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THEORY OF STRUCTURES
CHAPTER 3 : SLOPE DEFLECTION (FOR BEAM)
PART 1

by
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Chapter 3 : Part 1 – Slope Deflection

- Aims
  - Determine the end moment for beam using Slope Deflection Method.

- Expected Outcomes :
  - Able to indicate the degree of freedom.
  - Able to indicate the moment due to angular displacement.
  - Able to determine the moment due to linear displacement
  - Able to determine the fixed end moment
  - Able to write the slope deflection equation.

- References

  - Mechanics of Materials, R.C. Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, Hibbeler, 7th Edition, Prentice Hall
  - Structural Analysis, SI Edition by Aslam Kassimali,Cengage Learning
  - Structural Analysis, Coates, Coatie and Kong
  - Structural Analysis - A Classical and Matrix Approach, Jack C. McCormac and James K. Nelson, Jr., 4th Edition, John Wiley

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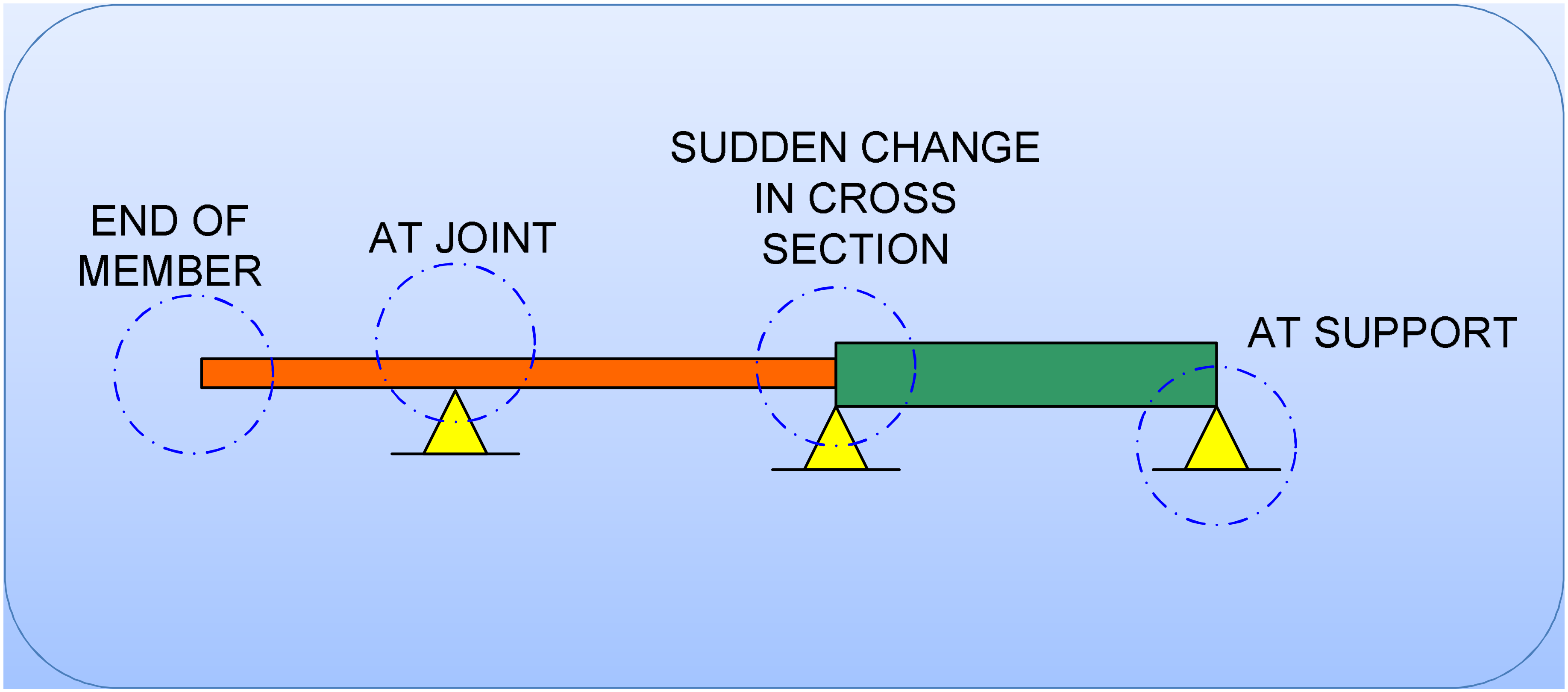
INTRODUCTION

- Introduced for analyzing statically indeterminate structure – reactions and internal forces.

- The method required the solution of simultaneous equations representing the overall system of equilibrium equation.

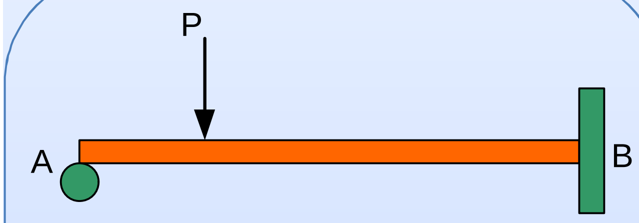
DEGREE OF FREEDOM

- When a structure is loaded, specified point on it call nodes, will undergo displacements.
- These displacement are referred to as the degree of freedom for the structure.
- To determined the number of degrees of freedom, we can imagine the structure to consist of a series member connected to nodes, which is usually located at JOINT, SUPPORT, and at the END OF MEMBER or where the member have SUDDEN CHANGE IN CROSS SECTION.



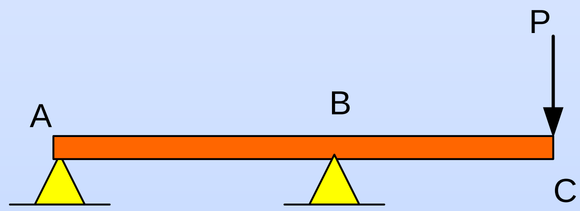
END OF MEMBER
AT JOINT
SUDDEN CHANGE IN CROSS SECTION
AT SUPPORT

EXAMPLE



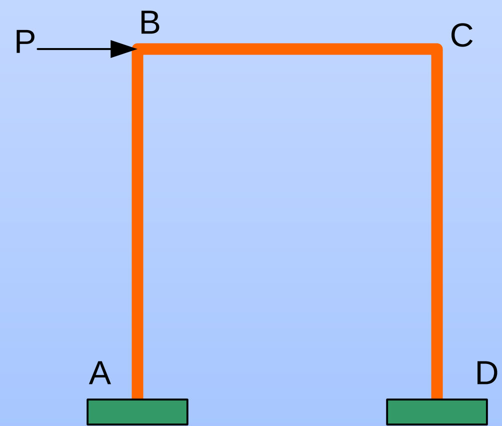
When load P is applied to the beam, will cause node A to rotate, while node B is completely restricted from moving.

