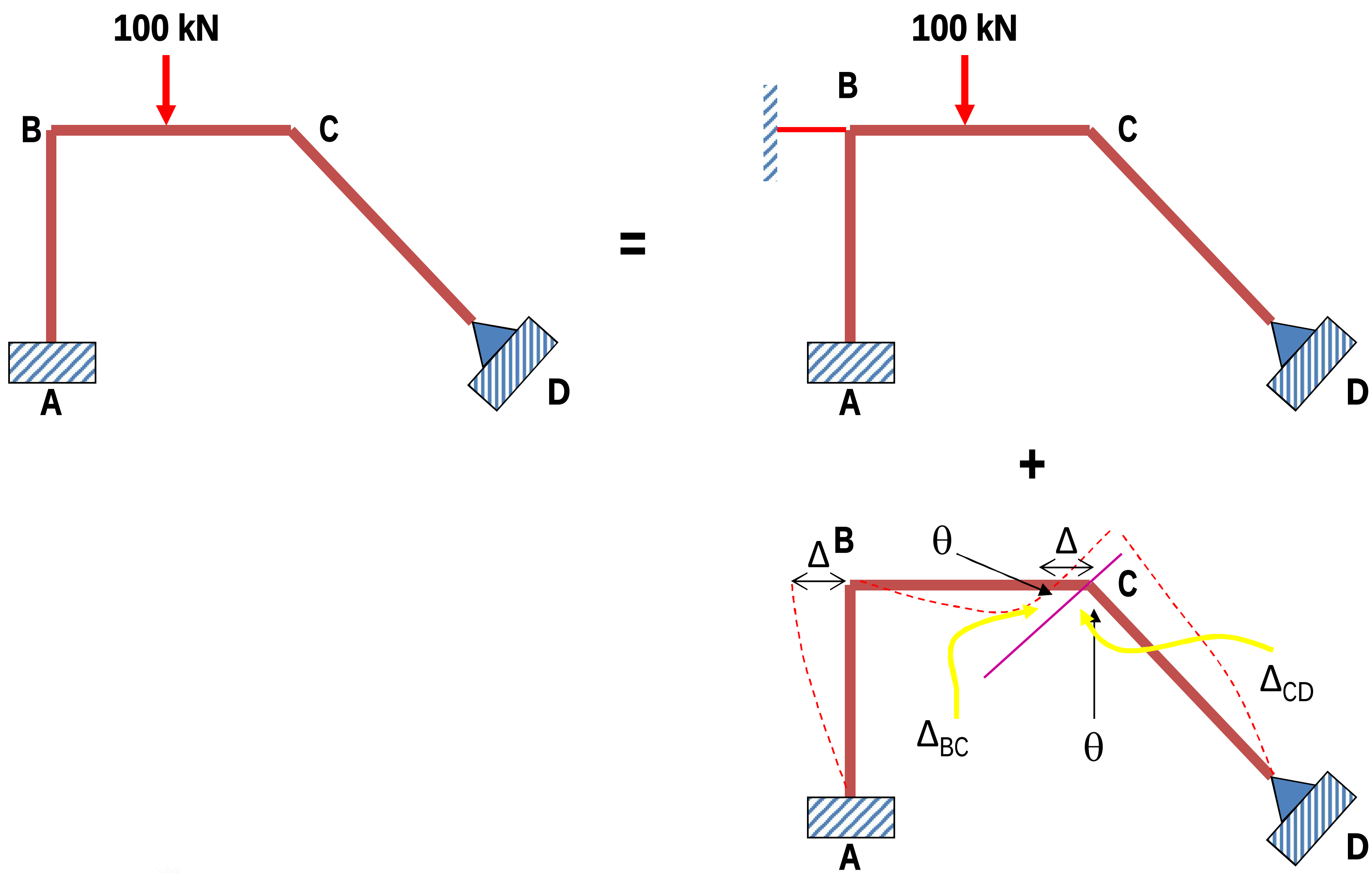


EXAMPLE 3

A portal frame ABCD as shown in Figure below is subjected to point load of 100 kN at member BC. EI is constant. Analyze using Moment Distribution Method.



