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THEORY OF STRUCTURES
CHAPTER 3 : SLOPE DEFLECTION (FOR FRAME)
PART 2

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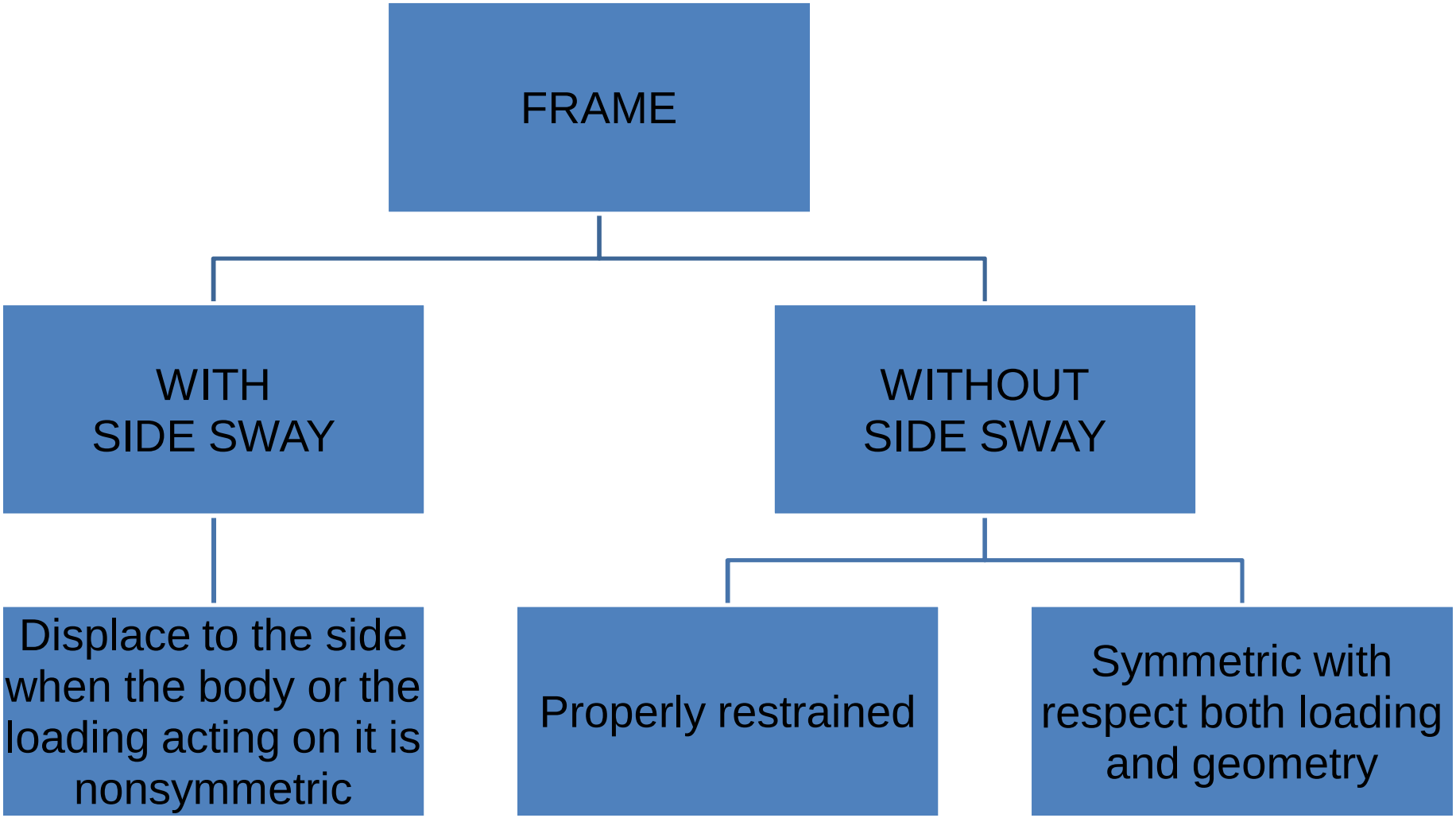
Chapter 3 : Part 2 – Slope Deflection

