REINFORCED CONCRETE
DESIGN 1

Design of Beam
(Examples and Tutorials)
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Example 1: Simply supported beam design - Rectangular

A rectangular reinforced concrete beam simply supported on two mansory walls 200 mm thick and 6 m apart. The beam has to carry a distributed permanent action of 10 kN/m (excluding beam self-weight) and variable action of 8 kN/m. The beam is inside building subject to a 1 hour fire resistance and design for 50 years design life. Design the beam with deflection and crack check. Illustrate the beam detailing. Use concrete characteristic strength, $f_{ck} = 30$ N/mm$^2$ and steel characteristic strength, $f_{yk} = 500$ N/mm$^2$. 
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**Specification**

- Effective span, $L = 6$ m
- Characteristic action:
  - Permanent, $g_k = 15$ kN/m
  - Variable, $q_k = 10$ kN/m
- Design Life = 50 years
- Fire resistance = R60
- Exposure classes = XC1

**Materials**

- Concrete Char. strength, $f_{ck} = 30$ N/mm$^2$
- Steel char. strength, $f_{yk} = 500$ N/mm$^2$
- Links char. strength, $f_{yk} = 500$ N/mm$^2$
- Reinforced concrete unit weight = 25 kN/m$^3$
- Use, $\phi_{bar 1} = 20$ mm
  - $\phi_{bar 2} = 12$ mm
  - $\phi_{links} = 8$ mm
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Beam Size
Overall depth, $h = \frac{L}{14} = \frac{6000}{13} = 462$ mm
Width, $b = 0.4 \times (h) = 0.4 \times 462 = 185$ mm

Try: $b \times h = 200 \times 500$ mm

Durability, Fire & Bond Requirements
Min. cover with regards to bond, $C_{\text{min},b} = 20$ mm
Min. cover with regards to durability, $C_{\text{min,dur}} = 15$ mm
Min. required axis distance for R60 fire resistance, $a_{sd}$

$$a_{sd} = 30 + 10 = 40$$ mm

Min. cover regards to fire, $C_{\text{min}} = a_{sd} - \phi_{\text{links}} - \phi_{\text{bar}} / 2$

$$= 40 - 8 - 20 / 2 = 22$$ mm

Allowance in design for deviation, $\Delta C_{\text{dev}} = 10$ mm

Nominal cover, $C_{\text{nom}}$

$$C_{\text{nom}} = C_{\text{min}} + \Delta C_{\text{dev}} = 22 + 10 = 32$$ mm

Use: $C_{\text{nom}} = 35$ mm
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**LOADING & ANALYSIS**

Beam self-weight $= 0.2 \times 0.5 \times 25 = 2.5 \text{ kN/m}$

Permanent load (Excluding self-weight) $= 10 \text{ kN/m}$

Total Charc. Permenant action $= 12.5 \text{ kN/m}$

Charc. Variable action $= 8 \text{ kN/m}$

Design Action, $w_d = 1.35 \text{ gk} + 1.5qk = 28.88 \text{ kN/m}$

Shear force, $V$

$V = w_dL/2 = 28.88 \times 6/2 = 86.64 \text{ kN}$

Bending Moment, $M$

$M = W_dL^2/8 = 28.88 \times 6^2/8 = 130 \text{ kNm}$
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**MAIN REINFORCEMENT**

Effective depth,

\[
d = h - C_{\text{nom}} - \phi_{\text{link}} - \phi_{\text{bar}}/2 = 500 - 35 - 8 - 20/2 = 447 \text{ mm}
\]

\[
d' = C_{\text{nom}} + \phi_{\text{link}} + \phi_{\text{bar}}/2 = 35 + 8 + 20/2 = 53 \text{ mm}
\]

