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# THEORY OF STRUCTURES
# CHAPTER 3 : MOMENT DISTRIBUTION (FOR BEAM)
# PART 3

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# Chapter 3 : Part 3 – Slope Deflection

- Aims
  - Determine the end moment for beam using Moment Distribution Method

- Expected Outcomes :
  - Able to do moment distribution for beams.

- References
  - Mechanics of Materials, R.C. Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, SI Edition by Aslam Kassimali,Cengage Learning
  - Structural Analysis, Coates, Coatie and Kong
  - Structural Analysis - A Classical and Matrix Approach, Jack C. McCormac and James K. Nelson, Jr., 4th Edition, John Wiley

# MOMENT DISTRIBUTION METHOD

## SIGN CONVENTION

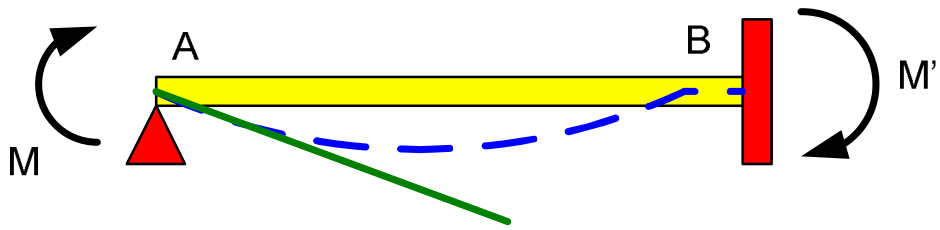
-Clockwise moment consider positive, whereas counterclockwise moment are negative.

## FIXED END MOMENTS (FEMs)

-Can be determined from the table.

$\theta_A$

# MEMBER STIFFNESS FACTOR




$$K = \frac{4EI}{L}$$

FAR END FIXED

*Stiffness factor at A can be defined as the amount of moment M required to rotate the end A of the beam = 1 rad*

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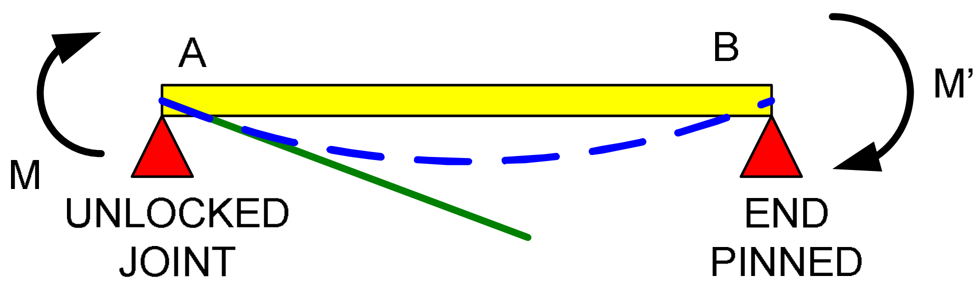
**MODIFICATIONS…..**

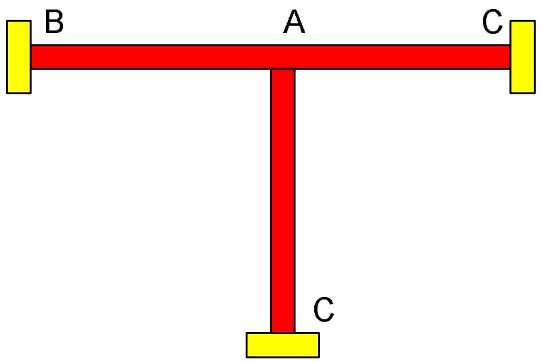



$$K = \frac{3EI}{L}$$

FAR END PINNED

# JOINT STIFFNESS FACTOR

-The total stiffness factor is a sum of the member stiffness factor at the joint.