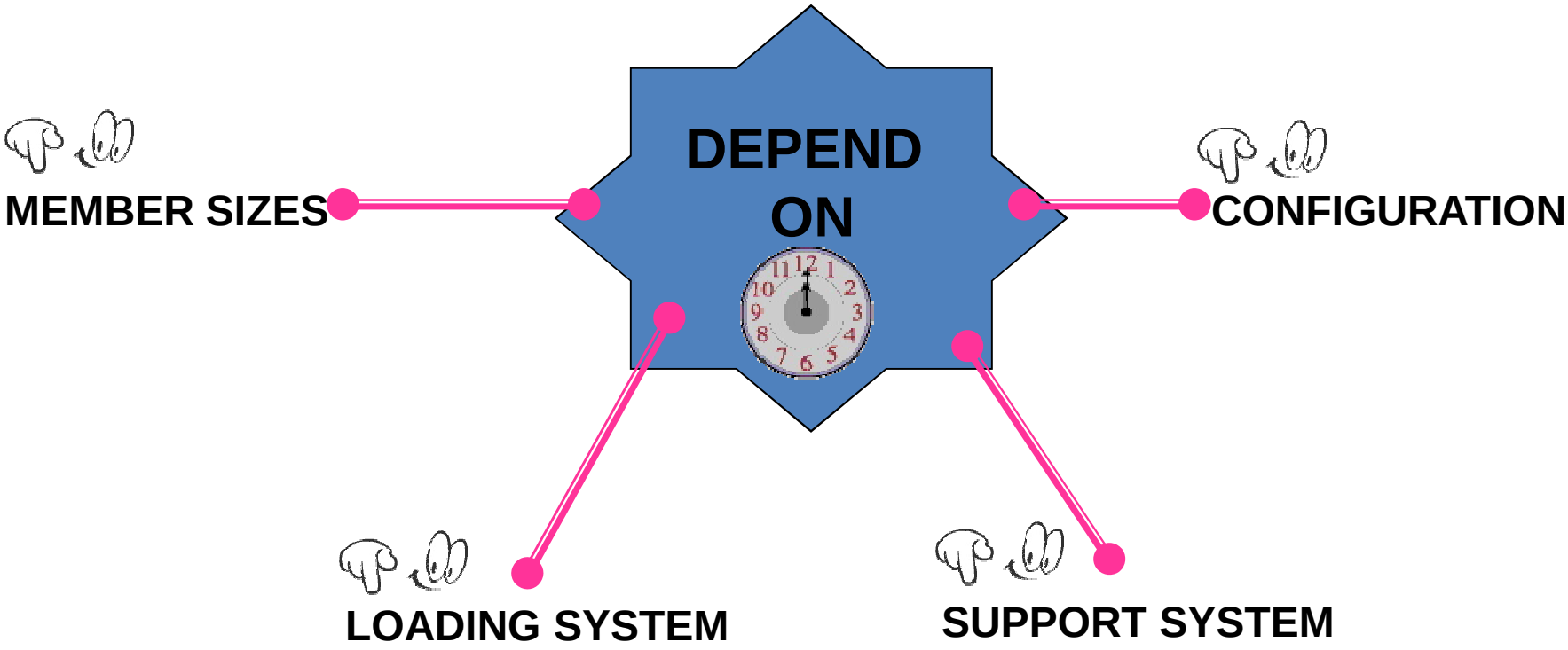
- Aims
  - Determine the end moment for frame using Slope Deflection Method.

