BEE1133 Circuit Analysis

Chapter 2B
Methods of Analysis (DC Circuits)

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Chapter Description

Aims

This chapter is aimed to:
1. Explain the Mesh Analysis technique in solving problem related to electric circuit

Expected Outcomes

Student should be able to
1. Identify the loop for circuit
2. Identify the supermesh in the circuit
3. Determine the equation of ohm’s law
4. Determine the KVL equation for each loop for solving the electric circuit problem.

References

Basic Concept

5.1 Mesh Analysis
5.2 Mesh Analysis with current source: Supernode
5.3 Nodal versus mesh analysis
Mesh Analysis

- Mesh Analysis assigns **UNKNOWN MESH CURRENTS** to all the meshes in the circuit.
- Finds the **UNKNOWN MESH CURRENTS** by performing KVL around all meshes.
- KVL: Summation of voltage in close loop equal to zero.
- We can find any **BRANCH CURRENTS** passing through any element in the circuit after finding the **UNKNOWN MESH CURRENTS**.
Mesh Current and Branch Current

Branches forming the outer most boundary of the circuit will have its **BRANCH CURRENT** the same as its **MESH CURRENT**
Remember!
Please understand on how to write the equation of voltage, V for each close loop
### General Step For Using Mesh Analysis

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
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</thead>
<tbody>
<tr>
<td>Assign loop for each meshes in clockwise</td>
<td>Decide the number of equation</td>
<td>Apply KVL for each mesh in the circuit.</td>
<td>Calculate the mesh current using Cramer/Calculator</td>
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<tr>
<td></td>
<td>The circuit ONLY consist of R and dependent source</td>
<td>The main loop voltage drops, ADDED and the voltage at the adjacent loop, SUBTRACTED.</td>
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<td>No. of loop=no. of KVL</td>
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<td>If the current source exist at the outer branch = No. of KVL -1 , no need to write the KVL equation, since the mesh current already given.</td>
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Methods of Analysis by N.R.H. Abdullah
Reminder!

Finding the voltage

Current entering the resistor, the polarity setting is always positive

KVL at $I_1$

$+20I_1$

main Loop (add)

-20$I_2$

adjacent Loop (subtract)

KVL at $I_2$

$20I_2$

main Loop (add)

-20$I_1$

adjacent Loop (subtract)
Circuit with dependent source

1 dependent source = additional 1 constraint equation

\[ 2i_x \rightarrow 2v_x \]

This is dependent voltage source

\[ i_x = i_1 - i_2 \]

\[ v_x = (i_1 - i_2) R_2 \]

Additional eq \rightarrow Constraint equation
Circuit with current source

Current Source

On the branch

the boundary between two meshes.

the outer most boundary of the circuit.

The mesh current = the branch current

Supermesh

KCL = combine 2 meshes eq.

KCL at Supermesh

Eq. Supremesh eq.
KVL at Supernode: \(-V_s + I_1R_1 + I_2R_2 + I_2R_3 = 0\)

Supernode: \(I_s = I_2 - I_1\)
\[ I_y = I_1 - I_2 \]
\[ I_x = I_2 - I_1 \]
The current flow through the branch is $I_x$. Therefore, $R$ is ignored or act as short circuit.

\[ I_x = I_2 - I_1 \]
Assume that we are trying to find the voltage across and the current through all the elements.
Step 1: Assign loop (clock wise)
Step 2: Decide no. of equation

No. of loop = 4
Supermesh = 4 - 1 (supermesh) + 1 (supermesh eq.)
Loop 4 = already know the current value. (no need to do KVL)
Dependent source = + 1 constraint eq
Step 3: Decide no. of equation

- 3 KVL Equations
  - Loop 1
  - Loop Supermesh (combine loop 2 and loop 3)
  - Loop 4

- 1 Supermesh Equation

- 1 Constraint Equation

Total = 5 Equations
Step 3: Decide no. of equation (cont)

- 3 KVL equation
  - Loop 1
    - $-5 + 10I_1 + 8(I_1 - I_2) = 0$
  - Loop Supermesh (combine loop 2 and loop 3)
    - $8(I_2 - I_1) + 4I_2 + 4(I_3 - I_4) = 0$
  - Loop 4
    - $I_4 = 2I_x$

- 1 Supermesh Equation
  - $I_3 - I_2 = 3$

- 1 Constraint Equation
  - $I_x = I_1 - I_2$
Step 4: Calculate the mesh currents

Solving the simultaneous equation by applying Cramer's Rule or using calculator
KVL at Loop 1
\[-5 + 10I_1 + 8(I_1 - I_2) = 0\]
\[I_1(10 + 8) - I_2(8) = 5\]
\[I_1(18) - I_2(8) = 5\]

KVL at Supermesh
\[8(I_2 - I_1) + 4I_2 + 4(I_3 - I_4) = 0\]
\[I_1(-8) + I_2(8 + 4) + 4I_3 + I_4(-4) = 0\]
\[I_1(-8) + I_2(12) + 4I_3 + I_4(-4) = 0\]

At Loop 4
\[I_4 = 2I_x\]

Supermesh Eq.
\[I_3 - I_2 = 3\]

Constraint Eq.
\[I_x = I_1 - I_2\]
Substitute (5) – (3)

\[ I_4 = 2(I_1 - I_2) \]
\[ 2I_1 - 2I_2 - I_4 = 0 \]  \(\textcolor{red}{6}\)

From 4
\[ I_3 = 3 + I_2 \]  \(\textcolor{red}{7}\)

Substitute (7) – (2)
\[ -8I_1 + 12I_2 + 4(3 + I_2) - 4I_4 = 0 \]
\[ -8I_1 + 12I_2 + 12 + 4I_2 - 4I_4 = 0 \]
\[ -8I_1 + 16I_2 - 4I_4 = -12 \]  \(\textcolor{red}{8}\)

Rearrange Eq. (1), (6), (8)
\[
\begin{bmatrix}
18 & -8 & 0 \\
2 & -2 & -1 \\
-8 & 16 & -4 \\
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_4 \\
\end{bmatrix}
=
\begin{bmatrix}
5 \\
0 \\
-12 \\
\end{bmatrix}
\]

\[ I_1 = \frac{3}{38} = 0.0789 \text{ A} \]
\[ I_2 = -\frac{17}{38} = -0.4474 \text{ A} \]
\[ I_4 = \frac{20}{19} = 1.053 \text{ A} \]
\[ I_3 = 3 + I_2 = 3 - 0.4474 = 2.553 \text{ A} \]
**Additional Question (Discuss with friend)**

1. Find the voltage at current source, $V_y$ and $V_z$.
2. Determine the power delivered by **all** source.
Nodal VS Mesh

Compare the number of Nodal equations to the number of Mesh equations required.

✓ Choose the less equation would be the better choice.
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