One degree of freedom, θ_A



Four degree of freedom,

$$\theta_A, \theta_B, \theta_C, \Delta_C$$



Three degree of freedom,

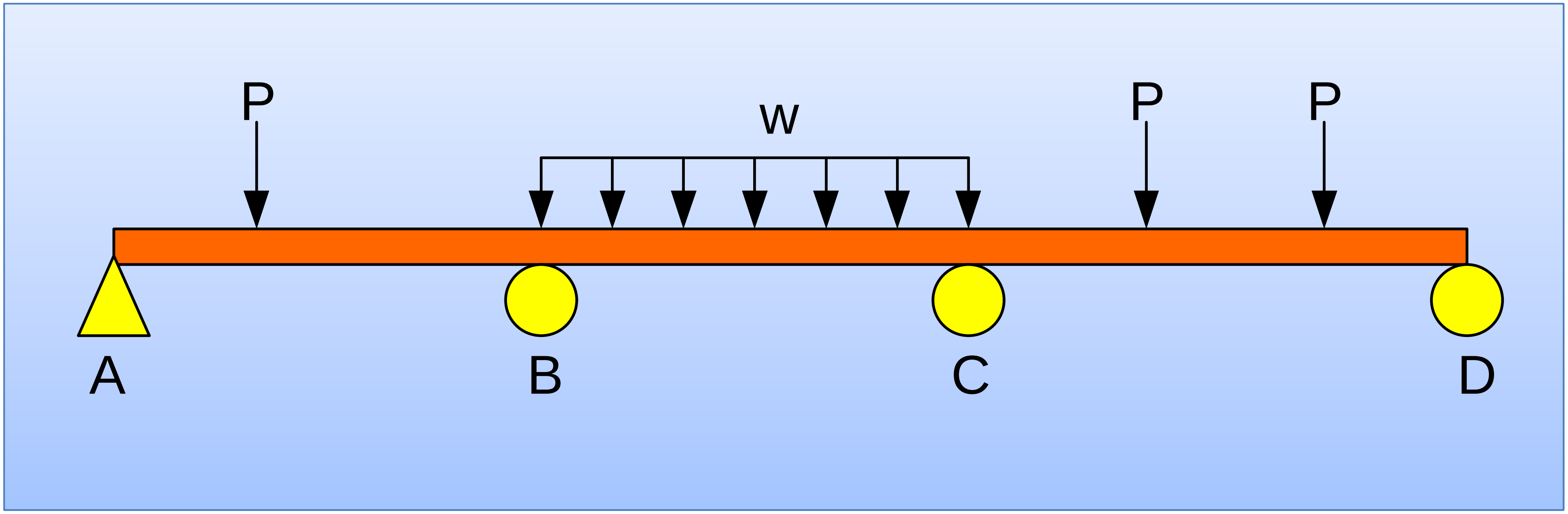
$$\theta_B, \theta_C, \Delta_B$$

Conclusion

Real beam		
θ	$\Delta = 0$	pinned
θ	$\Delta = 0$	roller
$\theta = 0$	$\Delta = 0$	fixed
θ	Δ	free
θ	$\Delta = 0$	Internal pinned
θ	$\Delta = 0$	Internal roller
θ	Δ	hinge

FOOD OF MIND

Find the number of degree of freedom



Ans : 4

SLOPE DEFLECTION EQUATION

M_{AB} - INTERNAL END MOMENT

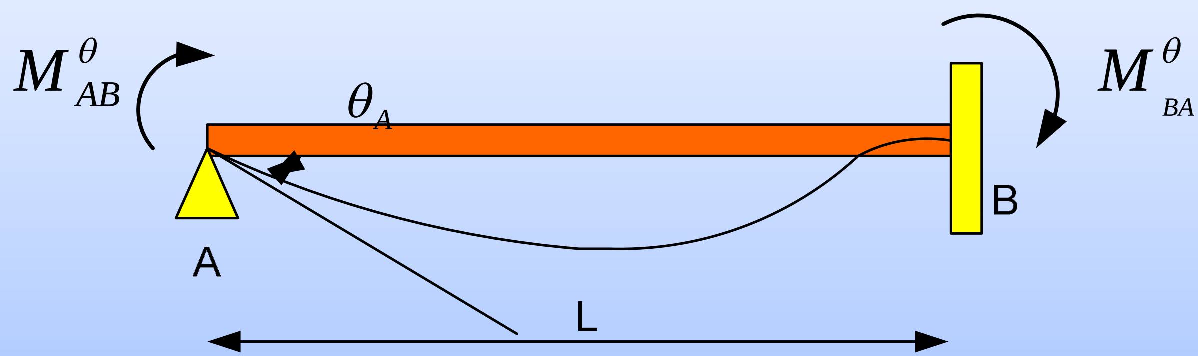
Angular displacement at A (Near)

Angular displacement at B (Far)