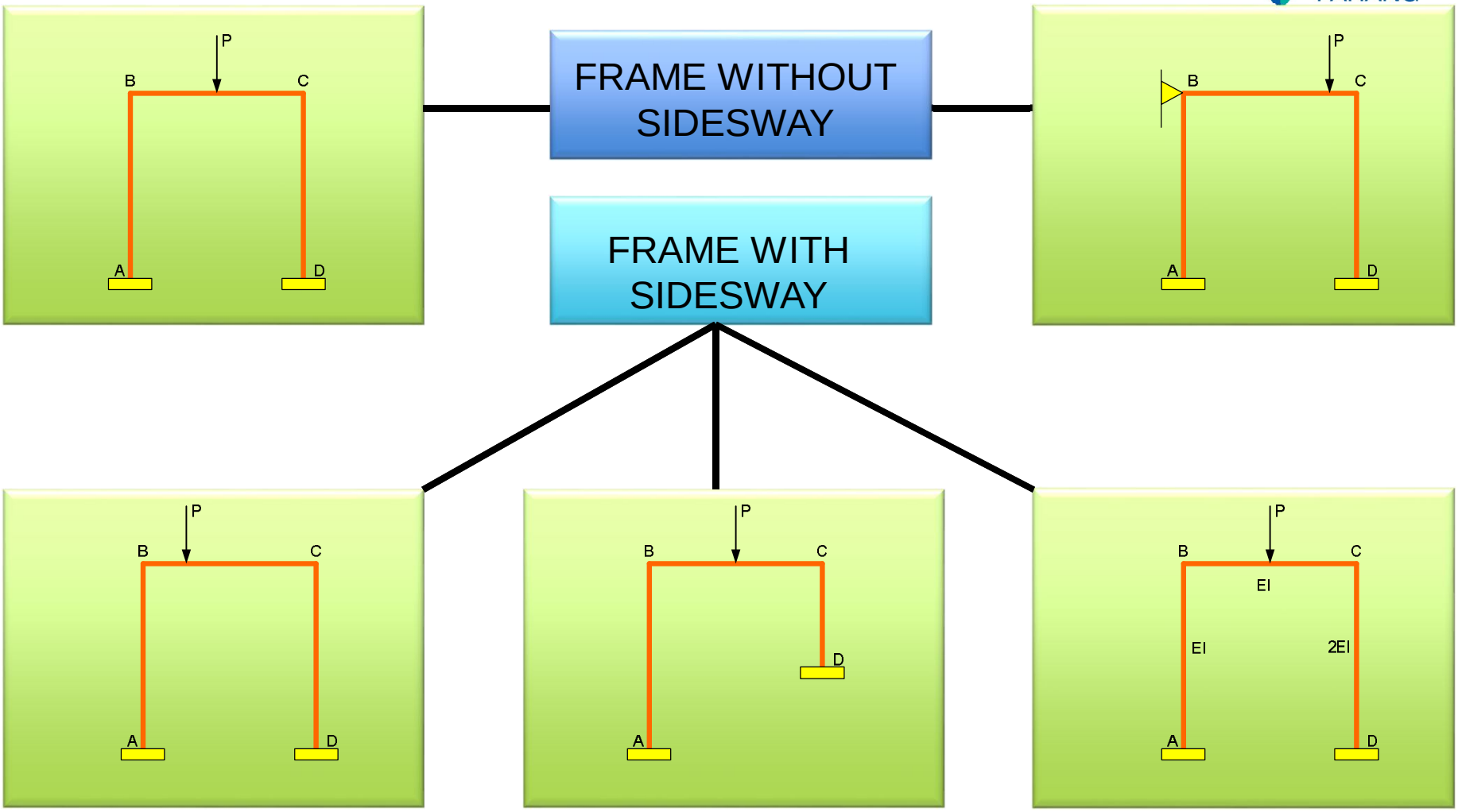
- Expected Outcomes :
  - Able to identify the frame – with or without side sway.
  - Able to determine end moment at critical points.

- References

  - Mechanics of Materials, R.C. Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, SI Edition by Aslam Kassimali,Cengage Learning
  - Structural Analysis, Coates, Coatie and Kong
  - Structural Analysis - A Classical and Matrix Approach, Jack C. McCormac and James K. Nelson, Jr., 4th Edition, John Wiley



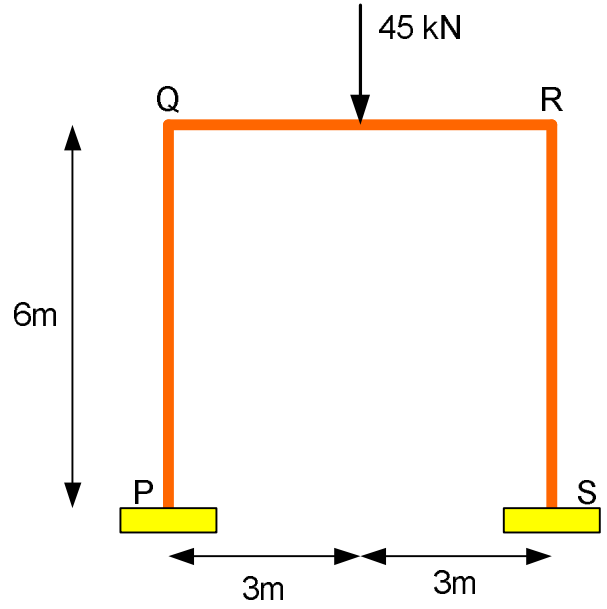




EXAMPLE 1

ANALYSIS - FRAME WITHOUT SIDESWAY.

By using SDM, determine the bending moment at critical points. Assume EI is constant.



SOLUTION

FIXED END MOMENT

$$M_{PQ}^{F} = M_{QP}^{F} = M_{RS}^{F} = M_{SR}^{F} = 0$$

$$M_{QR}^{F} = -\frac{PL}{8} = -\frac{45(6)}{8} = -33.75\,\text{kNm}$$

$$M_{RQ}^{F} = +\frac{PL}{8} = +\frac{45(6)}{8} = +33.75\,\text{kNm}$$

SLOPE DEFLECTION EQUATION $\theta_P = \theta_S = \Delta = 0$

$$M_{PQ} = \frac{4EI\theta_P}{L} + \frac{2EI\theta_Q}{L} - \frac{6EI\Delta}{L^2} + M^F_{PQ}$$

$$M_{PQ} = \frac{EI\theta_Q}{3}$$

$$M_{QP} = \frac{4EI\theta_Q}{L} + \frac{2EI\theta_P}{L} - \frac{6EI\Delta}{L^2} + M^F_{QP}$$