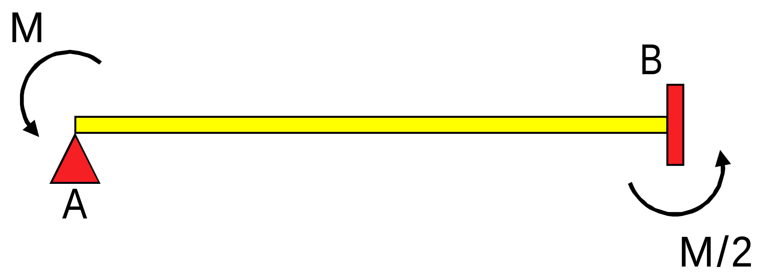



$$K_T = \sum K$$

$$= K_{AB} + K_{AD} + K_{AC}$$

## CARRY OVER

In MDM, we have to analyze the effects of applying imaginary moments at a specified point



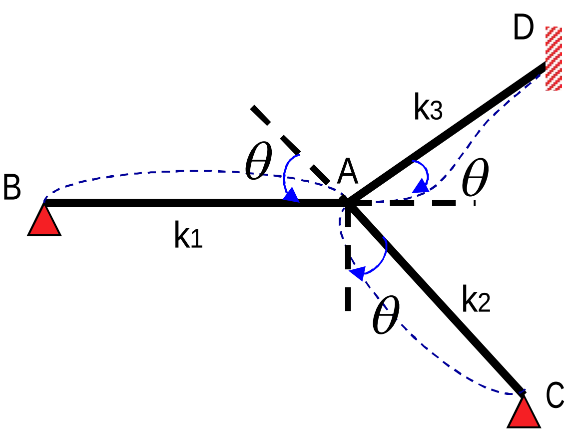

The beam in Figure, when it receives a moment M at A, will develop at B moment of M/2.

This M/2 is called the *carry over moment*

If the far end B were hinged, the CO will be zero

# DISTRIBUTION FACTOR (DF)

A moment which tends to rotate without translation a joint to which several members are connected will be divided amongst the connected members in proportion to their stiffnesses.




Assumption:
Connected at A
Rotation $\theta$ is given to joint A by external moment

i. The rotation of each member at A is obviously $\theta$

ii. The moments $M_{AB}$, $M_{AC}$, $M_{AD}$ (assuming up to M0 will be in ratio $k_1$;$k_2$:$k_3$

## DISTRIBUTION FACTOR (DF)

$$M_i = K_i \theta$$

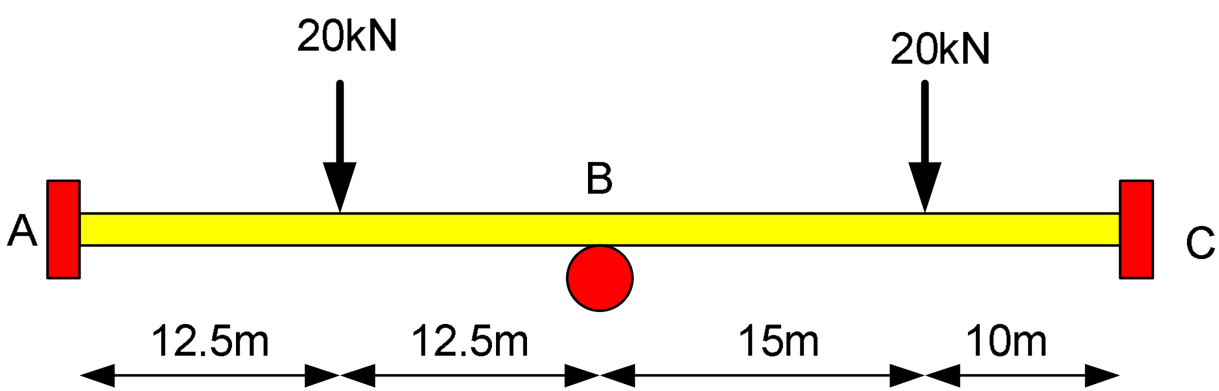
$$M = \theta \sum K_i$$

$$DF_i = \frac{M_i}{M} = \frac{K_i \theta}{\theta \sum K_i}$$

$$DF = \frac{K}{\sum K}$$

# EXAMPLE 1

**A continuous beam ABC is shown in figure below. Analyse the beam to for its end moment and draw the shear force and bending moment diagram. Assume EI is constant**



