CHAPTER 6

Control Charts for Attributes

Expected Outcomes

Know limitations of variable control charts and the different types of attribute charts.
Know the objectives of the p chart group and the applicable distribution.
Be able to construct a:
Fraction defective chart - fixed subgroup size
Fraction defective chart - variable subgroup size
Percent defective chart
Number defective chart
The term Attribute refers to those quality characteristics that conform to specifications or do not conform to specifications.

**Attribute are used:**

- Where measurements are not possible.
- Where measurements can be made but are not made because of time, cost, or need.
Types of Attribute Charts:

1. Nonconforming Units (based on the Binomial distribution): $p$ chart, $np$ chart.
The P Chart

- The $P$ Chart is used for data that consist of the proportion of the number of occurrences of an event to the total number of occurrences.
- It is used in quality to report the fraction or percent nonconforming in a product, quality characteristic, or group of quality characteristics.
The P Chart

Formula:

\[ p = \frac{np}{n} \]

- The fraction nonconforming, \( p \), is usually small, say, 0.10 or nonconforming is very small, the subgroup.
- Because the fraction output sizes must be quite large to produce a meaningful chart.
It can be used to control one quality characteristic, as is done with X bar and R chart,

Or to control a group of quality characteristics of the same type or of the same part,

Or to control the entire product.

It can be established to measure the quality produced by a work center, by a department, by a shift, or by an entire plant.
Objectives of the $P$ Chart:

1. Determine the average quality level: This information provides the process capability in terms of attributes.
2. Bring to the attention of management any changes in the average.
3. Improve the product quality: Ideas for quality improvement.
**P-Chart Construction for Constant Subgroup Size**

1. Select the quality characteristic(s):
   a) Single quality characteristic.
   b) Group of quality characteristics.
   c) A part.
   d) An entire product.
   e) A number of products.
   f) It can be established for performance control of an operator, work center, department, shift, plant, or corporation.
P Chart Construction for Constant Subgroup Size cont’d.

2. Determine the subgroup size and method:
   - The size of the subgroup is a function of the proportion nonconforming.
   - A minimum size of 50 is suggested as a starting point.
The P Chart

**P Chart Construction for** Constant Subgroup Size cont’d.

3. Collect the data:
   - At least 25 subgroups.
   - Different sources (Check sheet).
   - For each subgroup the proportion nonconforming is calculated by the formula

\[ P = np/n \]
The P Chart

P Chart Construction for Constant Subgroup Size

4. Calculate the trial central line and the control limits:

\[ \bar{p} = \frac{\sum np}{\sum n} \]

\[ UCL = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \]

\[ LCL = \bar{p} - 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \]
FIGURE 9 - A p Chart to Illustrate the Trial Central Line and Control Limits Using the Data from Table 9.
5. Establish the revised central line and control limits.

\[
P_{new} = p_0 = \frac{\sum np - np_d}{\sum n - n_d}
\]

\[
UCL = p_0 + 3\sqrt{\frac{p_0(1 - p_0)}{n}}
\]

\[
LCL = p_0 - 3\sqrt{\frac{p_0(1 - p_0)}{n}}
\]
FIGURE 9 - Continuing Use of the p Chart for Representative Values of the Proportion Nonconforming, p
The P Chart

*P* Chart Construction for **Variable** Subgroup Size

1. Collect the data.
2. Determine the trial central line and control limits: Since the subgroup size changes each day, limits must be calculated for each day.
FIGURE 9-5 Preliminary Data, Central Line, and Trial Control Limits

- Fraction Nonconforming
- UCL
- LCL
- \( p = 0.020 \)

March 29, 30, 31, 1, 2, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30

April
P Chart Construction for Variable Subgroup Size cont’d.

2. As the subgroup size gets larger, the control limits are closer together.

3. Establish revised central line and control limits:
**The P Chart**

**P Chart Construction for Variable Subgroup Size cont’d.**

- If $P_0$ is known, the process of data collection and trial control limits is not necessary.

- $P$ is the proportion (fraction) nonconforming in a single subgroup.

- $Pbar$ is the average proportion (fraction) nonconforming of many subgroups.
The P Chart

$P$ Chart Construction for **Variable** Subgroup Size cont’d.

- $Po$ is the standard or reference value of the proportion (fraction) nonconforming based on the best estimate of $PBar$.

- $\Phi$ is the population proportion (fraction) nonconforming.
Minimizing the Effect of Variable Subgroup Size

1. Control limits for an average subgroup size: By using an average subgroup size, one limit can be calculated and placed on the control chart.

\[ n_{av} = \frac{\sum n}{g} \]

\[ UCL = p_0 + 3 \sqrt{\frac{p_0(1 - p_0)}{n_{av}}} \]

\[ LCL = p_0 - 3 \sqrt{\frac{p_0(1 - p_0)}{n_{av}}} \]
Chart for May Data Illustrating Use of an Average Subgroup Size
The \( np \) Chart

Number Nonconforming Chart \((np)\):

- The \( np \) chart is easier for operating personnel to understand than the \( p \) chart.
- The limitation that this chart has is that the subgroup size needs to be constant.
The np Chart

Central Line = \( np_0 \)

Control Limits = \( np_0 \pm 3\sqrt{np_0(1 - p_0)} \)
Number Nonconforming Chart \((np)\):

- If the fraction nonconforming \(p_0\) is unknown, then it must be determined by collecting data, calculating trial control limits, and obtaining the best estimate of \(p_0\).
Number Nonconforming Chart (np Chart)
For an attribute this process is much simpler.

- The process capability is the central line of the control chart.
- Management is responsible for the capability.
- When the plotted point is outside the control limit, operating personnel are usually responsible.
FIGURE 9-10 Process Capability Explanation and Responsibility

- Operator Responsibility
- UCL
- Variation
- Management Responsibility
- Variation
- Subgroup
- LCL
- Percent Nonconforming (100p)
- n = 500
- n = 200
- n = 50
- 100p₀ = 5.0%