$$M_{QP} = \frac{2EI\theta_Q}{3}$$

$$M_{QR} = \frac{4EI\theta_Q}{L} + \frac{2EI\theta_R}{L} - \frac{6EI\Delta}{L^2} + M_{QR}^F$$

$$M_{QR} = \frac{2EI\theta_Q}{3} + \frac{EI\theta_R}{3} - 33.75$$

$$M_{RQ} = \frac{4EI\theta_R}{L} + \frac{2EI\theta_Q}{L} - \frac{6EI\Delta}{L^2} + M_{RQ}^F$$

$$M_{RQ} = \frac{2EI\theta_R}{3} + \frac{EI\theta_Q}{3} + 33.75$$

$$M_{RS} = \frac{4EI\theta_R}{L} + \frac{2EI\theta_S}{L} - \frac{6EI\Delta}{L^2} + M_{RS}^F$$

$$M_{RS} = \frac{2EI\theta_R}{3}$$

$$M_{SR} = \frac{4EI\theta_S}{L} + \frac{2EI\theta_R}{L} - \frac{6EI\Delta}{L^2} + M_{SR}^F$$

$$M_{SR} = \frac{EI\theta_R}{3}$$

EQUILIBRIUM AT JOINT

$$\sum M_Q = 0$$

$$M_{QP} + M_{QR} = 0$$

$$\frac{4EI\theta_Q}{3} + \frac{EI\theta_R}{3} = 33.75 \quad \text{(1)}$$

$$\sum M_R = 0$$

$$M_{RQ} + M_{RS} = 0$$

$$\frac{4EI\theta_R}{3} + \frac{EI\theta_Q}{3} = -33.75 \quad \text{(2)}$$

$$\begin{bmatrix} 4/3 & 1/3 \\ 1/3 & 4/3 \end{bmatrix} \begin{bmatrix} EI\theta_Q \\ EI\theta_R \end{bmatrix} = \begin{bmatrix} 33.75 \\ -33.75 \end{bmatrix}$$

BY USING CALCULATOR

$$EI\theta_Q = 33.75$$

$$EI\theta_R = -33.75$$

SUBSTITUTING INTO SDE

$$M_{PQ} = 11.25kNm$$

$$M_{QP} = 22.5kNm$$

$$M_{QR} = -22.5kNm$$

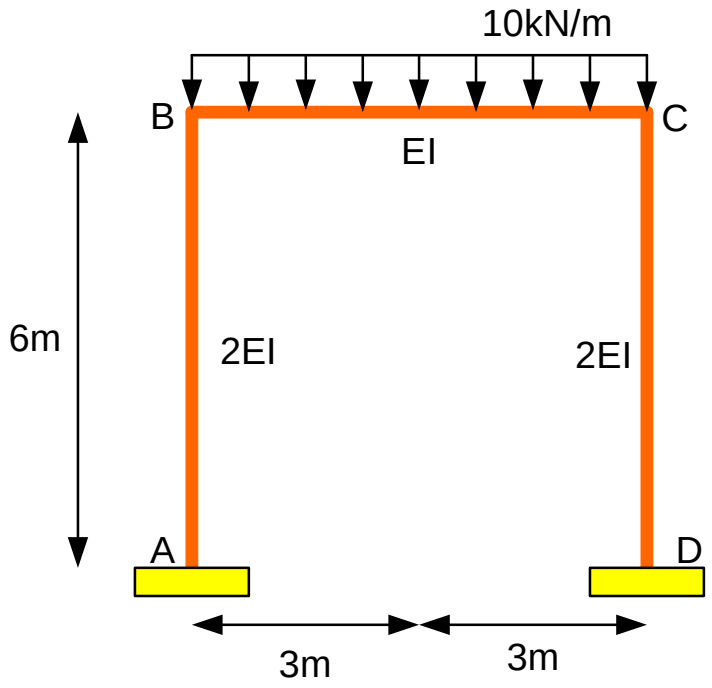
$$M_{RQ} = 22.5kNm$$

$$M_{RS} = -22.5kNm$$

$$M_{SR} = -11.25kNm$$

FOOD OF MIND

Determine end moment at critical point.