SOLUTION EXAMPLE 3:



Case 1:
Fixed End Moment (M^F): Non-sway Analysis

$$M_{AB}{}^{F} = M_{BA}{}^{F} = M_{CD}{}^{F} = M_{DC}{}^{F} = 0$$

$$-M_{BC}{}^{F} = M_{CB}{}^{F} = -75kNm$$

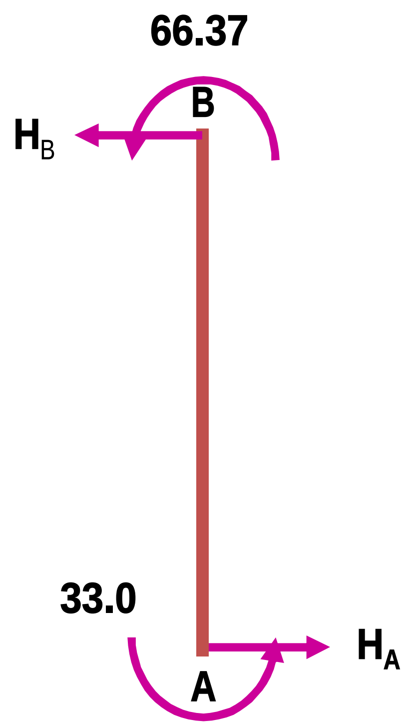
Distribution Factor (DF)

JOINT	MEMBER	K	ΣK	DF
A	AB	$\frac{4EI}{3}$	$\frac{4EI}{3}+\infty$	0
B	BA	$\frac{4EI}{3}$	$2EI$	0.67
	BC	$\frac{4EI}{6}$		0.33
C	CB	$\frac{4EI}{6}$	$\frac{19EI}{15}$	0.53
	CD	$\frac{3EI}{5}$		0.47
D	DC	$\frac{3EI}{5}$	$\frac{3EI}{5}$	1

Table Moment Distribution (Non-sway Analysis)

Member	AB	BA	BC	CB	CD	DC
DF	0	0.67	0.33	0.53	0.47	1
M^F	0	0	-75	75	0	0
Bal		50.3	24.7	-39.75	-35.25	
CO	25.2		-19.9	12.4		-17.63
Bal	0	13.3	6.6	-6.6	-5.8	17.63
CO	6.7		-3.3	3.3		
Bal	0	2.2	1.1	-1.7	-1.6	
CO	1.1		-0.85	0.55		
Bal	0	0.57	0.28	-0.29	-0.26	
End Moments	33.0	66.37	-66.37	42.91	-42.91	0