**_Solution…_**

i-Distribution Factor (DF)

| JOINT | MEMBER | K | $\sum K$ | DF |
|---|---|---|---|---|
| $A$ | $AB$ | $4EI/25$ | $4EI/25 + \infty$ | $0$ |
| $B$ | $BA$ | $4EI/25$ | $8EI/25$ | $0.5$ |
| | $BC$ | $4EI/25$ | | $0.5$ |
| $C$ | $CD$ | $4EI/25$ | $4EI/25 + \infty$ | $0$ |

## ii-Fixed End Moment

$$M^F_{AB} = \frac{-20(12.5)(12.5)^2}{25^2} = -62.5kNm$$

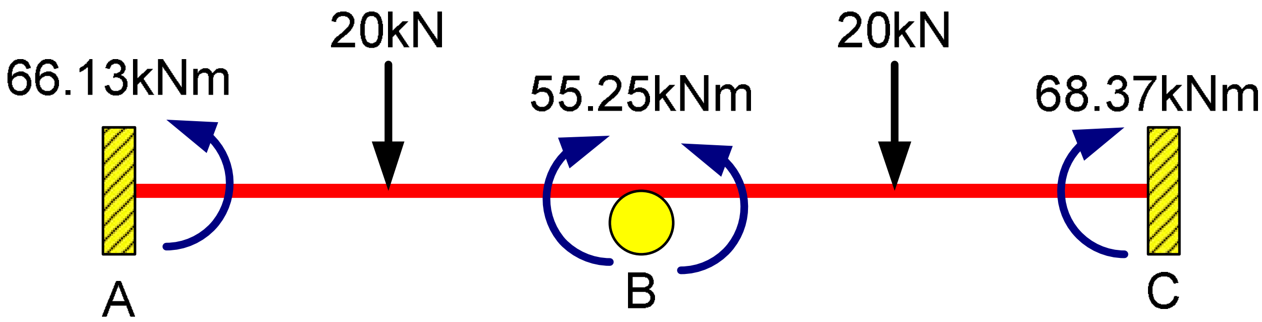
$$M^F_{BA} = +62.5kNm$$

$$M^F_{BC} = \frac{-(20)(15)(10)^2}{25^2} = -48kNm$$

$$M^F_{BC} = \frac{-(20)(15)^2(10)}{25^2} = +72kNm$$

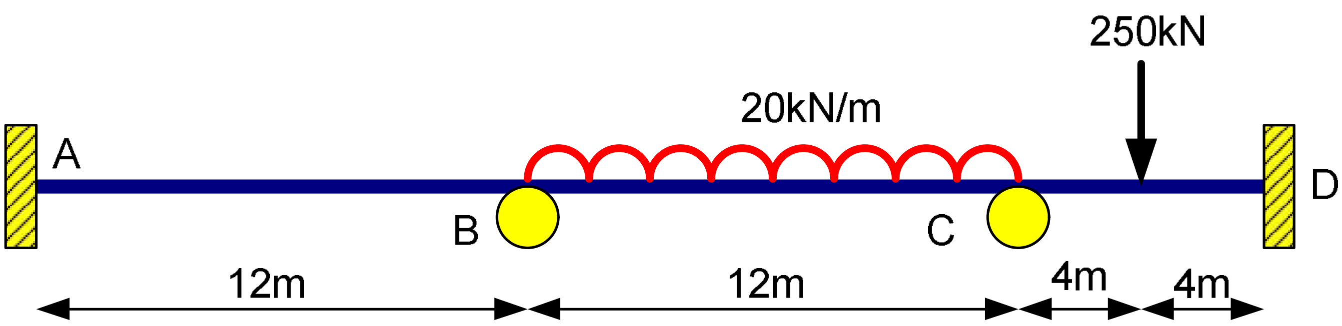
iii-Distribution Table

| JOINT | A | | B | | C |
|---|---|---|---|---|---|
| MEMBER | AB | | BA | BC | CB |
| D.F | 0 | | 0.5 | 0.5 | 0 |
| FEM | -62.5 | | 62.5 | -48 | 72 |
| BAL | 0 | | -7.25 | -7.25 | 0 |
| C.O | -3.63 | | 0 | 0 | -3.63 |
| BAL | 0 | | 0 | 0 | 0 |
| END MOMENT | -66.13 | | +55.25 | -55.25 | 68.37 |