Linear displacement

Fixed end moment

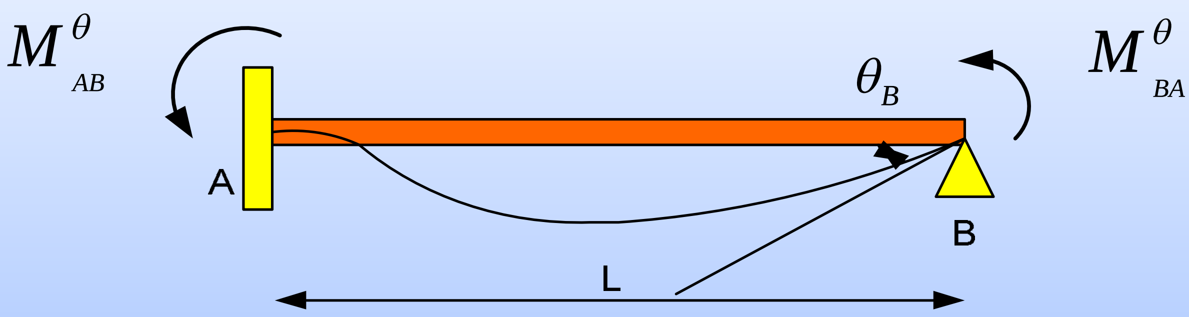
1. ANGULAR DISPLACEMENT AT A



$$M^{\theta}_{AB} = \frac{4EI}{L}\theta_A$$

$$M^{\theta}_{BA} = \frac{2EI}{L}\theta_A$$

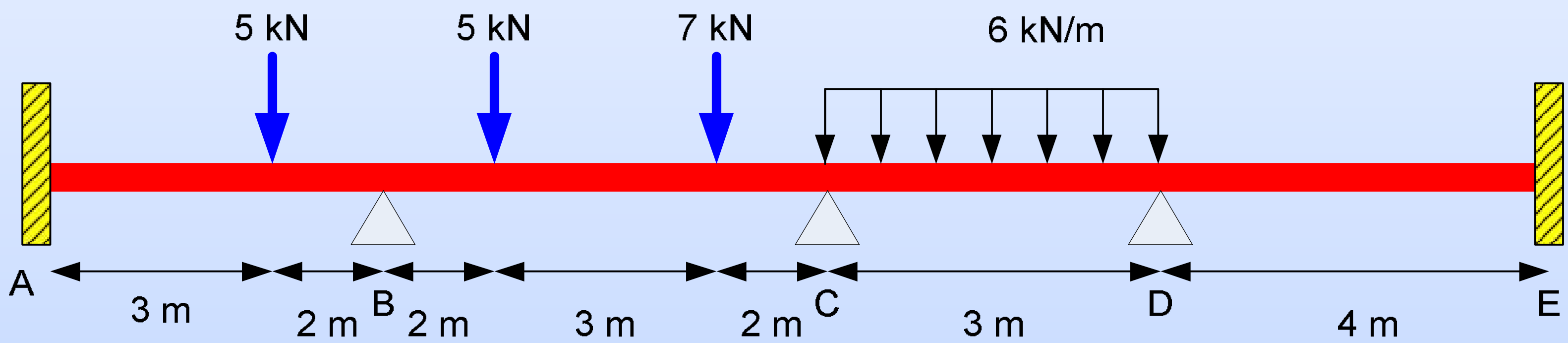
2. ANGULAR DISPLACEMENT AT B



$$M^{\theta}_{BA} = \frac{4EI}{L}\theta_B$$

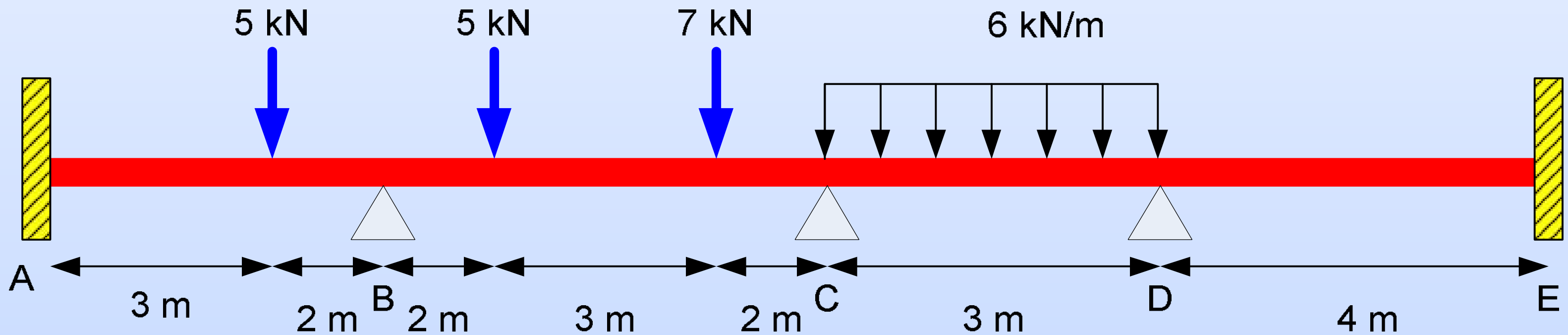
$$M^{\theta}_{AB} = \frac{2EI}{L}\theta_B$$

EXAMPLE.



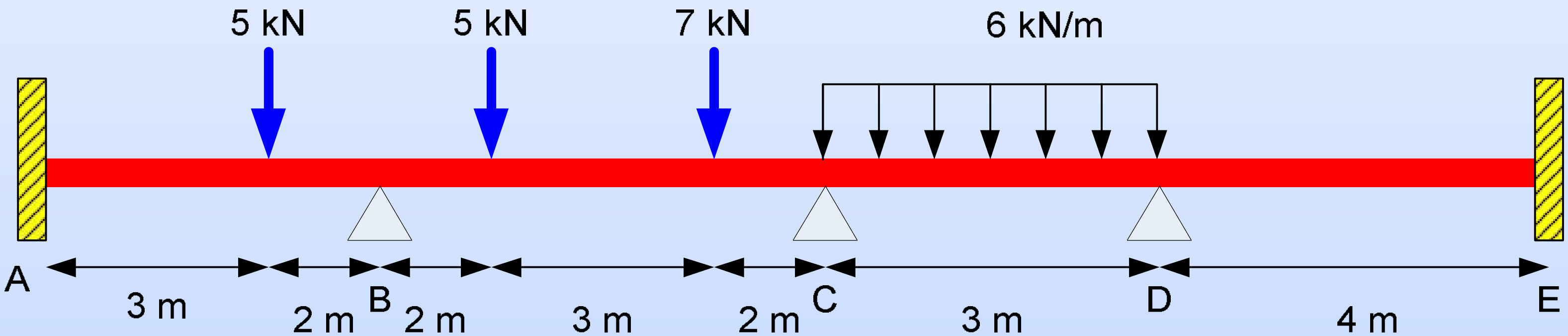
$$M_{AB}^{\theta} = \frac{4EI}{L}\theta_A \qquad M_{BA}^{\theta} = \frac{4EI}{L}\theta_B$$

$$\frac{2EI}{L}\theta_B \qquad \frac{2EI}{L}\theta_A$$



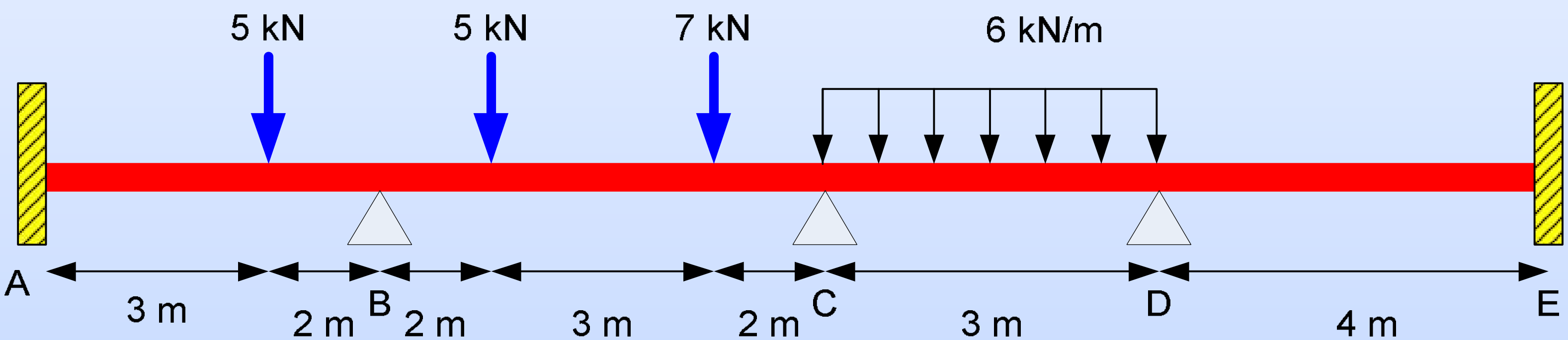
$$M^{\theta}_{BC} = \frac{4EI}{L}\theta_B \qquad M^{\theta}_{CB} = \frac{4EI}{L}\theta_C$$

$$\frac{2EI}{L}\theta_C \qquad \frac{2EI}{L}\theta_B$$



$$M_{CD}^{\theta} = \frac{4EI}{L}\theta_C \qquad \frac{2EI}{L}\theta_D$$

$$M_{DC}^{\theta} = \frac{4EI}{L}\theta_D \qquad \frac{2EI}{L}\theta_C$$



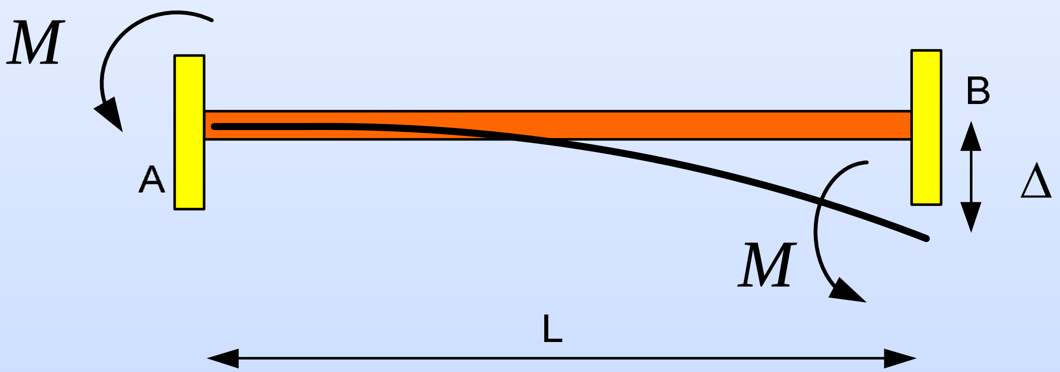
$$M^{\theta}_{DE} = \frac{4EI}{L}\theta_D$$