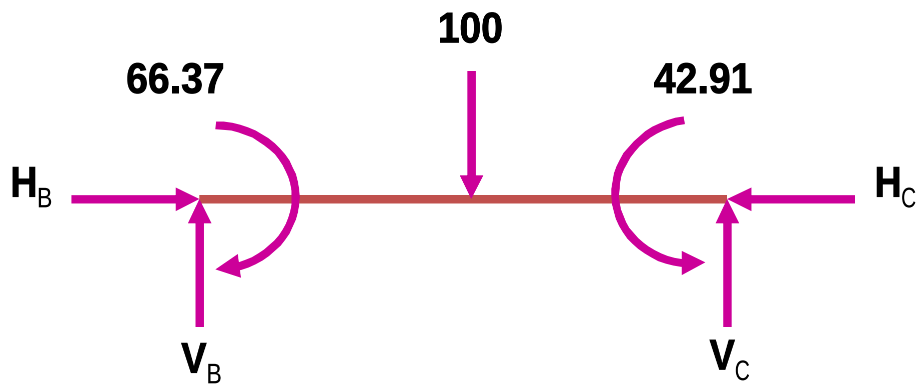
Horizontal Reactions



$$\sum M_A = 0,$$

$$-H_B(3) + 33.0 + 66.37 = 0$$

$$\therefore H_B = 33.12 kN$$



$$\sum M_B = 0,$$

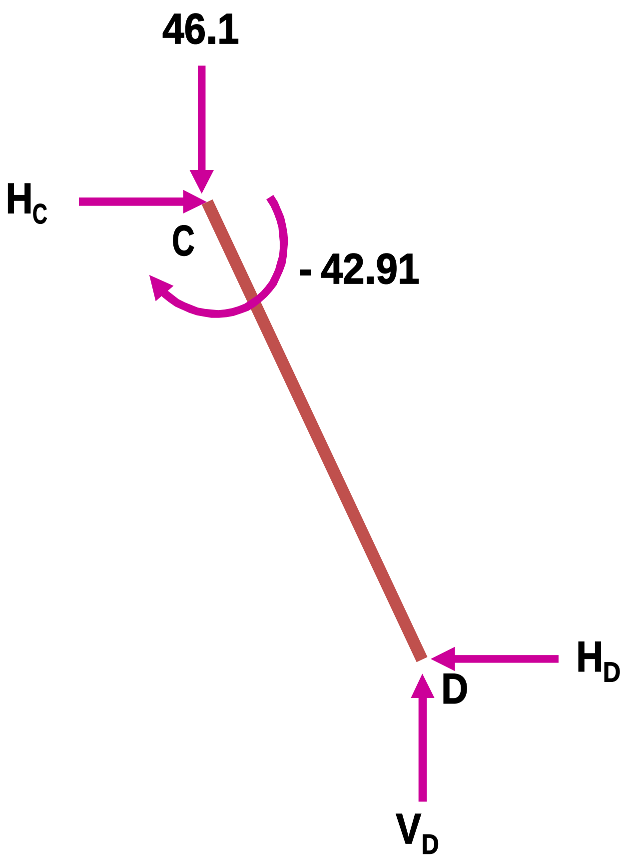
$$V_C(6) + 66.37 - (100 \times 3) - 42.91 = 0$$