66.13kNm
20kN
55.25kNm
20kN
68.37kNm
A
B
C

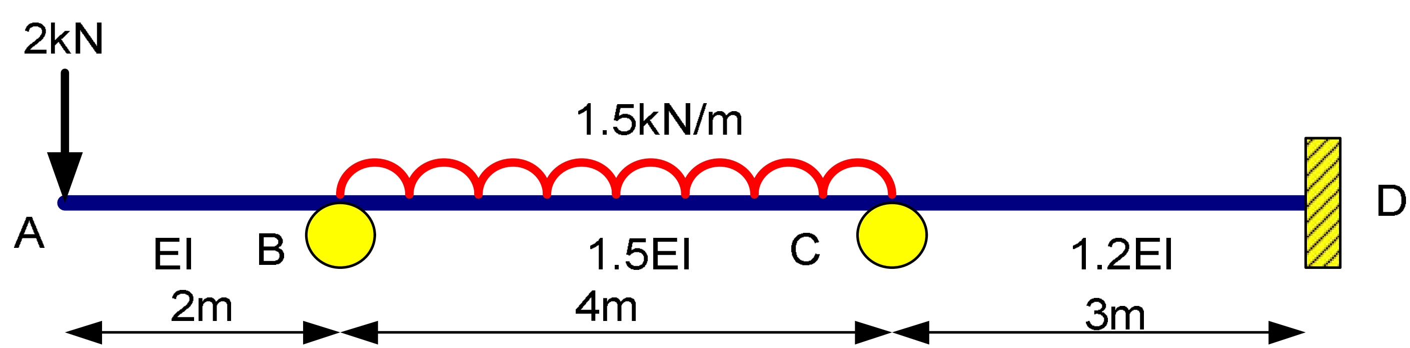
## *Food of mind*

Determine the internal moments at each support of the beam shown in figure below. EI is constant.



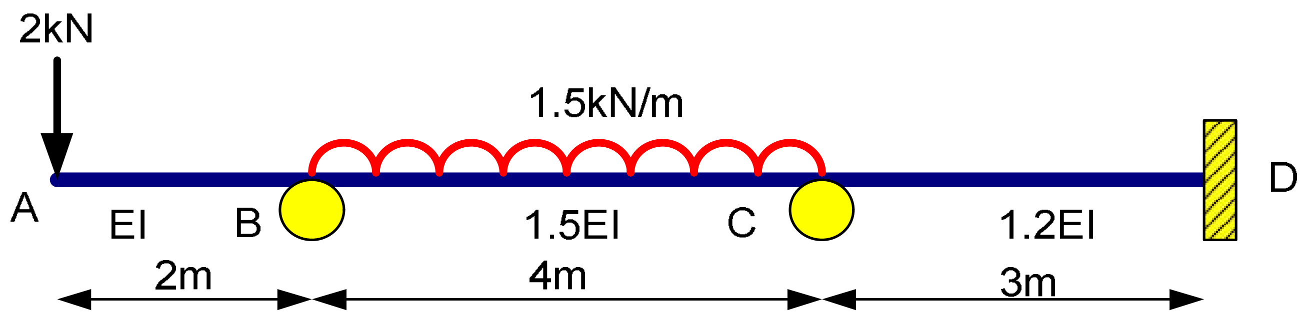
# EXAMPLE 2

Determine the internal moment at each support of the beam shown in figure below. The moment of inertial of each span is indicated.



*Solution..*

*The moment does not get distributed in the overhanging span AB, so the distributed factor* $(DF)_{AB} = 0$ *and* $(DF)_{BA} = 1.0$



**Solution…**

i-Distribution Factor (DF)

| JOINT | MEMBER | K | $\sum K$ | DF |
|---|---|---|---|---|
| A | AB | - | - | – |
| B | BA | - | $4(1.5EI)/4$ | 0 |
| | BC | $4(1.5EI)/4$ | | 1.0 |
| C | CB | $4(1.5EI)/4$ | $3.1EI$ | 0.48 |
| | CD | $4(1.2EI)/3$ | | 0.52 |
| D | DC | $4(1.2EI)/3$ | $4(1.2EI)/3 + \infty$ | 0 |

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# THANKS

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