$$\frac{2EI}{L}\theta_E$$

$$M^{\theta}_{ED} = \frac{4EI}{L}\theta_E$$

$$\frac{2EI}{L}\theta_D$$

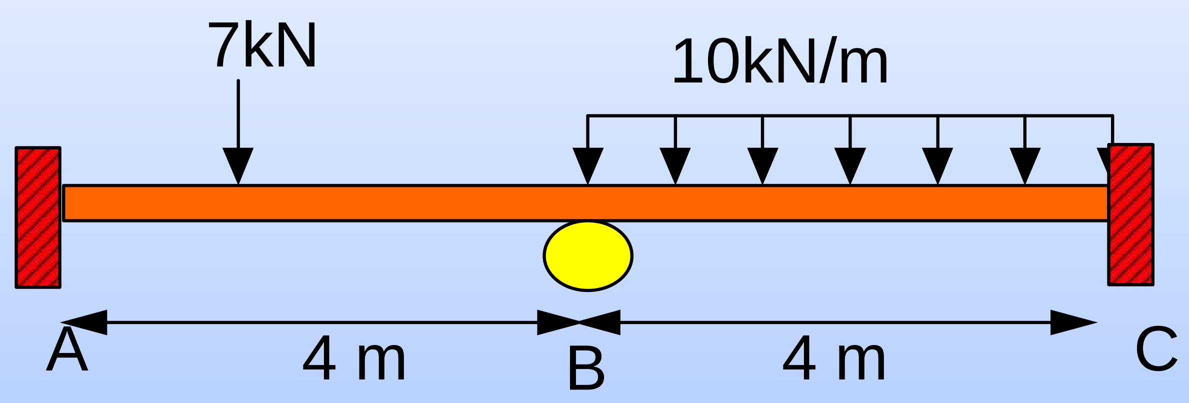
3. RELATIVE LINEAR DISPLACEMENT



$$M_{AB} = M_{BA} = M = -\frac{6EI}{L^2}\Delta$$

EXAMPLE 1

Determine the moment due to linear displacement for each members. Assume 2mm settlement occur at support B.



$$M_{AB}^{\Delta} = M_{BA}^{\Delta} = -\frac{6EI\Delta}{L^2}$$

$$= -\frac{6EI(+0.002)}{4^2}$$

$$= -0.003EI$$

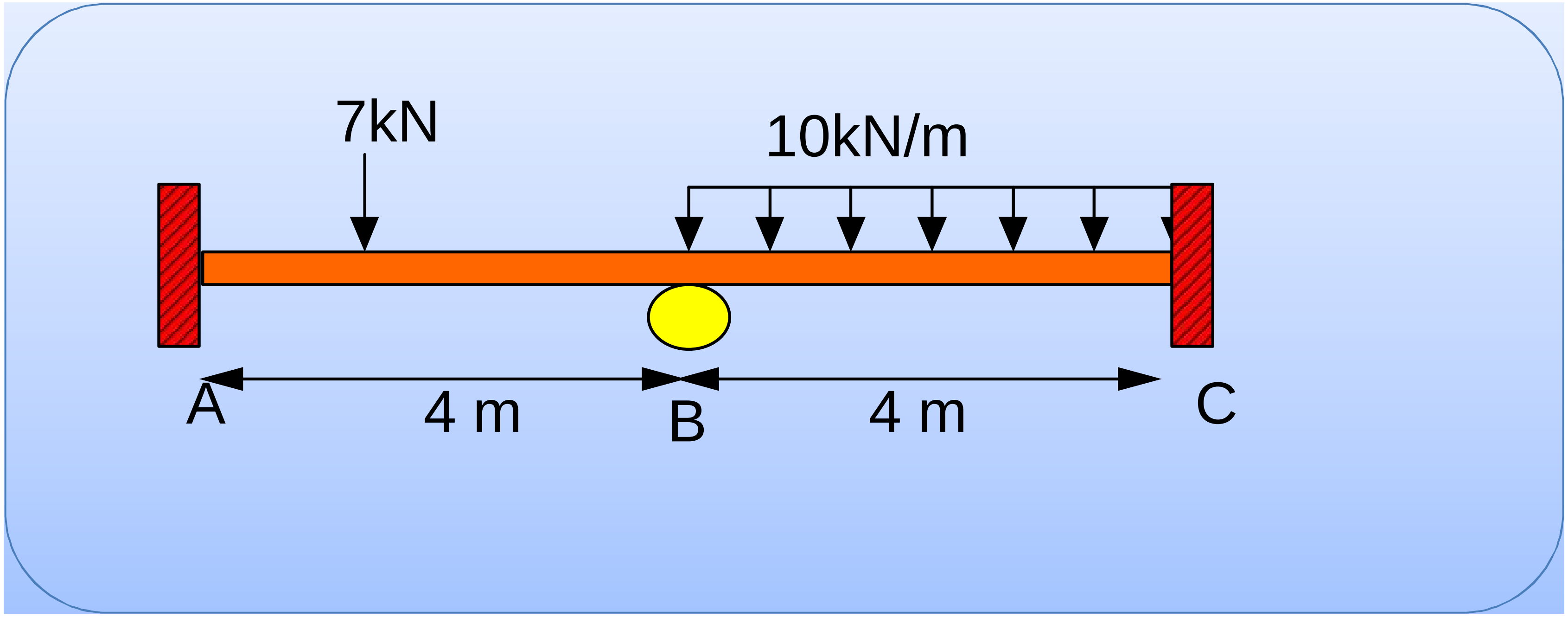
$$M_{BC}^{\Delta} = M_{BC}^{\Delta} = -\frac{6EI\Delta}{L^2}$$

$$= -\frac{6EI(-0.002)}{4^2}$$

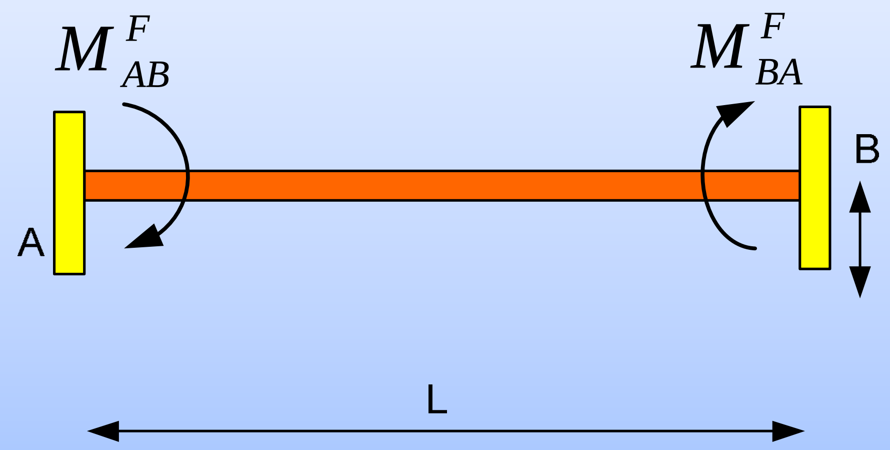
$$= +0.003EI$$

FOOD OF MIND

Determine the moment due to linear displacement for each members. Assume 2mm and 1mm settlement occur at support B and C respectively.