SOLUTION

FIXED END MOMENT

$$M_{AB}^{F} + M_{BA}^{F} + M_{CD}^{F} + M_{DC}^{F} = 0$$

$$M_{BC}^{F} = -M_{CB}^{F} = -30kNm$$

SLOPE DEFLECTION EQUATION $\theta_A = \theta_D = \Delta = 0$

$$M_{AB} = \frac{2(2EI)}{L}\left[2\theta_A + \theta_B - \frac{3\Delta}{L}\right] + M_{AB}^{F}$$

$$M_{AB} = \frac{2EI\theta_B}{3}$$

$$M_{BA} = \frac{2(2EI)}{L}\left[2\theta_B + \theta_A - \frac{3\Delta}{L}\right] + M_{BA}^F$$

$$M_{AB} = \frac{4EI\theta_B}{3}$$

$$M_{BC} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\Delta}{L}\right] + M_{BC}^F$$

$$M_{BC} = \frac{2EI\theta_B}{3} + \frac{EI\theta_C}{3} - 30$$

$$M_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\Delta}{L}\right] + M_{CB}^F$$

$$M_{CB} = \frac{2EI\theta_C}{3} + \frac{EI\theta_B}{3} + 30$$

$$M_{CD} = \frac{2(2EI)}{L}\left[2\theta_C + \theta_D - \frac{3\Delta}{L}\right] + M_{CD}^F$$

$$M_{CD} = \frac{4EI\theta_C}{3}$$

$$M_{DC} = \frac{2(2EI)}{L}\left[2\theta_D + \theta_C - \frac{3\Delta}{L}\right] + M^F_{DC}$$

$$M_{DC} = \frac{2EI\theta_C}{3}$$

EQUILIBRIUM AT JOINT

$$\sum M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$\sum M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$\begin{bmatrix} 2 & 1/3 \\ 1/3 & 2 \end{bmatrix} \begin{bmatrix} EI\theta_B \\ EI\theta_C \end{bmatrix} = \begin{bmatrix} 30 \\ -30 \end{bmatrix}$$