$$\therefore V_C = 46.1kN$$

$$\sum F_y = 0,$$

$$V_B + V_C - 100 = 0$$

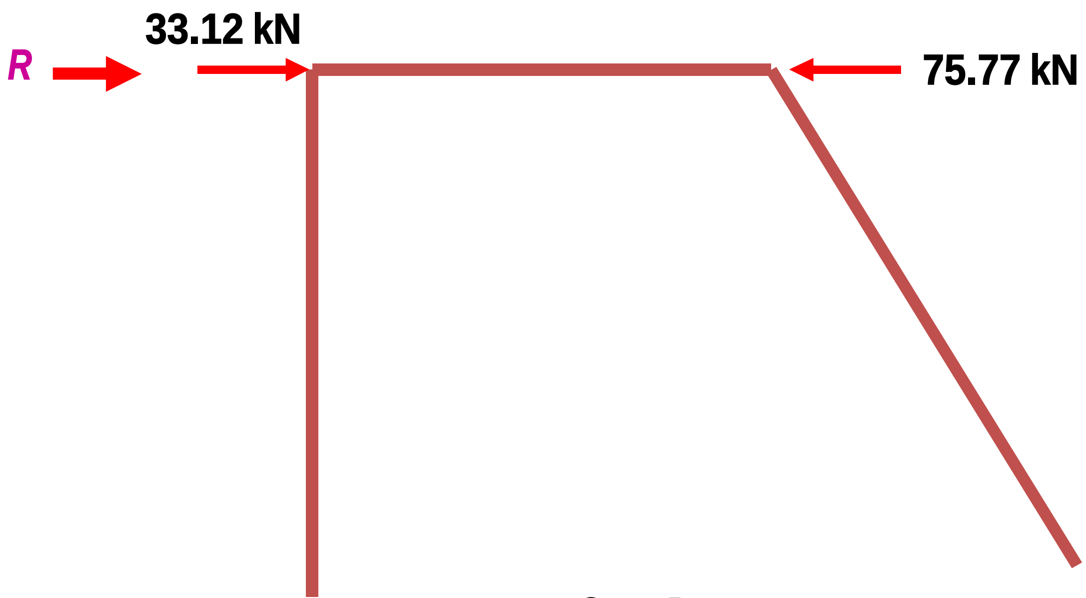
$$\therefore V_B = 53.9kN$$



$$\sum M_D = 0,$$

$$H_C(3) - (46.1 \times 4) - 42.91 = 0$$

$$\therefore H_C = 75.77kN$$



To find R

$$\sum^{\rightarrow +} F_H = 0$$

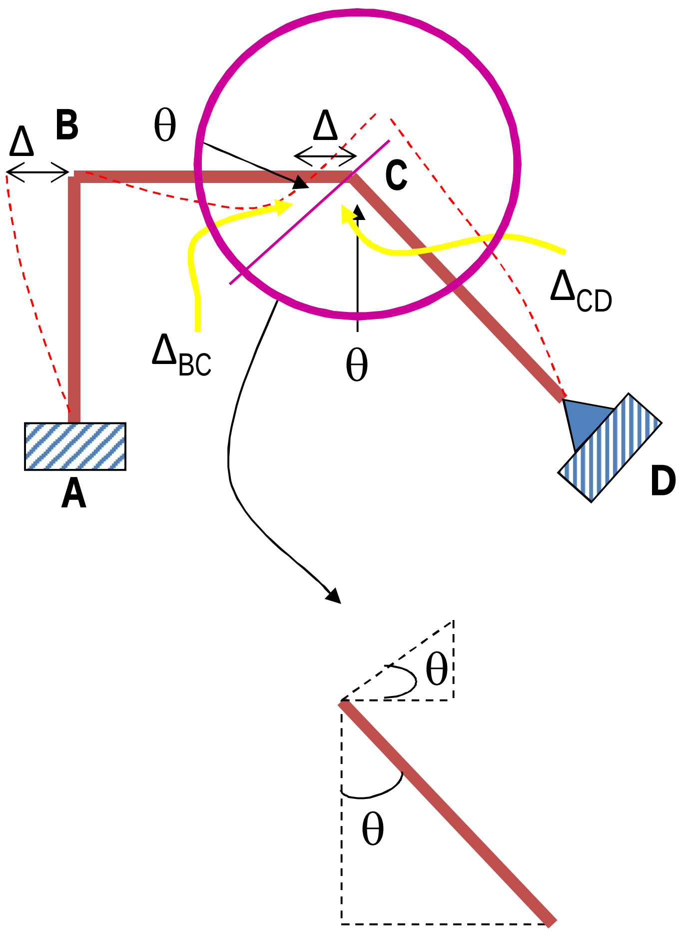
$$R + H_B - H_C = 0$$

$$R + 33.12 - 75.77 = 0$$

$$\therefore R = 42.65 kN$$

Case 2:

Fixed End Moment (M^F): Sway Analysis



$$\tan\theta = \frac{4}{3} = \frac{\Delta_{BC}}{\Delta} \Rightarrow \Delta_{BC} = \frac{4\Delta}{3}$$

$$\sin\theta = \frac{4}{5} = \frac{\Delta_{BC}}{\Delta_{CD}} \Rightarrow \Delta_{CD} = \frac{5\Delta}{3}$$

therefore,

$$M^S{}_{AB} = M^S{}_{BA} = \frac{6EI\Delta}{L^2} = \frac{6EI\Delta}{3^2} = \frac{6EI\Delta}{9}$$

$$M^S{}_{BC} = M^S{}_{CB} = -\frac{6EI\Delta}{L^2} = -\frac{6EI(\frac{4\Delta}{3})}{6^2} = -\frac{2EI\Delta}{9}$$

$$M^S{}_{CD} = \frac{3EI\Delta}{L^2} = \frac{3EI(\frac{5\Delta}{3})}{5^2} = \frac{EI\Delta}{5}$$

