REINFORCED CONCRETE DESIGN 1

Design of Staircase (Examples and Tutorials)

by

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Example 1: Straight staircase design

A reinforced concrete staircase for office use is shown in Figure 1. It is connected to a landing at upper part and supported by a beam at the end of the landing. At the end lower the stair supported by a beam and continuous with the floor slab. Design the staircase by using concrete grade 25 and strength of reinforcement of 500 N/mm². The imposed load is 2.5 kN/m² and finishes is 0.5 N/m². Nominal cover, $c_{nom}$ is 25 mm. The width of staircase is 1500 mm, the thickness of landing is 150 mm and the waist thickness (h) is 150 mm. Design the reinforcement for the stairs. Use diameter bar $= 10$ mm.
Example 1: Straight staircase design

Figure 1
Example 1: Straight staircase design

Load Analysis
Average thickness of flight, \( y \) = \( h(G^2 + R^2)^{1/2}/G \)
\[ = 150 \ (250^2 + 170^2)^{1/2}/250 \]
\[ = 181.4 \text{ mm} \]
Average thickness, \( t = y + (R/2) = 181.4 + (170/2) \)
\[ = 266.4 \text{ mm} \]

Actions
Landing permanent action,
Self-weight staircase = 0.15 x 25
\[ = 3.75 \text{ kN/m}^2 \]
Finishes = 0.5 \text{ kN/m}^2
\[ \text{Total } g_k \]
Variable action, \( q_k \) = 2.5 \text{ kN/m}^2
Example 1: Straight staircase design

Design Action, \( w \) = \( 1.35g_k + 1.5q_k \)
= 9.49 kN/m\(^2\)

Flight permanent action,
Self-weight staircase = \( 0.266 \times 25 \)
= 6.65 kN/m\(^2\)

Finishes = 0.5 kN/m\(^2\)

**Total \( g_k \)** = 7.15 kN/m\(^2\)

Variable action, \( q_k \) = 2.5 kN/m\(^2\)

Design Action, \( w \) = \( 1.35g_k + 1.5q_k \)
= 13.4 kN/m\(^2\)
Example 1: Straight staircase design

Total load, \( F = 13.4(2.5) + 9.49(1.4) = 46.79 \text{ kN} \)

\[ M_{ED} = \frac{wL}{10} \]
\[ = \frac{46.79(3.9)}{10} \]
\[ = 18.25 \text{ kNm} \]
Example 1: Straight staircase design

Effective depth,
\[ d = h - c_{\text{nom}} - 0.5\phi_{\text{bar}} = 150 - 25 - 10/2 = 120 \text{ mm} \]

\[ M_{\text{ED}} = 18.25 \text{ kNm} \]

\[ K = M/bd^2f_{\text{ck}} \]
\[ = 18.25 \times 10^6/(1000 \times 120^2 \times 25) \]
\[ = 0.052 < k_{\text{bal}} = 0.167 \quad \text{no. compression reinforcement required} \]

\[ z = d[0.5 + (0.25 - K/1.134)^{1/2}] \]
\[ = 0.98d > 0.95d \quad \text{use 0.95d} \]

\[ A_s = M/0.87f_{\text{yk}}z \]
\[ = 18.25 \times 10^6/(0.87 \times 500 \times 0.95 \times 120) \]
\[ = 368 \text{ mm}^2 \]

Provide H10-200
\[ (A_s = 393 \text{ mm}^2) \]
Example 1: Straight staircase design

Minimum and maximum reinforcement area,

\[ A_{s,\text{min}} = 0.26 \left( \frac{f_{\text{ctm}}}{f_{y,k}} \right) bd \]
\[ = 0.0151 \, bd > 0.013bd \]
\[ = 179.45 \, \text{mm}^2 \]

\[ A_{s,\text{max}} = 0.04 \, Ac \]
\[ = 0.04(1000)(150) \]
\[ = 6000 \, \text{mm}^2 \]