Solving by using Calculator

$$EI\theta_B = 18$$

$$EI\theta_C = -18$$

SUBSTITUTING INTO SDE

$$M_{AB} = 12kNm$$

$$M_{BA} = 24kNm$$

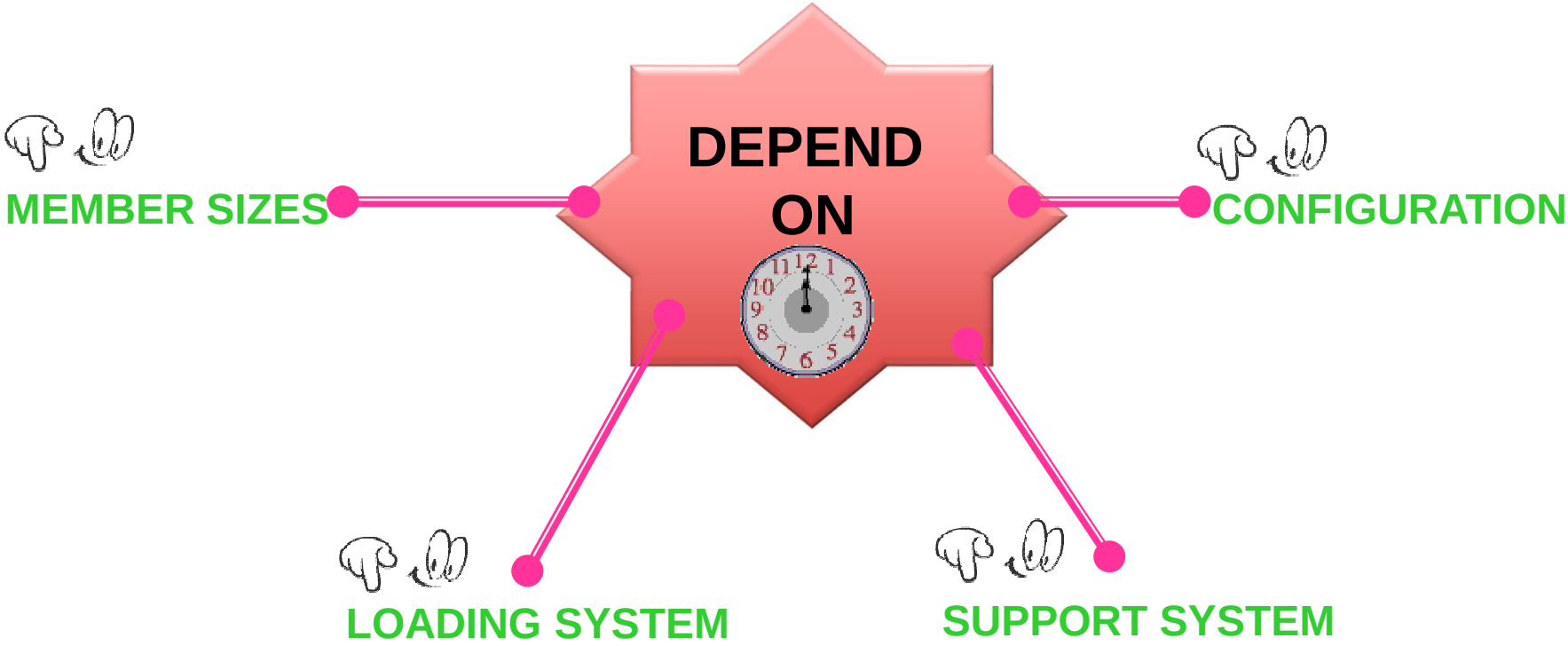
$$M_{BC} = -24kNm$$

$$M_{CB} = +24kNm$$

$$M_{CD} = -24kNm$$

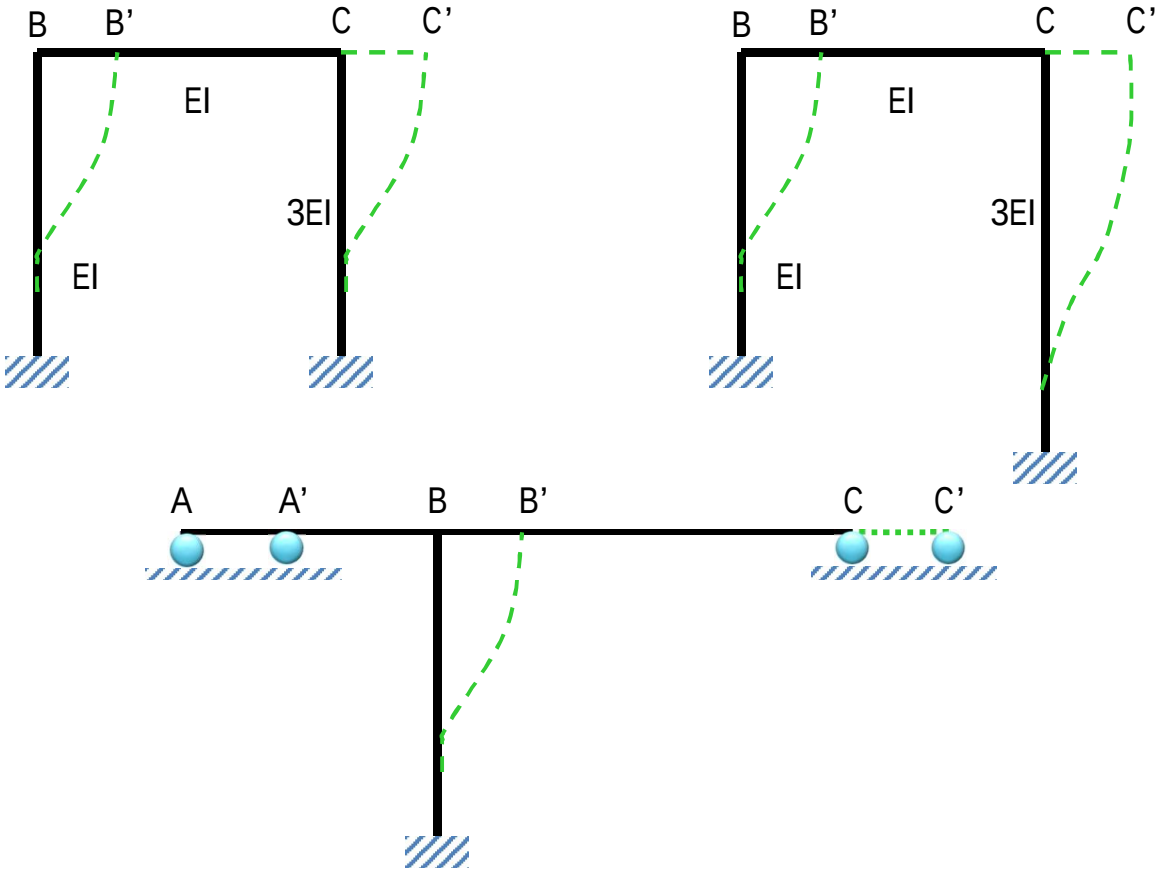
$$M_{DC} = -12kNm$$

ANALYSIS OF FRAME WITH SIDE SWAY.



