Chapter Description

• Expected Outcomes
  - Describe the concept of columns in terms of types of column, cross-sectional of shapes, industry standard application and slenderness ratio of column
  - Illustrate, explain and differentiate the failure mode of columns due to buckling
  - Explain the influence of support conditions
  - Relationship between the effective length and radius of gyration
  - Describe the compression member of long/slender column
  - Apply the Euler formula to determine the critical load for long columns
• A **column** in structural engineering is a vertical structural element that transmits, through compression, the weight of the structure above to other structural elements below.

• For the purpose of wind or earthquake engineering, **columns** may be designed to resist lateral forces.

• **Columns** are frequently used to support beams or arches on which the upper parts of walls or ceilings rest.

• A column is a relatively long, slender member loaded in compression.
• When a perfect column is subjected to a compressive axial force, the only deformation that takes place is a shortening of the column.
• For low values of F, if the column were to be deflected laterally by a force perpendicular to the column, and the lateral force were thereafter removed, the column would return to its straight position, even with the force F remaining in place.
• This indicates a condition of stability.
• If the load F were increased, there is a value of F for which, when the lateral load is removed, the column would remain in the deformed shape.
• This condition is referred to as buckling and the column is said to have failed from a structural standpoint.
• Buckling can also be described in simple terms as bending or bowing of a column due to a compressive load.
Slenderness ratio is a measure of how long the column is compared to its cross-section's effective width (resistance to bending or buckling). The **slenderness ratio** is the column's effective length divided by the radius of gyration.

\[
s = \frac{L}{r}
\]

Where \( r = \sqrt{I/A} \)
What is Buckling?

- When a slender member is subjected to an axial compressive load, it may fail by a condition called buckling.
- Buckling is not a failure of the material itself (as is yielding and fracture), but is due to geometric instability of the system.
- Note that buckling is not dependent on material strength.
Effective Length

• How a column is supported governs its **buckling strength**. The effective length \( L_e \) accounts for differences in the **end supports**.

**What is the effective length?**

• The **effective length** is the length the column **would be if it were to buckle** as a **pinned-pinned** column.
• A dimensionless coefficient \( K \), **effective-length factor**, is used to calculate \( L_e \)
The effective length is equal to the **distance between points** in the column where the **moment = 0** (between "pins"). This occurs when the curvature of the column changes.

The fixed-free column is "mirrored" through the fixed end to visualize $L_e = 2L$.

<table>
<thead>
<tr>
<th>End Condition</th>
<th>Pinned-Pinned</th>
<th>Fixed-Free</th>
<th>Fixed-Fixed</th>
<th>Fixed-Pinned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Length, $L_e$</td>
<td>$L$</td>
<td>$2L$</td>
<td>$0.5L$</td>
<td>$0.7L$</td>
</tr>
<tr>
<td>Relative Buckling</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strength ($\sim 1/ L_e^2$) for same $L$</td>
<td>1</td>
<td>0.25</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
Radius Of Gyration

• If all of the cross-sectional area $A$ were massed a distance $r$ away from the bending axis, the idealized lumped-area cross-section would have the same moment of inertia $I$ as the actual cross-section if:

$$I = Ar^2$$

• Distance $r$ is the radius of gyration. There generally two bending axes to consider, and thus two radius of gyration:

$$r_x = \sqrt{\frac{I_{xx}}{A}}; \quad r_y = \sqrt{\frac{I_{yy}}{A}}$$
• Columns are long slender members subjected to an axial compressive force. Lateral deflection on a column is called buckling. The maximum axial load that a column can support when it is on the verge of buckling is called the critical load $P_{cr}$.
• FOS is a safety margin given in design so that the member will not fail when the load is increased beyond the elastic limit or when the size in reduced.
• Normally, the factors of safety varies between 1.4 to 3.

\[ FOS = \frac{P_{ult}}{P_{all}} \quad \text{or} \quad FOS = \frac{\sigma_{ult}}{\sigma_{all}} \]
USE OF THE EULER FORMULA

- **Euler Buckling Formula**

- Both ends are pinned so they can **freely rotate** and **cannot resist a moment**. The critical load $P_{cr}$ required to buckle the pinned-pinned column is the

  $$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

  $P_{cr}$ = the euler buckling load
  $E$ = Young's modulus for the materials
  $I$ = the least second moment of area of the section
  $L_e$ = effective length
Assumptions / limitation of the Euler formula

- Axially loaded column
- Column is perfectly straight
- Isotropic and homogeneous material
- Material behaves within elastic properties
- Both ends of column support are pinned
Summary

- Long slender members subjected to an axial compressive force are called columns.
- Lateral deflection is called buckling.
- Maximum axial load a column can support when on the verge of buckling is called the critical load, $P_{cr}$. 
References