Secondary Reinforcement = 20% \(A_s\)
\[ A_s = 20\% \,(As) = 0.2 \,(503) = 100.6 \, \text{mm}^2 \]

Provide H10-400 \((A_s = 196.5 \, \text{mm}^2)\)
Example 1: Straight staircase design

**SHEAR**

![Diagram showing shear forces](image)

\[ R_A = 29.56 \text{ kN} \quad R_B = 17.22 \text{ kN} \]

Shear force, \( V_{ED} = 29.54 \text{ kN} \)

\[ V_{Rdc} = [0.12 k (100\rho f_{ck})^{1/3}] \cdot bd \]

\[ k = 1 + (200/d)^{1/2} \leq 2.0 \]

\[ = 2.29 > 2.0 \]

\[ \rho 1 = (A_{sl} / b_w d) = 503/(1000 \times 120) = 0.0033 \leq 0.02 \]

\[ = 0.0033 > 0.002 \]

\[ V_{Rdc} = 34.03 \text{ kN} \]

\[ V_{min} = [0.035k^{3/2} f_{ck}^{1/2}] \cdot bd = 59.4 \text{ kN} \]

Thus, \( V_{Rdc} = 54.9 \text{ kN} > V_{ED} = 29.56 \text{ kN} \), OK!
Example 1: Straight staircase design

Deflection

\[ \rho = \frac{A_{s, req}}{bd} = \frac{368}{(1000 \times 120)} = 0.0031 \]

\[ \rho_o = \sqrt{f_{ck}^{(10-3)}} = \sqrt{25^{(10-3)}} = 0.005 \times 10^{-3} \]

\( \rho_o > \rho \), structural system, \( K = 1.5 \)

\[ L/d = k \left( 11 + 1.5 \sqrt{f_{ck}} \left( \frac{\rho_o}{\rho} \right) + 3.2 \sqrt{f_{ck}} \left( \left( \frac{\rho_o}{\rho} \right) - 1 \right)^{3/2} \right) \]

\[ L/d = 1.5 \left( 11 + 12.1 + 7.68 \right) \]

\[ (L/d)_{\text{basic}} = 46.17 \]

Modification factor, \( A_{s, prov} / A_{s, req} = 393/368 = 1.07 < 1.5 \)

\[ (L/d)_{\text{allow}} = 46.17 \times 1.07 = 49.4 \]

\[ (L/d)_{\text{actual}} = \frac{3900}{120} = 32.5 < (L/d)_{\text{allow}}, \text{OK!} \]
**Check Crack**

Slab thickness, \( h = 150 \text{ mm} < 200 \text{ mm} \)

Main bar:

\[
S_{v,\text{max, slab}} = 3h \leq 400 \text{ mm}
\]

\[
= 3(150) \leq 400 \text{ mm} , \quad 450 \text{ mm} \geq 400 \text{ mm}, \quad \text{Use 400}
\]

Actual bar spacing = 200 mm < 400 mm, OK

Secondary bar:

\[
S_{\text{max, slab}} = 3.5h \leq 450 \text{ mm}
\]

\[
= 450 \text{ mm} , \quad \text{use 450 mm}
\]

Actual bar spacing = 400 < 450 mm, OK
Example 2: Open-well staircase design

A staircase of 1.5 m width for an office building with slab supported on a beam at the top and and on the landing of the flight at right angles at the bottom is shown in Figure 2. The riser and goings of the stairs are 160 mm and 250 mm, respectively. The variable load is 3.0 kN/m² and the permenant action from finishes, baluster and railing about 1.0 kN/m². Materials used in this construction consist of concrete with characteristic strength, $f_{ck} = 30$ N/mm² and steel strength, $f_{yk} = 500$ N/mm². The thickness of the landing is 150 mm and waist thickness (h) is 150 mm. Design the stairs if the concrete cover = 25 mm and the main bar diameter, $\phi_{main} = 10$ mm.
Example 2: Open-well staircase design