$$assume \quad EI\Delta = 45, therefore \quad :$$

$$M^{S}_{AB,BA} : M^{S}_{BC,CB} : M^{S}_{CD,DC}$$

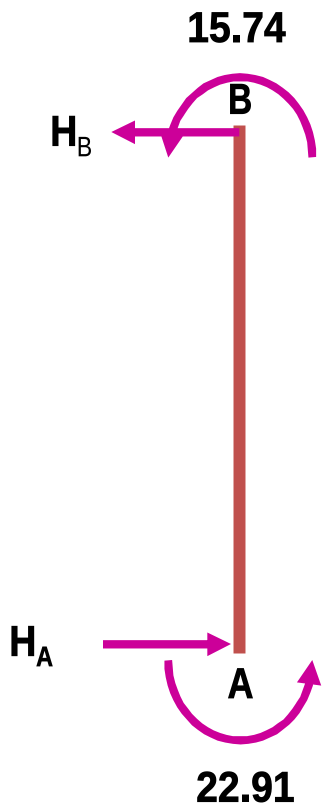
$$\frac{6EI\Delta}{9} : -\frac{2EI\Delta}{9} : \frac{EI\Delta}{5}$$

$$30 \quad : \quad -10 \quad : \quad 9$$

Table Moment Distribution (Sway Analysis)

Member	AB	BA	BC	CB	CD	DC
DF	0	0.67	0.33	0.53	0.47	1
M^F	30	30	-10	-10	9	0
Bal CO	 -6.7	-13.4	-6.6 0.27	0.53 -3.3	0.47	 0.24
Bal CO	0 -0.09	-0.18	-0.09 0.88	1.75 -0.5	1.55	
Bal CO	0 -0.3	-0.59	-0.29 0.14	0.27 -0.15	0.23	
Bal	0	-0.09	-0.05	0.08	0.07	
Assume Sway Moment	22.91	15.74	-15.74	-11.32	11.32	0