4. FIXED END MOMENT

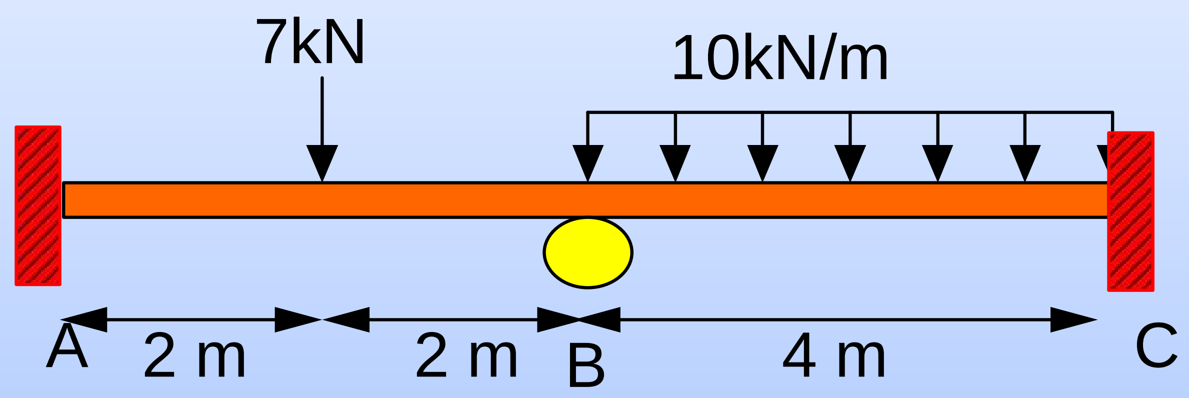


Moment cause by external load whilst the both support are fixed.

- REFER TABLE

EXAMPLE 2

Determine the Fixed End Moment for each members.



$$M_{AB}^{F} = -\frac{PL}{8}$$

$$= -\frac{7(4)}{8}$$

$$= -3.5kNm$$

$$M_{BA}^{F} = +\frac{PL}{8}$$

$$= +\frac{7(4)}{8}$$

$$= +3.5kNm$$

$$M_{CB}^{F} = -\frac{wL^2}{12}$$

$$= -\frac{10(4)^2}{12}$$

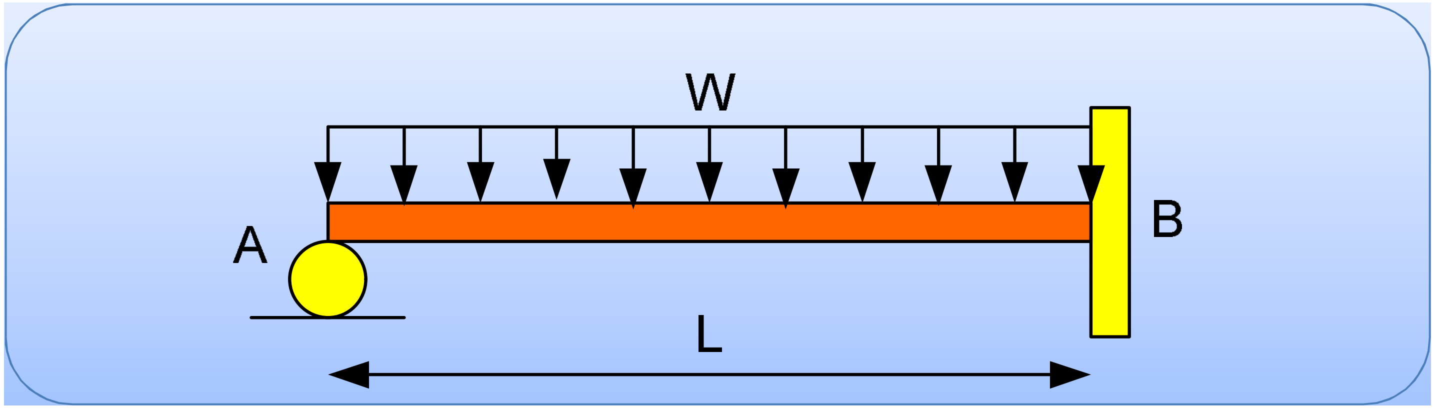
$$= -13.33kNm$$

$$M_{CB}^{F} = +\frac{wL^2}{12}$$

$$= +\frac{10(4)^2}{12}$$

$$= +13.33kNm$$

SLOPE DEFLECTION EQUATION



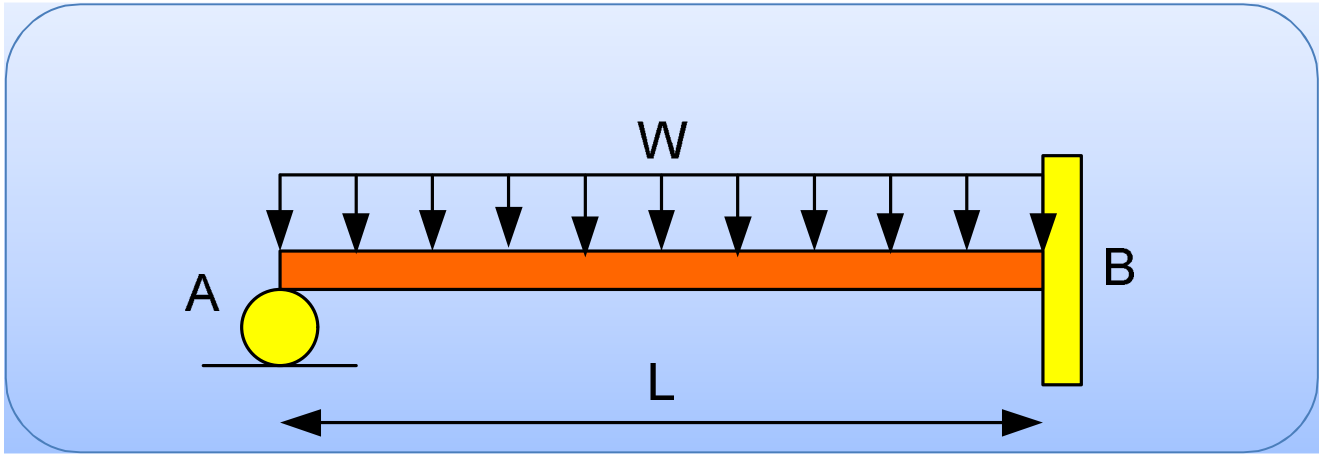
$$M_{AB} = \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2} - M_{AB}^F$$

Angular displacement at A

Angular displacement at B

Fixed end moment

Linear displacement



$$M_{BA} = \frac{4EI\theta_B}{L} + \frac{2EI\theta_A}{L} - \frac{6EI\Delta}{L^2} + M_{BA}^F$$