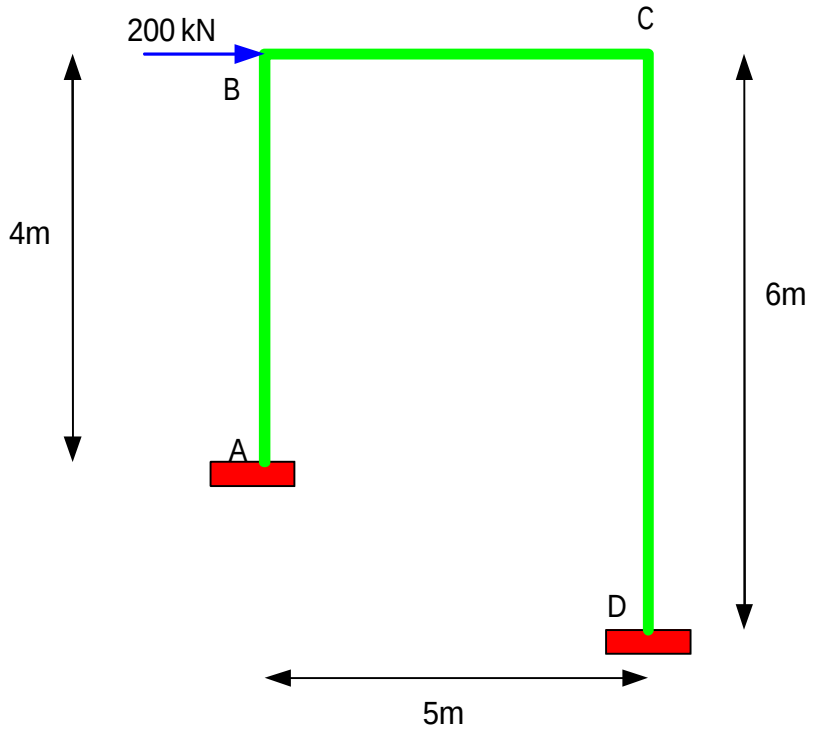
FUNDAMENTAL ASSUMPTIONS

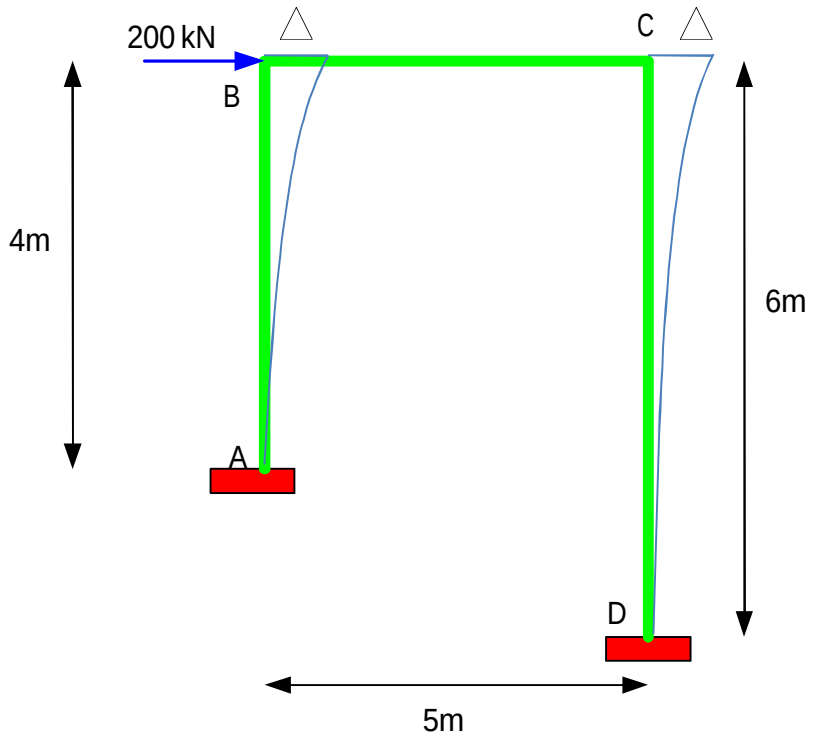
1. That axial deformation is ignored
2. That transverse end displacement do not affect the member length



EXAMPLE 2

DETERMINE END MOMENT AT CRITICAL POINT.
ASSUME *EI* IS CONSTANT.





SOLUTION

Fixed End Moment

$$M_{AB}^{F} = M_{BA}^{F} = M_{BC}^{F} = M_{CB}^{F} = M_{CD}^{F} = M_{DC}^{F} = 0$$

SLOPE DEFLECTION EQUATION

$$M_{AB} = \frac{EI\theta_B}{2} - \frac{3EI\Delta}{8}$$

$$M_{BA} = EI\theta_B - \frac{3EI\Delta}{8}$$

$$M_{BC} = \frac{4EI\theta_B}{5} + \frac{2EI\theta_C}{5}$$

$$M_{CB} = \frac{5EI\theta_C}{5} + \frac{2EI\theta_B}{5}$$

$$M_{CD} = \frac{2EI\theta_C}{3} - \frac{EI\Delta}{6}$$

$$M_{DC} = \frac{EI\theta_C}{3} - \frac{EI\Delta}{6}$$

Equilibrium at joint

$$\sum M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$EI\theta_B - \frac{3EI\Delta}{8} + \frac{4EI\theta_B}{5} + \frac{2EI\theta_C}{5} = 0$$

$$\frac{9EI\theta_B}{5} + \frac{2EI\theta_C}{5} - \frac{3EI\Delta}{8} = 0 \qquad \text{1}$$