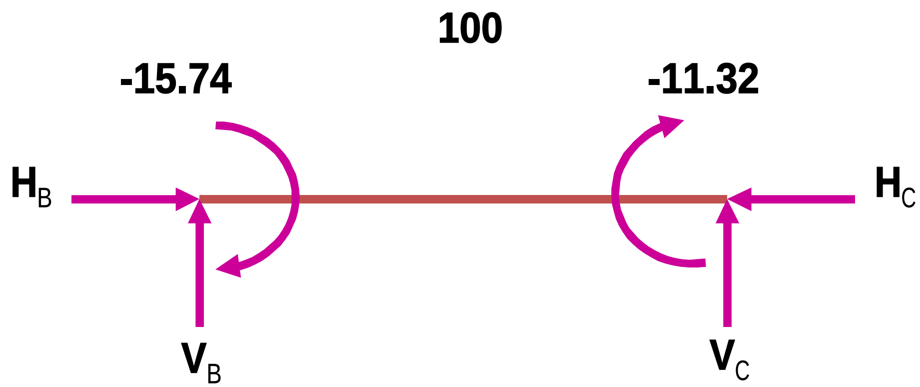
Horizontal Reactions



$$\sum M_A = 0,$$

$$-H_B(3) + 22.91 + 15.74 = 0$$

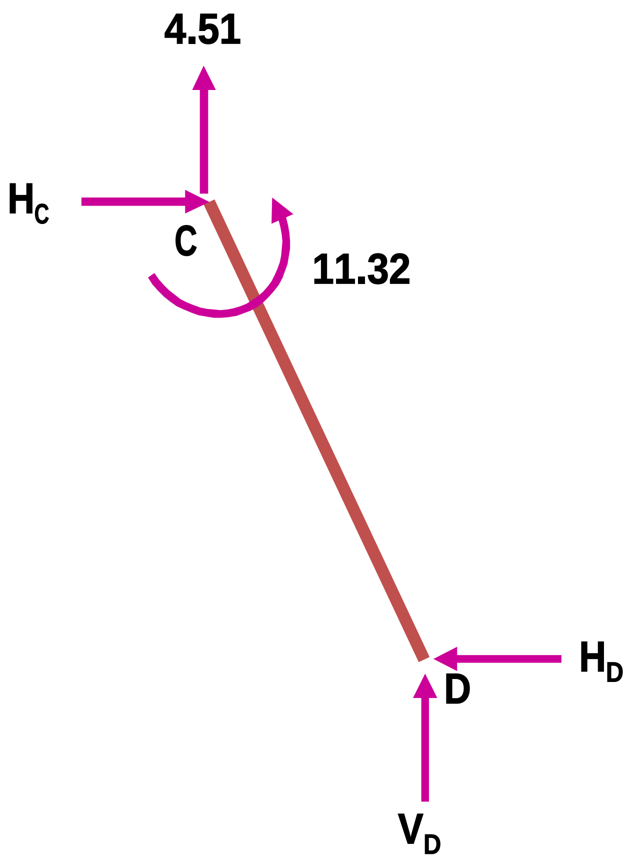
$$\therefore H_B = 12.88 kN$$



$$\sum M_B = 0,$$

$$-V_C(6) - 15.74 - 11.32 = 0$$

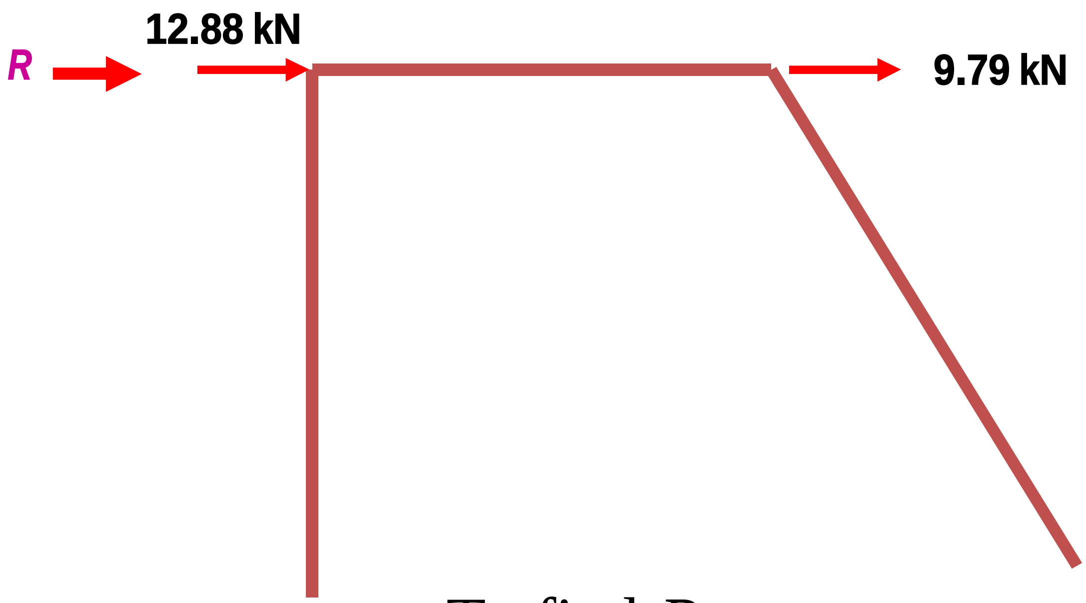
$$\therefore V_C = -4.51kN$$



$$\sum M_D = 0,$$

$$H_C(3) + 11.32 + (4.51 \times 4) = 0$$

$$\therefore H_C = -9.79kN$$



To find R

$$\sum^{\rightarrow +} F_H = 0$$

$$R + H_B + H_C = 0$$

$$R + 12.88 + 9.79 = 0$$

$$\therefore R = 22.67 kN$$

Correction Factor and Final Moment

$$\therefore ASM = \frac{R}{R'} = \frac{42.65}{22.67} = 1.88$$

A		B		C		D
Assume sway moment	22.91	15.74	-15.74	-11.32	11.32	0
Actual sway moment (ASM)	43.07	29.59	-29.59	-21.28	21.28	
(Non-sway moment)	33.0	66.37	-66.37	42.91	-42.91	0
Final Moments	76.07	95.96	-95.96	21.63	-21.63	0

****ASM x Assume sway moment**

THANKS

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