Figure 2
Example 2: Open-well staircase design

Figure 2 (continue)
Example 2: Open-well staircase design

Load Analysis
Average thickness, \( y = \frac{h(G^2 + R^2)^{1/2}}{G} \)
\[
= 150 \ (250^2 + 160^2)^{1/2}/250 \\
= 178 \text{ mm}
\]

Average thickness, \( t = \frac{y + (y + R)}{2} = 258 \text{ mm} \)

Actions
Landing
Slab self-weight \( = 0.15 \times 25 = 3.75 \text{ kN/m}^2 \)
Permanent load excluding self-weight \( = 1.00 \text{ kN/m}^2 \)
Characteristic permanent action \( = 4.75 \text{ kN/m}^2 \)
Characteristic variable action \( = 3.00 \text{ kN/m}^2 \)
Example 2: Open-well staircase design

Design action $n_d$  = 1.35 (4.75) + 1.5 (3.0) = 10.91 kN/m²

**Flight**
Slab self-weight = 0.258 x 25 = 6.45 kN/m²
Permanent load excluding self-weight = 1.00 kN/m²
Characteristic permanent action = 7.45 kN/m²
Characteristic variable action = 3.00 kN/m²
Design action $n_d$  = 1.35 (7.45) + 1.5 (3.0) = 14.60 kN/m²
Example 2: Open-well staircase design

Analysis

Total action = (5.46 x 1.625) + (14.6 x 2.625) = 47.2 kN/m

\[ M_{ED} = FL/10 \]

\[ = 47.2 \times (4.25)/10 \]

\[ = 20.06 \text{ kNm} \]
Example 2: Open-well staircase design

Shear force,

\[ V_A = \frac{[(14.6 \times 2.625 \times 2.94) + (5.46 \times 1.625 \times 0.81) - 20.06]}{4.25} = 23.48 \text{ kN/m} \]

\[ V_B = \frac{[(14.6 \times 2.625) + (5.46 \times 1.625) - 23.48]}{4.25} = 23.72 \text{ kN/m} \]
Example 2: Open-well staircase design

**MAIN REINFORCEMENT**

Effective depth,

\[ d = h - c_{\text{nom}} - 0.5\phi_{\text{bar}} = 150 - 25 - 10/2 = 120 \text{ mm} \]

\[ K = \frac{M}{bd^2f_{ck}} \]

\[ = 20.1 \times 10^6/(1000*120^2*25) \]

\[ = 0.056 < k_{\text{bal}} = 0.167, \text{ no. compression reinforcement required} \]

\[ z = d[0.5 + (0.25 - K/1.134)^{1/2}] \]

\[ = 0.95d \leq 0.95d \text{ use } 0.95d \]

\[ A_s = \frac{M}{0.87f_{yk}z} \]

\[ = 20.1 \times 10^6/(0.87 \times 500 \times 0.95 \times 120) \]

\[ = 405.3 \text{ mm}^2 \]
Example 2: Open-well staircase design

Minimum and maximum reinforcement area,

\[ A_{s, \text{min}} = 0.26 \left( \frac{f_{ctm}}{f_{yk}} \right)bd \]
\[ = 0.0145 \text{ bd} > 0.013 \text{bd} \]
\[ = 181 \text{ mm}^2 \]

\[ A_{s, \text{max}} = 0.04 \text{ Ac} \]
\[ = 0.04(1000)(150) \]
\[ = 6000 \text{ mm}^2 \]

Secondary bar = 0.2 x 405.3= 81.1 mm\(^2\)/m

Main Reinforcement ➔ Provide: H10-175 (449 mm\(^2\))

Secondary Reinforcement ➔ Use: H10 – 400 (196 mm\(^2\))
Example 2: Open-well staircase design