$$\sum M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

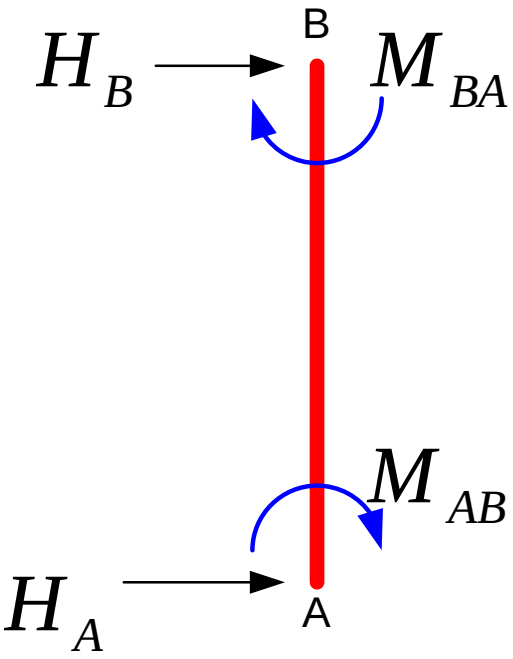
$$\frac{4EI\theta_C}{5} - \frac{2EI\theta_B}{5} + \frac{2EI\theta_C}{3} + \frac{EI\Delta}{6} = 0$$

$$\frac{2EI\theta_B}{5} + \frac{22EI\theta_C}{15} - \frac{EI\Delta}{6} = 0 \quad \text{——— 2}$$

$$\sum H = 0$$

$$H_A + H_D + 200 = 0$$

Consider member AB

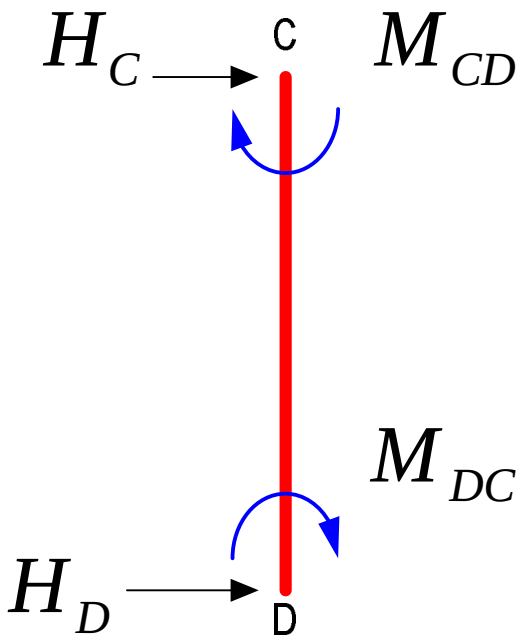


$$\sum M_B = 0$$

$$M_{AB} + M_{BA} - H_A(4) = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{4}$$

Consider member CD



$$\sum M_C = 0$$

$$M_{CD} + M_{DC} - H_D(6) = 0$$

$$H_D = \frac{M_{DC} + M_{CD}}{6}$$

$$\frac{M_{BA}+M_{AB}}{4}+\frac{M_{DC}+M_{CD}}{6}=-200$$

$$6M_{BA}+6M_{AB}+4M_{DC}+4M_{CD}=-4800$$

Insert the value……

$$9EI\theta_B+\frac{12EI\theta_C}{3}-\frac{35EI\Delta}{6}=-4800 \quad \text{(3)}$$

MATRIX FORM

$$\begin{bmatrix} 9/5 & 2/5 & -3/8 \\ 2/5 & 22/15 & -1/6 \\ 9 & 12/3 & -35/6 \end{bmatrix} \begin{bmatrix} EI\theta_B \\ EI\theta_C \\ EI\Delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -4800 \end{bmatrix}$$

Solving by using calculator

$$EI\theta_B = 243.78$$

$$EI\theta_C = 75.66$$

$$EI\Delta = 1250.86$$

Substituting into SDE

$$M_{AB} = \frac{243.78}{2} - \frac{3(1250.88)}{8} = -347.16kNm$$

$$M_{BA} = 243.78 - \frac{3(1250.88)}{8} = -225.29kNm$$

$$M_{BC} = \frac{4(243.78)}{5} + \frac{2(75.66)}{5} = +225.29kNm$$

$$M_{CB} = \frac{4(75.66)}{5} + \frac{2(243.78)}{5} = +158.04kNm$$

$$M_{CD} = \frac{2(75.66)}{3} - \frac{1250.88}{6} = -158.04kNm$$

$$M_{DC} = \frac{75.66}{3} - \frac{1250.88}{6} = -183.26kNm$$

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THANKS

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