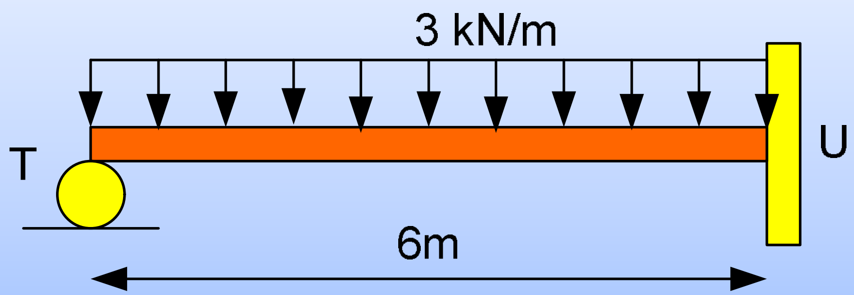
Angular displacement at B

Angular displacement at A

Linear displacement

Fixed end moment

Write down the slope deflection equation

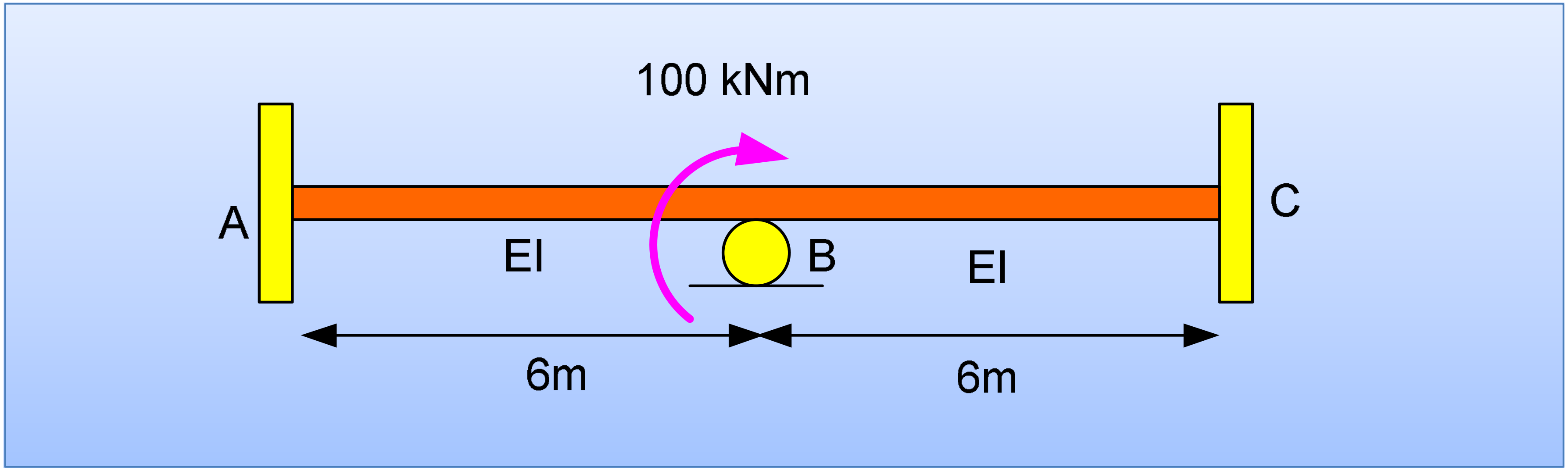


$$M_{TU} = \frac{4EI\theta_T}{L} + \frac{2EI\theta_U}{L} - \frac{6EI\Delta}{L^2} + M_{TU}^F$$

$$M_{UT} = \frac{4EI\theta_U}{L} + \frac{2EI\theta_T}{L} - \frac{6EI\Delta}{L^2} + M_{UT}^F$$

EXAMPLE 3

Analyse the two span continuous beam as shown below for the bending moment at the support point or member end using SDM. The relative flexural rigidity for both span are identical ie EI is constant and the beam is subjected to a point moment of 100kNm at B



Solution.

Fixed End Moment

$$M_{AB}^{F} = M_{BA}^{F} = M_{BC}^{F} = M_{CB}^{F} = 0$$

Slope Deflection Equation

$$\theta_A = \theta_C = 0$$

$$\Delta = 0$$

$$M_{AB} = \cancel{\frac{4EI\theta_A}{L}} + \frac{2EI\theta_B}{L} - \cancel{\frac{6EI\Delta}{L^2}} + \cancel{M_{AB}^{F}} \qquad = \frac{EI\theta_B}{3}$$

$$M_{BA} = \frac{4EI\theta_B}{L} + \cancel{\frac{2EI\theta_A}{L}} - \cancel{\frac{6EI\Delta}{L^2}} + \cancel{M_{BA}^{F}} \qquad = \frac{2EI\theta_B}{3}$$

$$M_{BC} = \frac{4EI\theta_B}{L} + \frac{2EI\theta_C}{L} - \frac{6EI\Delta}{L^2} + M_{BC}^F \qquad = \frac{2EI\theta_B}{3}$$

$$M_{CB} = \frac{4EI\theta_C}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2} + M_{CB}^F \qquad = \frac{EI\theta_B}{3}$$

EQUILIBRIUM AT JOINT

$$\sum M_B = 100$$

$$M_{BA} + M_{BC} = 100$$

$$\frac{2EI\theta_B}{3} + \frac{2EI\theta_B}{3} = 100 \qquad \theta_B = \frac{75}{EI}$$

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SUBSTITUTING INTO SDE

$$M_{AB} = +25kNm$$

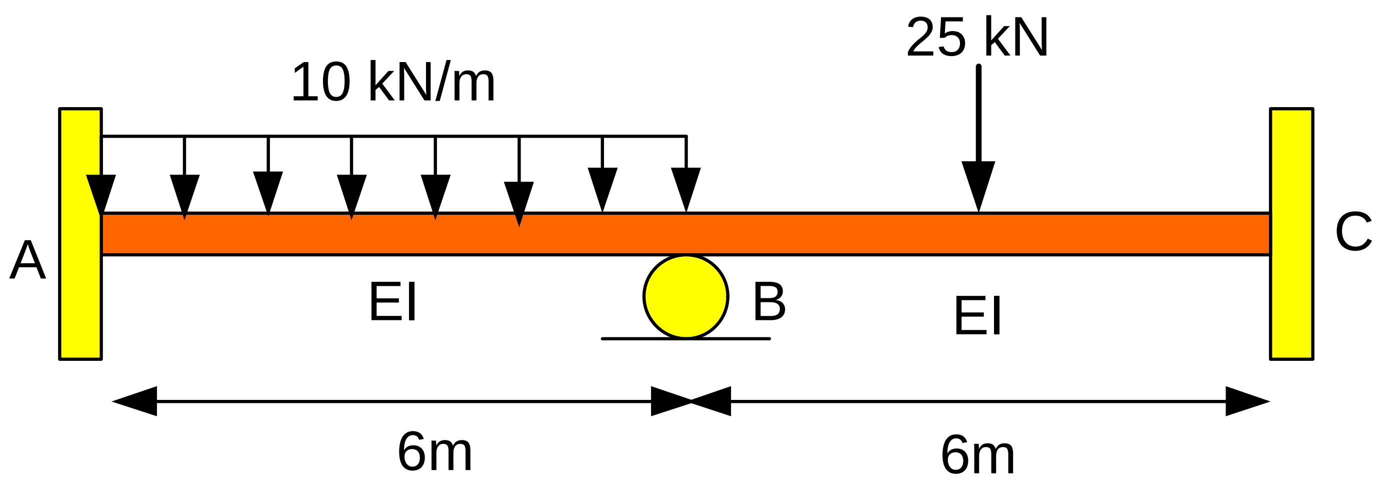
$$M_{BA} = +50kNm$$

$$M_{BC} = +50kNm$$

$$M_{CB} = +25kNm$$

EXAMPLE 4

In figure below, the two span continuous beam shown earlier is now subjected two in span loads of UDL having an intensity of 10 kN/m over span AB and a point load of 25kN at the mid span of BC. EI is constant. Determine the support moment at A, B and C. Draw the bending moment diagram.