Design bending moment, \(M_{\text{Ed}} = 193 \text{ kNm}\)

\[
K = \frac{M}{bd^2f_{ck}}, \quad K_{\text{bal}} = 0.167
\]

\[
= \frac{130 \times 10^6}{(200 \times 447^2 \times 30)}
\]

\[
= 0.108 < K_{\text{bal}} : \text{No compression reinforcement required}
\]

\[
z = d[0.5 + (0.25 - K/1.134)^{1/2}]
\]

\[
= 0.89 \times 447 = 398 \text{ mm}
\]

\[
x = (d - z)/0.4 = 191 \text{ mm}
\]
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Area of tension steel

\[ A_s = \frac{M}{0.87f_{yk}z} \]
\[ = \frac{130 \times 10^6}{(0.87 \times 500 \times 398)} \]
\[ = 751 \text{ mm}^2 \]

Use:

3H20 (As = 943 mm\(^2\))

Minimum and maximum reinforcement area,

\[ A_{s_{\text{min}}} = 0.26 \left( \frac{f_{\text{ctm}}}{f_{yk}} \right) bd \]
\[ = 0.26 \left( \frac{2.9}{500} \right) bd > 0.0013bd \]
\[ = 0.0015bd > 0.0013bd \text{ use } 0.0015bd \]
\[ = 135 \text{ mm}^2 \]

\[ A_{s_{\text{max}}} = 0.04Ac = 0.04 \times b \times h = 0.04 \times 200 \times 500 \]
\[ = 4000 \text{ mm}^2 \]
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SHEAR REINFORCEMENT

Design shear force, $V_{Ed} = 86.64$ kN

Concrete strut capacity

$V_{Rd, max} = \frac{0.36b_w f_{ck}}{w} \left(1 - \frac{f_{ck}}{250}\right) \left(\cot \theta + \tan \theta\right)$

$= 655$ kN \quad \theta = 22 \text{ deg} \quad \cot \theta = 2.5$

$= 950$ kN \quad \theta = 45 \text{ deg} \quad \cot \theta = 1.0$

$V_{Ed} < V_{Rd, max} \cot \theta = 2.5$

$V_{Ed} < V_{Rd, max} \cot \theta = 1.0$

--- Therefore angle $\theta = 22^0$
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Shear links

\[ A_{sw} / s = \frac{V_{Ed}}{0.78 f_{yk}d \cot \theta} = \frac{0.513 V_{Ed}}{f_{yk}d} \]

\[ = 0.513 \times 86.64 \times 10^3 / (500 \times 447) \]

\[ = 0.199 \]

Use links: H8. \( A_{sw} = 101 \text{ mm}^2 \)

Spacing, \( s = \frac{A_{sw}}{0.199} = 101/0.199 \)

\[ = 508 \text{ mm} \]

Maximum spacing, \( S_{vmax} = 0.75d = 335 \text{ mm} < 508 \text{ mm} \): use 300 mm

Use: H8 - 300
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Additional longitudinal reinforcement

Additional tensile force,

\[ F_{td} = 0.5 \times V_{Ed} \cot \theta = 0.5 \times 86.64 \times 10^3 \times 2.5 \]

\[ = 108.3 \text{ kN} \]

Additional tension reinforcement,

\[ A_s = \frac{F_{td}}{0.87 \times f_{yk}} = \frac{108.3 \times 10^3}{(0.87 \times 500)} \]

\[ = 249 \text{ mm}^2 \]

Use :1H20 (As = 314 mm²)

To be added to the As near support (after curtailment)
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**DEFLECTION**

Percentage of required tension reinforcement

\[ \rho = \frac{A_{s}}{bd} = \frac{751}{(200 \times 447)} = 0.0084 \]

Reference reinforcement ratio.

\[ \rho_{o} = (f_{ck})^{1/2} \times 10^{-3} = (30)^{1/2} \times 10^{-3} = 0.0055 \]

Factor for structural system, \( K = 1.0 \)

\[ \rho_{o} = 0.0055 < \rho = 0.0084 \]