SHEAR

Shear force, \( V_{ED} = 23.72 \) kN

\[ V_{Rdc} = [0.12 \times k \times (100 \times \rho_1 \times f_{ck})^{1/3}] \times b \times d \]

\[ k = 1 + (200/120)^{1/2} \leq 2.0 \]

\[ = 2.29 \leq 2.0 \]

\[ \rho_1 = (A_{sl} / b \times w \times d) = 449/(1000 \times 120) \leq 0.02 \]

\[ = 0.0037 \leq 0.002 \]

\[ V_{Rdc} = 64.2 \) kN \]

\[ V_{min} = [0.035k^{3/2} \times f_{ck}^{1/2}] \times b \times d = 65.1 \) kN \]

So, \( V_{Rdc} = 65.1 \) kN > \( V_{ED} = 23.72 \) kN, OK! No shear reinforcement required.
Example 2: Open-well staircase design

**Deflection**

\[ \rho = \frac{A_{s,req}}{bd} = \frac{405.3}{(1000 \times 120)} = 0.0034 \]

\[ \rho_o = \sqrt{f_{ck}^{(10-3)}} = \sqrt{25^{(10-3)}} = 0.005 \]

\( \rho < \rho_o \), structural system, \( K = 1.3 \)

\( L/d = k (11 + 1.5\sqrt{f_{ck} (\rho_o / \rho)} + 3.2\sqrt{f_{ck} ((\rho_o / \rho) - 1)^{3/2}}) \)

\( L/d = 1.3 (11 + 11.03 + 5.17) \)

\( (L/d)_{basic} = 35.35 \)

Modification factor, \( \frac{A_{s,prov}}{A_{s,req}} = \frac{449}{405.3} = 1.11 < 1.5 \)

\( (L/d)_{allow} = 35.35 \times 1.11 = 39.24 \)

\( (L/d)_{actual} = \frac{4250}{120} = 35.42 < (L/d)_{allow} \), OK!
Example 2: Open-well staircase design

**Check Crack**

Slab thickness, \( h = 150 \text{ mm} < 200 \text{ mm} \)

Main bar:
\[ S_{\text{max, slab}} = 3h \leq 400 \text{ mm} \]
\[ = 3(150) = 450 \text{ mm} \], use 400 mm

Max. bar spacing = 175 mm < 400 mm  OK.

Secondary bar:
\[ S_{\text{max, slab}} = 3.5h \leq 450 \text{ mm} \]
\[ = 3.5(150) = 525 \text{ mm} \], use 450 mm

Max. bar spacing = 400 < 450 mm, OK
Tutorial 1: Staircase design

Design a straight reinforced concrete stairs supported by reinforced concrete beams at both ends. Landing slabs at both ends of the stairs are cast together connecting the stairs. Using the following information, design the staircase.

Concrete grade: C30/C37
Steel grade: 500
Perm. load from finishes, baluster and railing: 1.5 kN/m²
Variable load: 3.5 kN/m²
C_{nom} = 25 mm
Bar size = 10 mm
R = 180 mm
G = 250 mm
h = 120 mm
Figure 3 shows the plan view of open well stair support by beam at the end of its landing. The risers are 160 mm, goings are 250 mm as shown in Figure 4, and story height is 3.5 m. Goings are provided with 3 cm thick marble finish on cement mortar that weighs 1.0 kN/m². The landings are surface finished with terrazzo tiles on sand filling that weighs 1.2 kN/m². The stair is to be designed for a variable action of 3.0 kN/m². Design the staircase by providing the reinforcement. Check the shear, deflection, and crack and illustrate the curtailments of the staircase. Use of concrete strength, $f_{ck} = 30$ N/mm², steel strength, $f_{yk} = 500$ N/mm² and diameter of bar, $\phi_{\text{main}} = 12$ mm. Nominal cover, $c_{\text{nom}}$ for this stairs, is 25 mm.
Tutorial 2: Staircase design

Figure 3
Tutorial 2: Staircase design

Figure 4
End of Examples and Tutorials