SOLUTION

Step # 1

FIXED END MOMENT

$$M_{AB}^{F} = \frac{-WL^2}{12} = \frac{-10(6)^2}{12} = -30kNm$$

$$M_{BA}^{F} = +30kNm$$

$$M_{BC}^{F} = \frac{-PL}{8} = \frac{-25(6)}{8} = -18.75kNm$$

$$M_{CB}^{F} = +18.75kNm$$

Step # 2

SLOPE DEFLECTION EQUATION $\theta_A = \theta_C = \Delta = 0$

$$M_{AB} = \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2} + M_{AB}^F$$

$$M_{AB} = \frac{2EI\theta_B}{6} - 30$$

$$M_{BA} = \frac{4EI\theta_B}{L} + \frac{2EI\theta_A}{L} - \frac{6EI\Delta}{L^2} + M_{BA}^F$$

$$M_{BA} = \frac{2EI\theta_B}{3} + 30$$

$$M_{BC} = \frac{4EI\theta_B}{L} + \cancel{\frac{2EI\theta_C}{L}} - \cancel{\frac{6EI\Delta}{L^2}} + M_{BC}^F$$

$$M_{BC} = \frac{2EI\theta_B}{3} - 18.75$$

$$M_{CB} = \cancel{\frac{4EI\theta_C}{L}} + \frac{2EI\theta_B}{L} - \cancel{\frac{6EI\Delta}{L^2}} + M_{CB}^F$$

$$M_{CB} = \frac{EI\theta_B}{3} + 18.75$$

Step # 3

Equilibrium Equation

$$\sum M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI\theta_B}{3} + 30 + \frac{2EI\theta_B}{3} - 18.75 = 0$$