Use

\[ \frac{l}{d} = K \left[ 11 + 1.5 \sqrt{f_{ck}} \frac{\rho_{o}}{\rho - \rho'} + \frac{1}{12} \sqrt{f_{ck}} \sqrt{\frac{\rho'}{\rho}} \right] \]

\[ \frac{l}{d} = 1.0 \left[ 11 + (1.5 \times (30)^{1/2} \times 0.655) + 3.2 \times (30)^{1/2} \times (0)^{1/2} \right] \]

\[ = 1.0 \left[ 11 + 5.38 + 0 \right] = 16.38 \]
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Therefore basic span-effective depth ratio, \( l/d = 16.38 \)

Modification factor for steel area provided,
- Tension = \( A_{spov} / A_{req} = 943 / 751 = 1.26 < 1.5 \)

Therefore allowable span effective depth, \((l/d)_{allowable}\)

\[
(l/d)_{allowable} = 16.38 \times 1.26 = 20.64
\]

Actual span effective depth, \((l/d)_{actual}\)

\[
(l/d)_{actual} = 6000/447 = 13.42
\]

\((l/d)_{allowable} > (l/d)_{actual} \Rightarrow OK\)
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CRACKING

Limiting crack width, \( w_{\text{max}} = 0.3 \text{ mm} \)

Steel stress, \( f_s \)

\[
f_s = \frac{f_y k}{1.15} \times \frac{G k + 0.3 Q k}{(1.35 G k + 1.5 Q k)} \times \frac{1}{\delta} = \frac{500}{1.15} \times \frac{12.5 + 0.3 \times 8}{(1.35(12.5) + 1.5(8))}
\]

\[= 224 \text{ N/mm}^2\]

Max. allowable bar spacing  = 200 mm

Actual bar spacing, \( s \)  = \[
\frac{h - 2(C_{\text{nom}}) - 2(\phi_{\text{links}}) - 20}{2}
\]

\[= [200 - 2(35) - 2(8) - 20]/2 = 47 \text{ mm}\]

Actual = 47 mm < Allow, 200 mm \( \Rightarrow \) OK
Tutorial 1: Simply supported beam

Figure below shows a 5.5 m simply supported beam with size of 150 x 350 mm. The characteristic permanents and variable action acting on the beam are 3.0 kN/m (excluding self-weight and brick wall) and 3.5 kN/m. This beam has to support the brick wall with 3.3 m height.

\[ \begin{align*}
G_k &= 3.0 \text{ kN/m} \\
Q_k &= 3.5 \text{ kN/m}
\end{align*} \]
The properties of the beam are as follows:

- Unit weight of concrete, \( \sigma_c \) = 25 kN/m\(^3\)
- Unit weight of brick wall, \( \sigma_{bw} \) = 2.6 kN/m\(^2\)
- Concrete characteristic strength, \( f_{ck} \) = 25 N/mm\(^2\)
- Steel characteristic strength, \( f_{yk} \) = 500 N/mm\(^2\)
- Nominal cover, \( c_{nom} \) = 25 mm
- Use main reinforcement, \( \phi_{main} \) = 20 mm
- Use shear reinforcement, \( \phi_{link} \) = 6 mm
Tutorial 1: Simply supported beam

By using the data above,

a) Calculate design load, bending moment and shear forced for the beam

b) Provide the main reinforcement for the beam

c) Provide the shear reinforcement for the beam

d) Check the beam due to the deflection

e) Sketch the detailing for this beam
Design the continuous beam (3/A-D) in figure below with beam detailing.
The beam and slab will be casted monolithically. Use the following data to design the beam.

\[ f_{ck} = 30 \text{N/mm}^2 \]
\[ f_{yk} = 500 \text{N/mm}^2 \]
Cover, \( c = 25 \text{ mm} \)
\[ \phi_{\text{main reinforcement}} = 8 \text{ mm} \]
\[ q_{k,\text{slab}} = 1.5 \text{kN/m}^2 \]
Slab finishing = 1.0 kN/m²
Slab thickness, \( h_{\text{slab}} = 150 \text{ mm} \)
beam size, \( b \times h = 150 \times 400 \text{ mm} \)
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