$$EI\theta_B = -8.38$$

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Step # 4

SUBSTITUTING INTO SDE

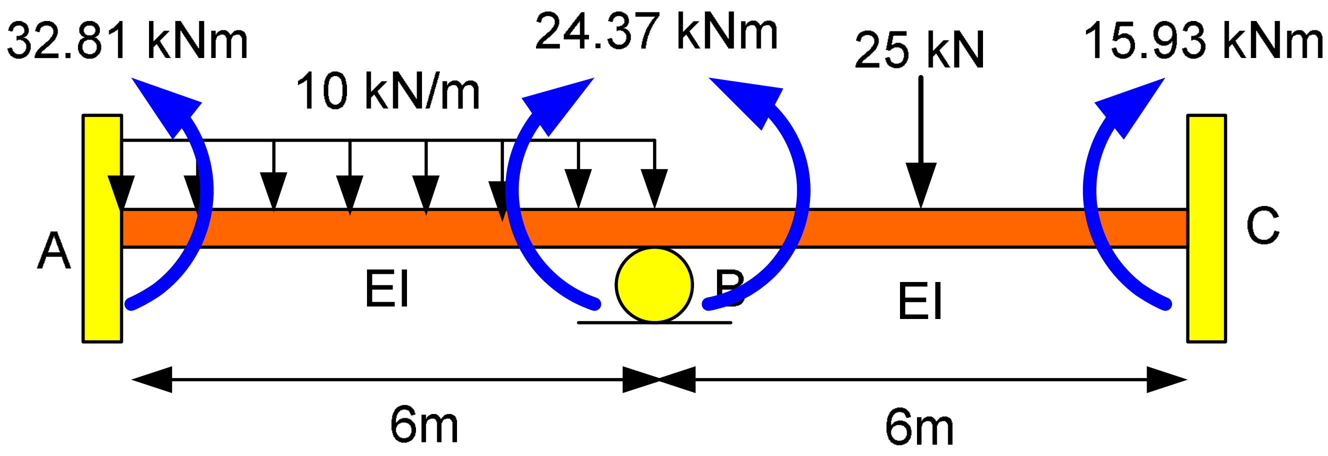
$$M_{AB} = -32.81 kNm$$

$$M_{BA} = +24.37 kNm$$

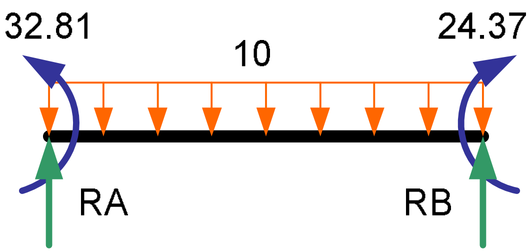
$$M_{BC} = -24.38 kNm$$

$$M_{CB} = +15.93 kNm$$

MOMENT PROFILE



Span AB

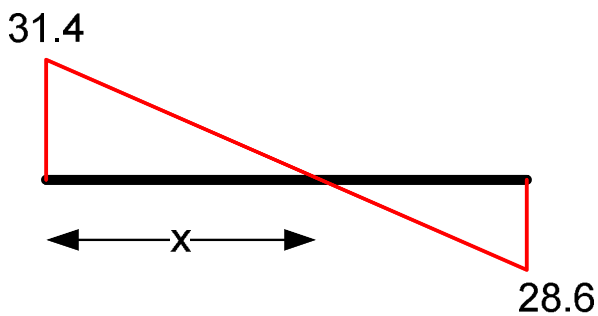


$$\sum M_A = 0$$

$$-32.81 + 24.37 + \frac{10(6)^2}{2} - R_B(6) = 0$$

$$R_B = 28.6kN$$

$$\therefore R_A = 31.4kN$$



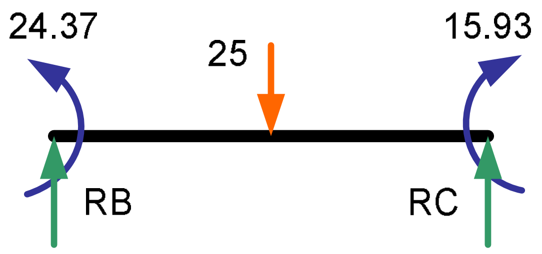
$$\frac{x}{31.4} = \frac{6}{60}$$

$$x = 3.14m$$

$$Area = \frac{1}{2}(3.14)(31.4) = 49.3kNm$$

$$Max\ Moment = -32.81 + 49.3 = 16.49kNm$$

Span BC

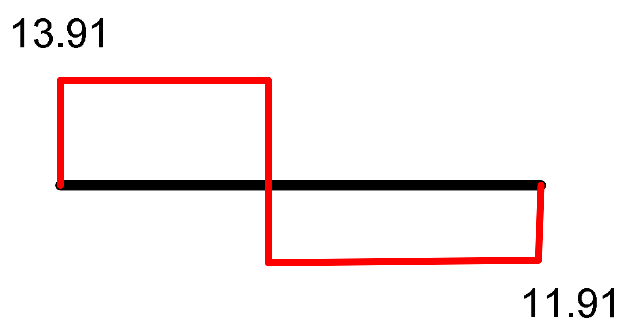


$$\sum M_B = 0$$

$$-24.37 + 15.93 + 25(3) - R_C(6) = 0$$

$$R_C = 11.09kN$$

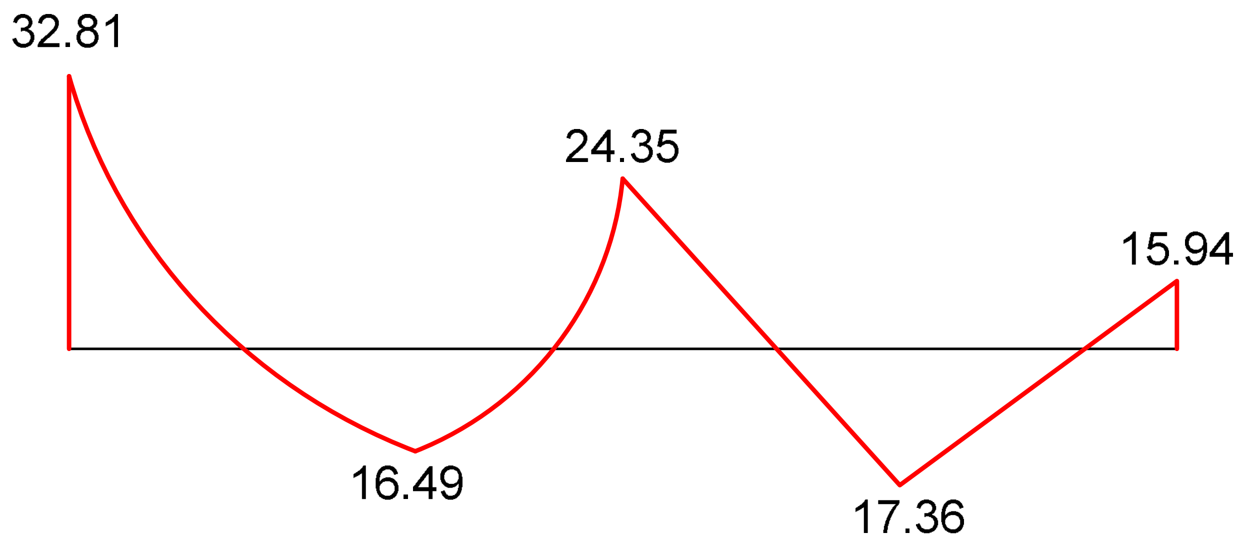
$$\therefore R_B = 13.91kN$$



$$Area = (3)(13.91) = 41.73kNm$$

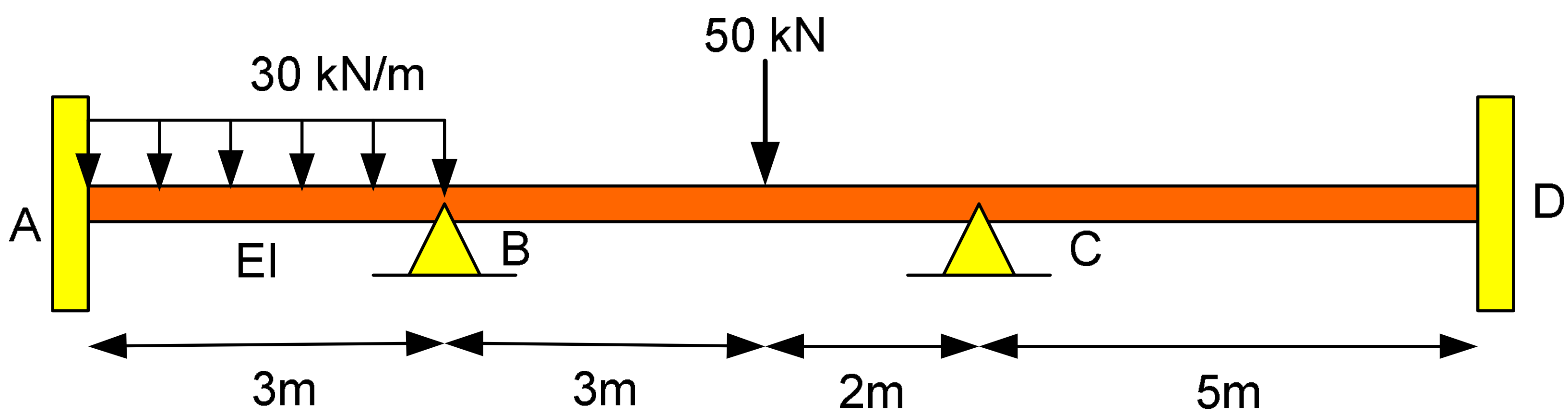
$$Max\ Moment = -24.37 + 41.73 = 17.36kNm$$

BMD



EXAMPLE 5

Continuous beam with settlement at support B and C. Determine the end moment in the figure if there is 1mm downward movement at support B and C.
Take EI is 75E3 kNm3 for all span



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SOLUTION

FIXED END MOMENT

$$M_{AB}^{F} = -22.5kNm$$

$$M_{BA}^{F} = +22.5kNm$$

$$M_{BC}^{F} = -24kNm$$

$$M_{CB}^{F} = +36kNm$$

$$M_{CD}^{F} = 0$$

$$M_{DC}^{F} = 0$$

MOMENT DUE TO SHRINKAGE.

$$M_{AB}^{\delta} = \frac{-6EI\Delta}{L^2} = \frac{-6\left(75\times10^3\right)(0.001)}{3^2} = -50kNm$$

$$M_{BA}^{\delta} = \frac{-6EI\Delta}{L^2} = \frac{-6\left(75\times10^3\right)(0.001)}{3^2} = -50kNm$$

$$M_{BC}^{\delta} = \frac{-6EI\Delta}{L^2} = \frac{-6\left(75\times10^3\right)(0)}{5^2} = 0$$

$$M_{CB}^{\delta} = \frac{-6EI\Delta}{L^2} = \frac{-6(75\times10^3)(0)}{5^2} = 0$$

$$M_{CD}^{\delta} = \frac{-6EI\Delta}{L^2} = \frac{-6(75\times10^3)(-0.001)}{5^2} = +18kNm$$

$$M_{DC}^{\delta} = \frac{-6EI\Delta}{L^2} = \frac{-6(75\times10^3)(-0.001)}{5^2} = +18kNm$$

Slope Deflection Equation.

$$M_{AB} = \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2} + M_{AB}^F$$

$$M_{AB} = \frac{2\left(75\times10^3\right)\theta_B}{3} - 50 - 22.5$$

$$M_{AB} = 5000\theta_B - 72.5$$

$$M_{BA} = \frac{4EI\theta_B}{L} + \frac{2EI\theta_A}{L} - \frac{6EI\Delta}{L^2} + M_{BA}^F$$

$$M_{BA} = \frac{4(75\times10^3)\theta_B}{3} - 50 + 22.5$$

$$M_{AB} = 10,000\theta_B - 27.5$$

$$M_{BC} = \frac{4EI\theta_B}{L} + \frac{2EI\theta_C}{L} - \frac{6EI\Delta}{L^2} + M_{BC}^F$$

$$M_{BC} = \frac{4(75\times10^3)\theta_B}{3} + \frac{4(75\times10^3)\theta_C}{3} - 24$$

$$M_{BC} = 60,000\theta_B + 30,000\theta_C - 24$$

$$M_{CB} = 60,000\theta_C + 30,000\theta_B + 36$$

$$M_{CD} = 60,000\theta_C + 18$$

$$M_{DC} = 30,000\theta_C + 18$$

EQUILIBRIUM AT JOINT

$$\sum M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$160,000\theta_B + 30,000\theta_C = 51.5 \qquad (1)$$

$$\sum M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$30{,}000\theta_B + 120{,}000\theta_C = -54 \quad (2)$$

$$\begin{bmatrix} 160{,}000 & 30{,}000 \\ 30{,}000 & 120{,}000 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 51.5 \\ -54 \end{bmatrix}$$

Solving by using calculator

$$\theta_B = 4.262 \times 10^{-4}$$

$$\theta_C = -5.566 \times 10^{-4}$$

SUBSTITUTING INTO SDE

$$M_{AB} = -51.19 kNm$$

$$M_{BA} = +15.12 kNm$$

$$M_{BC} = -15.13 kNm$$

$$M_{CB} = +15.39 kNm$$

$$M_{CD} = -15.40 kNm$$

$$M_{DC} = +1.30 kNm$$

THANKS

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